Uncertainty relations in terms of Tsallis entropy.

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Quantum-mechanical uncertainty relations for position \(x\) and momentum \(p\) in the form of inequalities involving Shannon or Renyi entropies had been derived by I. Białynicki-Birula some time ago [1,2]. Here we present derivation and discussion of the analogous uncertainty relations emerging from Tsallis entropy \(H_\alpha\), which (for \(\alpha \geq \beta\) when \(H_\alpha^{(p)} \leq H_\beta^{(x)}\)) have the following inequality form

\[
H_\alpha^{(p)} + H_\beta^{(x)} \geq \frac{1}{1-\alpha} \left[ \left( \frac{\beta}{\alpha} \right) \frac{\beta}{2} \left( \frac{2\beta}{\hbar} dx dp \right)^\frac{\alpha-1}{2} - 1 \right].
\]  

(1)

The simplicity of these relations indicate that Tsallis entropy is suitable characteristic of uncertainties quantum measurements. The resulting limitations on the information content characterizing quantum system generalize results obtained from Shannon entropies (to which they converge when \(\alpha \to 1\) and \(\beta \to 1\). In particular, we shall pay special attention to the following observation: whereas standard uncertainty relation, \(dx dp > \hbar/2\), is not a statement about accuracy of our measuring instruments, the entropic uncertainty relations do depend on it because they explicitly contain the volume of the corresponding phase space, \(dx dp\), determined by the accuracy of measuring instruments. It turns out that for all entropies considered so far (Shannon, Renyi and Tsallis) the sum of entropies \(H_\alpha^{(p)} + H_\beta^{(x)}\) becomes negative for large relative size of the phase-space area \(dp dx/\hbar\). However, whereas for Shannon and Renyi entropies this sum tends to \(-\infty\) when \(dp dx \to +\infty\), in the case of Tsallis entropy it is negative but remains finite and equals to \(-1/(\alpha - 1)\). This fact makes the Tsallis entropy more attractive for formulations of uncertainty relations because only with it one gets the sum of entropies limited in the sense mentioned above. One can then renormalize the respective probability counting and obtain \(H_\alpha^{(p)} + H_\beta^{(x)} > 0\) for all phase-space areas \(dx dp\).

All these is connected with the fact that, when the size of an object \(dp dx\) is bigger than the cell dimension \(\hbar\), it can be found in many cells at the same time, i.e., events are not mutually exclusive as in the case of standard entropy counting.