

**Abstracts Booklet**



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**Editors**

**G. Kaniadakis and A.M. Scarfone**

**Abstracts**

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## Topical workshops

- Biophysics (F. Seno)  
Complex Networks (G. Caldarelli and A. Scala)  
Econophysics and Sociophysics (P. Argyrakis and S. Solomon)  
Mathematical Physics (P. Damianou and C. Sophocleous)  
Nanophysics (F. Kusmartsev)

## Preface

The present volume contains the abstracts of the invited talks and of selected contributed oral or poster presentations submitted to the International Conference on ΣΤΑΤΙΣΤΙΚΗ ΦΥΣΙΚΗ held at Larnaca in Cyprus, from July 11 to 15, 2011.

The oral and the poster presentations are more than 350, among them more than 100 are invited talks.

The conference has covered the following topics:

A) Foundations and theoretical aspects of classical, quantum and relativistic statistical physics and thermodynamics.

B) Mathematical aspects and methods: formalism, rigorous results, exact solutions, connections with the methods of high energy physics, string theory, mathematical statistics and information theory.

C) Nonequilibrium systems: classical, quantum and relativistic transport theory, Boltzmann and Fokker-Planck kinetics, nonlinear kinetics, dynamical systems, relaxation phenomena, random systems, chemical reactions, pattern formation etc.

D) Applications to physical systems: quantum systems, soft condensed matter, plasmas, fluids, surfaces and interfaces, disordered and glassy systems etc.

E) Interdisciplinary applications of statistical physics

F) Numerical Physics: Numerical methods and computer simulations in statistical physics

Furthermore topical workshops on Biophysics, Complex Networks, Econophysics and Sociophysics, Mathematical Physics and Nanophysics have been organized as parallel events.

G. Kaniadakis and A.M. Scarfone  
(Editors of the Abstract Booklet)



## Gauge-theoretic structure in quantum thermodynamics

S. Abe†

†Mie University

Developments in manipulating microscopic systems have been drawing fresh interest in quantum thermodynamics and thermodynamics of small systems. This is physically relevant, from nano-scales to scales of a few particles. Extracting the work from a small quantum thermodynamic system by controlling external conditions is currently one of primary issues. Especially, it is important to consider such a problem in non-equilibrium situations, if a variety of examples such as condensed matter systems and (bio)molecular motors are taken into account. Any physical idea suggesting new theoretical and experimental methods of controlling non-equilibrium small systems must be welcome, today. Here, we wish to discuss the problem of extracting the work from a quantum-mechanical thermodynamic system. We show that the gauge-theoretic structure naturally emerges in a system that is driven by the change of an external condition which are represented by a set of slowly varying parameters contained in both the Hamiltonian and the boundary conditions. The emergent gauge field is defined in the parameter space and transforms, in fact, as an Abelian gauge field under a class of redefinitions of the system density matrix. The strength of the gauge field is found to identically vanish, if the system is in an equilibrium state, depending only on the Hamiltonian operator, analogously to the canonical density matrix. In other words, the non-vanishing gauge field strength implies that the system is in a non-equilibrium quasi-stationary state. This is a novel characterization of the non-equilibrium nature of a quantum thermodynamic state. The work done through a cyclic process a long a closed curve in the parameter space is described in terms of the flux of the field penetrating the surface enclosed by the closed curve. Then, we apply this general formalism to a simple (but illustrative) example of a single spin-1/2 in a slowly rotating external magnetic field. We discuss a specific finite-time cyclic process and evaluate the extractable work when the power output is at its maximum value. The present work offers a method of "geometrically" controlling quantum thermodynamic systems. Also, we make a comment on the use of a master equation of a certain type in order to further clarify the property of the finite-time process.

[1] S. Abe, *Gauge theory of quantum-thermodynamic engine*, (preprint, 2011).

