Study of the $\beta \to \beta'$ transformation for a generalized hydrogen atom in Tsallis statistics.

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In this article the $\beta \to \beta'$ transformation method proposed in Tsallis statistics is applied and studied for a hydrogen atom in different space dimensions (the generalized hydrogen atom [1]) in contact with a heat reservoir. For the difficulties with Boltzman-Gibbs (BG) statistics in describing systems with the long range interactions [2], the Tsallis statistics has been applied to study the specific heat of a hydrogen atom in contact with a heat reservoir using the un-normalized $q$ expectation value formalism [3]. In our recent paper [4], we extended the method presented by Tsallis et al [3] on the behavior of the specific heat of a hydrogen atom in contact with a heat reservoir in different space dimensions ($D$). In the next study on the $\beta \to \beta'$ transformation method proposed by Tsallis et al [6], we showed that [5] the re-normalized temperature $T$ for a hydrogen atom in contact with a heat reservoir, which is a function of intermediate temperatures $T'$, does not behave as a monotonically increasing function for the accepted range of $q$. Therefore the $\beta \to \beta'$ transformation is not applicable for 3D-hydrogen atom. In this article, our previous work [5] is extended and applied to the hydrogen atom in different space dimensions. The results indicate that the general behavior of $T$ as a function of $T'$, is almost independent of the space dimensions. It linearly increases with slope of one, passing through the origin for $0 < T' < (1-q)(\frac{16D}{D^2-1})^\frac{D}{2}$. Then it decreases for $(1-q)(\frac{16D}{D^2-1})^\frac{D}{2} < T' < (1-q)(\frac{4}{D-1})^\frac{D}{2}$ and approaches zero as $T' \to (1-q)(\frac{4}{D-1})^\frac{D}{2}$. The rate of approach depends on the space dimensions and it increases as the dimension increases especially for larger values of $q (\frac{1}{2} < q < \frac{3}{2})$. There also exist several local maxima in the decreasing range. The number of these maxima increases as the dimension increases. The behavior is more pronounced for small values of $q (0 < q \leq \frac{1}{2})$ and for lower intermediate temperatures. It is easy to show that the local maxima correspond to the anomalous behavior of the specific heat [4] calculated in the un-normalized $q$ expectation value formalism.