Generating function approach to thermodynamics based on time averages.

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We derive the statistical-mechanical expressions obtained via the method of time averages [1,2] by considering a renewal stochastic process. Our approach is based on the introduction of a generating function for the n-th event occurrence. This involves weighting the probability distribution function $\psi_n(t)$ for the n-th event taking place at time $t$ by a factor $z^n$. For identical independent events (IIEs) the $\psi_n(t)$ are determined in turn from the distribution function for a single event $\psi(t)$ occurring at time $t$, and this is provided in the usual straightforward way by repeated convolutions of $\psi(t)$. When the weight parameter $z$ is assigned a functional dependence on the system’s relevant parameters the generating function is seen to represent the partition function of a statistical-mechanical system from which a free energy (or Massieu entropy function) is obtained via standard procedures. Within the consideration of IIEs, and according to the specific choice of the functional form of $z$ on, say, the configurational energy of the system, we obtain, amongst other possibilities, the Boltzmann-Gibbs or the Tsallis general expressions. Boltzmann-Gibbs statistics is obtained by assuming an exponential dependence inside $z$ while Tsallis statistics follows from a $q$-deformed exponential form. Thus, the type statistical-mechanical structure produced by our generating function approach is determined by the manner in which the renewal events are weighted and not by the nature of the renewal process itself. We further illustrate our formalism by considering the Weierstrass renewal processes of IIEs introduced by Hughes, Montroll and Schlesinger that consists of a single event distribution $\psi(t)$ composed of an infinite number of exponentially decaying processes [3]. Depending on the relative presence of low frequency events the Weierstrass process exhibits either exponential or power law time decay in the limit of infinite events $n \to \infty$. In the former case our generating function approach recovers the Botzmann-Gibbs statistics. For the latter case we determine the resultant statistical-mechanical structure.