## $\kappa$ -deformed random matrix theory based on Kaniadakis entropy

A.Y. Abul-Magd†

†Department of Basic Sciences, Faculty of Engineering, Sinai University, Sinai, Egypt

Random-matrix theory (RMT) is often used to describe quantum systems whose classical counterparts have chaotic dynamics [1]. This is the statistical theory of random matrices, whose entries fluctuate as independent Gaussian random numbers. The weight function for the random-matrix ensembles,  $P(H)$  is obtained from the principle of maximum entropy by postulating the existence of the trace of the square of the Hamiltonian matrix. The resulting expression reads  $P(H) = Z^{-1}(\eta) \exp[-\eta \text{Tr}(H^\dagger H)]$  where  $\eta$  is a parameter related the mean level density and  $Z^{-1}(\eta)$  is a normalizing constant. The corresponding ensemble is invariant with respect to the change of the base in which the matrix elements are expressed. The conventional approach is to use Shannon's definition for the entropy for ensembles of random matrices. Generalizations of RMT by using other forms of entropy have also been considered. Among these forms is the Kaniadakis entropy, characterized by the index  $\kappa$  (Boltzmann-Gibbs entropy is recovered in the limit  $\kappa \rightarrow 0$ ) [2]. Kaniadakis'  $\kappa$ -framework is based on  $\kappa$ -exponential and  $\kappa$ -logarithm functions. We have proposed the non-Gaussian deformations ( $\kappa \neq 0$ ) of the conventional orthogonal ensemble of random matrices in an earlier publication [3]. We here elaborate further on the generalization of RMT within the framework of the Kaniadakis statistics. The Kaniadakis entropy is maximized again under the constraints of normalization of  $P(H)$  and fixed mean value of  $\text{Tr}(H^\dagger H)$ . One arrives to the following distribution  $P(\kappa, H) = Z_\kappa^{-1}(\eta) \exp_\kappa[-\eta \text{Tr}(H^\dagger H)]$ , here  $Z_\kappa$  is a normalization constant. This distribution function, as in the conventional RMT, depends on the matrix elements in the combination  $\text{Tr}(H^\dagger H)$ . Therefore, the random-matrix ensembles described by these distribution functions are base independent. We consider systems with and without time reversal invariance. We derive analytical expressions for the joint eigenvalue distributions for the  $\kappa$ -deformed ensembles by applying the principle maximum entropy to Kaniadakis entropy. Using these expressions, we introduce a new generalized form of the Wigner surmise for the level spacing distribution that accurately approximates the result of RMT. The high accuracy of Wigner's surmise in describing chaotic systems justifies the use of the obtained  $\kappa$ -deformed surmise for describe the transition of the nearest-neighbor spacing distribution from chaos to order. The so-generalized surmise is tested by comparing with the results of a number of previous numerical experiments. It is demonstrated that the Kaniadakis approach leads to accurate description for the spacing distributions in the early stages of the transition from chaos to integrability. Integrable systems have known eigenvalues which form a preferred basis for the Hamiltonian matrix. One cannot expect a base invariant theory to describe nearly integrable systems. The comparison between the  $\kappa$ -deformed Wigner surmise with the experimental spacing distribution can be used to decide quantitatively, when mixed systems under consideration no more can be described by a base independent RMT approach.

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[2] G. Kaniadakis, Phys. Lett. A **288**, 283 (2001); Phys. Rev. E **66** 056125; **72**, 036108 (2005).

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## Dynamics of spin qubits in graphene

F. Aghel†, M.M. Golshan

†Physics Department, Payamnoor University, Shiraz, Iran

A major need for the development of quantum computers and, generally, quantum information processing, is to establish physical entities as the fundamental qubits [1]. The electronic spin states, as opposed to its charge states, are much less sensitive to the unavoidable electromagnetic noises, and thus have been proposed for such applications. In fact, the electronic spin states in graphene nanoribbons, nanotubes and nanodots, with the potential of constructing networks, have been shown to form promising qubits. The development of qubits based on Pi-electrons in graphene relies on unique properties of graphene: it exhibits spin transport at room temperature, long spin relaxation length, tunable spin transport, etc. [2]. The behavior of electronic spins, as qubits, under different conditions, is thus crucial in the applications with the aim of developing quantum information processing. In the present work, therefore, we report the dynamical behavior of electronic spin states in graphene, with Rashba spin-orbit coupling (RSOC), under the influence of an external magnetic field normal to the graphene (along the z-axis). The so-called Pi-electrons in graphene mainly possess three degrees of freedom, namely, the spin, the pseudospin (locating the Pi-electrons at sublattices) and the internal ones (within the sublattices). The state of the Pi-electrons is then described as a tensor product of three independent Hilbert spaces of aforementioned degrees of freedom. In the present work we report the dynamical behavior of Pi-electron's spin components for graphene sheets grown on different substrates [3,4]. Although the RSOC in an isolated sheet of graphene is rather weak, it has been well established that the RSOC in graphene may be manipulated using different substrates. It is then the main aim of this article to investigate the effect of RSOC on the dynamical behavior of spin's different components. In so doing we first present the Hamiltonian, with RSOC, that governs the pi-electrons spin behavior. Assuming the Pi-electrons in graphene form a pure ensemble, we calculate the time evolution of the density operator, initially formed by a pure coherent spin state. The ensemble averages of the spin components, as functions of time, are subsequently determined. We then show that one of the spin components in the sheet has always a vanishingly small average value, while that of the other, the ensemble average oscillates between [-1,1]. On the other hand, for the spin component normal to the sheet, the ensemble average oscillates between 0 and 1, with no negative values. It is thus concluded that graphene, with RSOC, acts as a spin filtering (polarizing) device.

