q-Gaussian approximants mimic non-extensive statistical mechanical expectation for many-body probabilistic model with long-range correlations.

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q-Gaussian distributions seem to provide a class of distributions which may successfully model a wide variety of natural phenomena [1] and which serve as an attractor for certain correlated systems [2]. However, it is not always obvious what underlying generating mechanism gives rise to these models. Possible mechanisms do exist, however. For example [3] describes Student-t distributions (a special case of q-Gaussians) as a family of scale mixtures of normal laws. In this talk we introduce and study a strictly scale-invariant probabilistic N-body model with symmetric uniform identically distributed random variables. Correlations are induced through a transformation of a multivariate Gaussian distribution with covariance matrix decaying out from the unit diagonal, as $\frac{\rho}{r^\alpha}$, for $r = 2, 3, \ldots N$, where r indicates the displacement from the diagonal; $\alpha \geq 0$ characterizes the range of the correlations; and $0 \leq \rho \leq 1$ characterizes the strength of the correlation. We show numerically that, as N increases towards infinity, the sum of the N random variables quickly approaches a nontrivial limiting distribution which mimics (but is not equivalent to) to a compact support q-Gaussian distribution with $q(\rho, \alpha) \leq 1$.
In the particular case of $\alpha = 0$ we obtain $q = \frac{1-(5/3)\rho}{1-\rho}$. This result was analytically derived in a recent paper by Hilhorst and Schehr [4] addressing the present model. Our present results with these q-Gaussian approximants precisely mimic the behavior that was expected in the frame of non-extensive statistical mechanics. The fact that the $N \to \infty$ limiting distributions are not exactly, but only approximately, q-Gaussians reveal that the present random variables are not exactly, but only approximately, q-independent, in the sense of the q-generalized central limit theorem recently proved by Umarov, Steinberg and Tsallis. Short range interaction ($\alpha > 1$) and long range interactions ($\alpha < 1$) are discussed. Other simple mechanisms which lead to the production of q-Gaussians, such as mixing, are discussed as well.

[1] M. Gell-Mann and C. Tsallis (Eds.), *Nonextensive EntropyInterdisciplinary Applications*, (Oxford University Press, Oxford, 2004).
[2] S. Umarov, C. Tsallis and S. Steinberg, *A generalization of the central limit theorem consistent with nonextensive statistical mechanics*, Milan J. Math. (2006), doi:10.1007/s00032-008-0087-y.
[3] V.E. Bening and V.Yu. Korolev, *On an Application of the Student Distribution in the Theory of Probability and Mathematical Statistics*, Theor. Prob. Appl. **49**, 377 (2005).
[4] H.J. Hilhorst and G. Schehr, *A note on q-Gaussians and non-Gaussians in statistical mechanics*, J. Stat. Mech. P06003 (2007).