Topological phases in the Kitaev honeycomb lattice model on torus.

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We investigate low energy spectral properties of quantum lattice models which are believed to form topologically ordered states known also as topological phases. Our particular focus is on the Kitaev honeycomb spin-1/2 lattice model [1]. In the absence of external magnetic field, the model is exactly solvable and its phase diagram exhibits a gapless phase and an abelian topological phase whose effective description is given by the $\mathbb{Z}_2 \times \mathbb{Z}_2$ topological field theory. As known from the perturbation theory, a weak magnetic field has no dramatic effect on the abelian topological phase but turns the gapless phase into a non-abelian topological phase whose effective description is given by the SU(2)$_2$ theory. The quasiparticle excitations of this phase are nonabelian anyons satisfying the Ising fusion rules.

We particularly study the Kitaev honeycomb lattice on torus [2]. We describe symmetries of this model and review the perturbative mapping of its abelian topological phase onto the $\mathbb{Z}_2 \times \mathbb{Z}_2$ square lattice model known as the toric code. We provide the classification of finite size effects on the model low-energy spectral properties [3]. In this context, special attention is given to the thin-torus limit and related conformal field theory data. We then investigate properties of the model's vortex excitations. We complete this part with discussion of topological degeneracy of the model [2].

We then proceed to numerical investigation of the non-abelian topological phase in the perturbative limit of weak magnetic field [4] and beyond. The weak field is modeled by an effective three body interaction term which does not commute with the bare Hamiltonian but commutes with the vortex operators. In this regime, we observe that the magnetic field is able to induce level crossing of states belonging to the same vortex sector. We also investigate the model in strong field regime modeled by the full Zeeman term which allows for dynamics of vortices.

We conclude with discussion of the topological phase transitions in the model and brief review of other lattice models whose low energy spectra provide realization of topological field theories.