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## On-Off instability in a dynamo flow

A. Alexakis†

†Ecole Normale Supérieure

An investigation of the dynamo instability close to the threshold produced by a turbulent flow is presented. The investigation is carried out with the use of direct numerical simulations of the full system and by simple low-dimensional analytical models. The talk will focus on the on-off intermittency behavior of the dynamo its presence or absence in the dynamical systems examined and how it depends on flow properties in the linear and the non-linear regime. It is shown in the numerical simulations that the Lorentz force drastically alters the statistics of the turbulent fluctuations of the flow and reduces their amplitude. As a result, much longer bursts of on phases are observed than is expected based on the amplitude of the fluctuations in the kinematic regime of the dynamo. For large Reynolds numbers, the duration time of the on phase follows a power law distribution, while for smaller Reynolds numbers the Lorentz force completely kills the noise and the system transits from a chaotic state into a laminar time periodic flow. The behavior of the on-off intermittency as the Reynolds number is increased is also examined and decrease of the range of existence is observed. We try to explain some of the observed behaviors by the effect of multiplicative noise on a system described by two modes close to a bifurcation point. The bifurcation is assumed stationary and noise acts as random coupling between these modes. An analytic formula that predicts the onset of instability is derived, and the domain of existence of on-off intermittency is calculated based on an eigenvalue problem. This approach, confirmed by numerical simulations of the Langevin equations, allows quantifying the possible effects of the noise. The stability and the on-off behavior are shown to be very sensitive to deviations of the deterministic system from the case where both modes grow with equal rate and the system displays a continuous symmetry associated to rotation in phase space. In general, a noise term that breaks this continuous symmetry will increase the domain of instability of the system while a noise term that preserves the symmetry reduces the domain of instability. The system is investigated further by examining a nonlinear coupling with the noise term. The connections with dynamo experiments and theoretical modeling are discussed.

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## Conductance and noise in fully epitaxial magnetic tunnel junctions

