Non-Euclidean normed statistical mechanics.

G. Livadiotis
Section of Astrophysics, Astronomy and Mechanics, Physics Department, University of Athens, Greece.

A possible generalization of statistical mechanics within the framework of non-Euclidean metrics induced by $L^q$-norms, is presented. By specifying a generalized formal scheme of means characterization, emerging through the optimization of $L^q$-normed variance, we deduce the non-Euclidean expectation values [1]. The non-Euclidean adaptations of the classical extensive Boltzmann-Gibbs (BG) and of the non-extensive Tsallis statistics [2], are derived separately. In particular, the non-Euclidean Canonical probability distributions are respectively given by an exponential, and by a deformed-exponential, of a power-law dependence on energy. Moreover, it is proven that the connection with thermodynamics arises through the known classical formalisms. The presented non-Euclidean statistics is founded on the concept of non-Euclidean expectation values of physical quantities, such as internal energy. The non-Euclidean entropy, defined as the non-Euclidean expectation value of the Tsallis-Shannon’s information measure, is discussed, but not followed through with. On the contrary, the adopted Tsallis statistics ($L^Q$-normed entropy), together with the non-Euclidean BG statistics ($L^q$-normed internal energy), are combined to the overall non-Euclidean Tsallis statistics. In BG statistics and in the absence of correlations, both entropy and internal energy are additive. However, in Tsallis statistics they cannot be both additive, since specific correlations that recover the additivity in entropy, cause the non-additivity in internal energy, and visa versa [3]. Similarly, in non-Euclidean BG statistics, relevant correlations that recover the additivity in internal energy, cause the non-additivity in entropy. Thereafter, we examine whether entropy and internal energy can be both additive within the framework of non-Euclidean Tsallis statistics. Finally, we refer to an application to chemical kinetics, implemented by generalizing the Arrhenius equation for the temperature dependence of the rate constant. Glass-forming materials characterized as “fragile” (e.g. ionic liquids), exhibit a non-Arrhenius temperature dependence, that is a non-exponential decay, in the low-temperature/high-activation-energy region, that alters to the classical exponential decay in the high-temperature/low-activation-energy region [4]. Such a behavior seems to be justified by the Canonical probability distribution, governed by the non-Euclidean Thermostatistics.