Quantization methods and quantum simulations of random walks and of Lie walks.

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Rules for quantizing the walker+coin parts of a classical random walk are provided by treating them as interacting quantum systems, forming a quantum statistical model of a particle (walker) immersed in a bath of other particles (coins). Quantum walks following the so called U- and $\varepsilon$- quantization rules are presented. The former rule involves unitary transformation of the quantum coin system, while the latter one involves a completely positive trace preserving map. This last map acts on the quantum coin system, and is motivated and justified by the fact that it allows to consider coin systems as quantum systems interacting with external agents. Asymptotic statistics of walker’s position exhibiting enhanced diffusion rates, as compared to classical ones, are analytically treated. The setting of these walks is further shown to provide the framework for implementing the mechanism of quantum simulation among quantum systems. Lie group theoretical extensions of these walks are provided. The case of U(n) valued coin systems, is detailed by using techniques of the root and weight systems from the theory of Lie group classification and representations. The resulting polypodic quantum walks have dimensionality of walker space determined by the rank of the group, and walker “footprint size” (polipodicity), determined by the dimensionality of the irreducible representation chosen. Furthermore directional and boundary aspects of the accelerated diffusion rate of the walker, are also reported, and their associations with problems of quantum computing are also discussed.