F. Aliev†

†Universidad Autonoma de Madrid

Magnetic tunnel junctions (MTJs) are nowadays one of the most active areas of material science and spintronics. Here, we review our recent studies of conductance and low frequency noise as a function of applied bias, magnetic state and temperature in different types of MTJs [1-4]. The shot noise measurements are used to resolve between direct and sequential tunnelling [1,2]. Fully epitaxial Fe/C/MgO/Fe(001) MTJs exhibit record low Hooge factors being at least one order of the magnitude smaller than previously reported [3]. Interface engineering by using Vanadium doping of Fe electrodes allows to relax misfit defects inside MgO barrier, dramatically reducing  $1/f$  noise and enhancing tunnelling magnetoresistance [4]. Investigation of electronic transport in epitaxial Fe(100)/MgO/Fe/MgO/Fe double magnetic tunnel junctions with soft barrier breakdown (hot spots) [5] reveals quasi-periodic changes in the resistance as a function of bias voltage which point out formation of quantum well states in the middle Fe continuous free layer. We introduce "1/f noise band spectroscopy" to characterize band structure of ferromagnetic electrodes in MTJs involved in the spin dependent tunnelling. Besides, the shot noise measurements on symmetric and asymmetric fully epitaxial double magnetic tunnel junctions are shown to be effective method to determine spin diffusion length in ferromagnetic central layer. Finally, we use low frequency noise measurements to investigate magnetic inhomogeneities in submicron FeCoB/MgO/FeCoB magnetic tunnel junctions. Field training minimises the influence of metastable domain states and allows reproducible characterization of inhomogeneities of about  $10^5 \mu B$  located either in the hard or soft layers through measurements of their random telegraph noise (RTN). Asymmetric with bias RTN in the antiparallel state indicates influence of the spin torque effect on the stability of magnetic inhomogeneities in the soft layer at current densities below  $5 * 10^5 A/cm^2$ . Our findings represent a significant step towards the control over low frequency noise in submicron magnetic tunnel junctions. Collaboration with C. Tiusan, D.Herranz, A. Gomez-Ibarlucea, R. Guerrero, G. Reiss, M. Hehn, and S. Andrieu, R.Villar, J. Moodera, T.Santos, Y. Tserkovnyak, F.Greullet, F.Bonell, V. Dugaev, and J. Barnas is gratefully acknowledged. The work has been supported in parts by Spanish-French Integrated Action project, Spanish MICINN and European Science Foundation (FONE-SPINTRA).

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## Design principles of biological circuits

U. Alon†

†Weizmann institute

Cells function by means of complex networks of biological interactions. Understanding such networks is a challenge for science, because complex networks appear in diverse fields from sociology to ecology and neuroscience. This talk will describe the discovery that transcription regulation networks are made of a handful of recurring interaction patterns known as network motifs. Each of the network motifs is an elementary circuit, which carries out defined dynamical functions: filters, pulse generators, response accelerators, logic gate arrays etc. These functions were demonstrated by dynamic experiments in living cells. Combinations of network motifs are able to carry out more complicated functions, which can be deduced from the function of each individual motif. The same small set of network motifs seems to characterize the transcription networks of all organisms studied so far, from E. coli to humans. This constitutes a degree of simplicity, because the vast majority of possible patterns do not seem to occur in transcription networks. Network motifs are discovered again and again in different organisms by convergent evolution, perhaps because they are the simplest patterns that can robustly carry out their respective functions. We will focus on new types of motifs related to fold-change detection (FCD): a response whose entire shape, including amplitude and duration, depends only on fold changes in input and not on absolute levels. Thus, a step change in input from, for example, level 1 to 2 gives precisely the same dynamical output as a step from level 2 to 4, because the steps have the same fold change. We show that FCD is necessary and sufficient for sensory search to be independent of multiplying the input field by a scalar. Thus, the FCD search pattern depends only on the spatial profile of the input and not on its amplitude. Such scalar symmetry occurs in a wide range of sensory inputs, such as source strength multiplying diffusing/convecting chemical fields sensed in chemotaxis, ambient light multiplying the contrast field in vision, and protein concentrations multiplying the output in cellular signaling systems. FCD entails two features found across sensory systems, exact adaptation and Weber's law, but that these two features are not sufficient for FCD. Finally, we present a wide class of mechanisms that have FCD, including certain nonlinear feedback and feed-forward loops. Bacterial chemotaxis displays feedback within the present class and hence, is expected to show FCD. This can explain experiments in which chemotaxis searches are insensitive to attractant source levels. This suggests a connection between properties of biological sensory systems and scalar symmetry stemming from physical properties of their input fields.

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## Probing Dirac fermions in graphene: from Landau levels to fractional quantum Hall effect

