Critical scaling in standard biased random walks.

C. Anteneodo and W.A.M. Morgado
Departamento de Física, PUC-Rio, Brasil.

Random walk theory has allowed to deal with a diversity of problems in a number of areas of physics, as well as in many other theoretical and applied fields. In any phenomenon where random walks are relevant, a fundamental quantity is the number of distinct sites visited, since it furnishes the extent of the active territory. Indeed, it is crucial in processes ranging from reaction kinetics to population dynamics, and also in technical applications such as in search strategies. The vast literature on coverage mainly deals with two dimensions, although there are also many works about the standard symmetric onedimensional random walk. Meanwhile, as far as we know, little or no attention has been paid to the asymmetric onedimensional case, despite of its importance in biased or anisotropic processes such as electrophoresis, polymer translocation through pores, and Brownian ratchets. However, as we will show, the asymmetric onedimensional problem presents its own peculiar features and nontrivial scaling properties.

In this work, we scrutinize the spatial coverage produced by a single discrete-time random walk, with an asymmetric jump probability \( p \neq 1/2 \) and nonuniform steps (different step lengths, \( l^+ \) and \( l^- \), occur in opposite directions), moving on an infinite one-dimensional lattice. Mutually prime \( l^+ \), \( l^- \) are considered. Analytical calculations are complemented with Monte Carlo simulations. We will show that, for appropriate step sizes, the model displays a critical phenomenon, at \( p = p_c \). In fact, the two anisotropic ingredients play competing roles and a nontrivial changeover between different coverage regimes, dependent on \( p \), may take place. The scaling properties as well as the main features of the fragmented coverage occurring in the vicinity of the critical point will be exhibited. In particular, in the limit \( p \to p_c \), the distribution of fragment lengths is scale-free, with nontrivial exponents. Moreover, a box counting procedure allows to reveal that the spatial distribution of cracks (unvisited sites) defines a fractal set over the spanned interval. Thus, from the perspective of the covered territory, a very rich critical phenomenology is revealed in a simple onedimensional standard model.