E.Y. Andrei<sup>†</sup>

<sup>†</sup>Department of Physics, Rutgers University

Graphene is a one-atom thick membrane of crystalline Carbon arranged in a honeycomb structure. One of the consequences of graphene's two dimensionality and of its lattice symmetries is a low energy excitation spectrum which is conical and electron-hole-symmetric resulting in charge carriers that mimic massless Dirac fermions. Because all the atoms in graphene are on the surface the intrinsic properties of the charge carriers in actual graphene samples are easily obscured by environmental disturbances. We have developed methods to decouple graphene from external perturbations which include the use of a perfectly matched and pristine substrate (graphite) or eliminating the substrate altogether. I will describe experiments which show that when graphene is decoupled from external disturbances the intrinsic properties of its charge carriers become apparent. The experiments employed scanning tunneling microscopy and spectroscopy (STM and STS), and electrical transport. By using STS to probe the charge carriers in graphene on graphite we find that the density of states is linear and vanishes at the Dirac point as expected for massless Dirac fermions. In the presence of a magnetic field we directly observe the quantized Landau level sequence which unequivocally identifies the massless Dirac fermions through its square-root dependence on field and level index, and through the presence of a unique zero-energy Landau level[1]. By using Landau level spectroscopy we obtained the Fermi velocity, the quasiparticle lifetime and evidence for electron-phonon interactions. Furthermore, we find that if graphene is exposed to the potential induced by a second graphene layer which is placed underneath it with a relative twist, its spectrum undergoes a qualitative transformation. Instead of the purely massless Dirac fermion spectrum of the decoupled graphene layers, the twisted stacked layers exhibit prominent Van Hove singularities whose position is controlled by the twist-angle [2,3]. Using transport measurements on suspended graphene devices we demonstrated ballistic transport on micron-size length scales and found evidence of strong electron-electron interactions manifest in the observation of a robust fractional quantum Hall effect and a magnetically-induced insulating phase[4].

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## Nonlinear hydrodynamic corrections to exothermic chemical wave fronts

C. Antoine<sup>†</sup>, A. Lemarchand, B. Nowakowski, G. Dumazer

<sup>†</sup>Université Pierre et Marie Curie - Paris 6, Laboratoire de Physique Théorique de la Matière Condensée, Paris, France

The interest in the front speed problem of travelling waves in combustion, plasma physics, chemical kinetics and biology has been continuously growing since the pioneering works of Fisher and Kolmogorov, Petrovskii, Piskunov (FKPP) in 1937. The reaction, diffusion and convection processes at stake can generally be modelled by a system of nonlinear parabolic differential equations whose study has already given many insightful results, for propagation in both homogeneous and heterogeneous media, for many different kinds of reactive kinetics and initial conditions, and at both the macroscopic and microscopic scales. For exothermic chemical wave fronts in fluids, the reaction-diffusion equations are strongly coupled to the hydrodynamic balance equations through the temperature dependence of reaction rates, the convective terms, and  $Q$ , the heat released by the reaction of interest. In return, several non-reactive, thermal waves precede or follow the reactive interface, perturbing the medium where the chemical wave evolves. Although many of the features of these travelling waves have been intensively studied, for example in combustion physics, the front speed problem for the reactive interface has not been fully tackled yet. We recently obtained some constraints on the dynamics of FKPP wave fronts in perfect and van der Waals gases, and proved the existence of a forbidden interval of stationary wave front speeds, at both the macroscopic and microscopic scales (using the Direct Simulation Monte Carlo (DSMC) method, which is a direct simulation of the Boltzmann equation). A transition has been also observed, for increasing values of  $Q$ , between two different dynamical regimes: One controlled by the reaction-diffusion processes (similar to diffusion flames in combustion physics), and one controlled by  $Q$  and the hydrodynamic conservation laws (the so-called Chapman-Jouguet detonation waves). These regimes are obtained respectively for  $Q \ll Q^*$  and  $Q \gg Q^*$ , where  $Q^*$  is the critical heat release which crucially depends on the activation energy of the main reaction of interest. Our goal, in this presentation, is to explore the wave front dynamics close to this transition, and to show how a nonlinear stability analysis can help characterizing the hydrodynamic corrections to the FKPP wave front dynamics.

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## Optimal receptor-cluster size determined by intrinsic and extrinsic noise

G. Aquino†, D. Clausznitzer, S. Tollis, R.G. Endres

†Division of Molecular Biosciences, Imperial College London

Biological cells sense external chemical stimuli in their environment using cell-surface receptors. To increase the sensitivity of sensing, receptors often cluster. This process occurs most noticeably in bacterial chemotaxis[1], a paradigm for sensing and signaling in general. While amplification of weak stimuli is useful in the absence of noise, its usefulness is less clear in the presence of extrinsic input noise and intrinsic signaling noise (mainly due to fluctuations in receptors methylation, in the case of bacteria). Here, exemplified in a bacterial chemotaxis system, we combine the allosteric Monod-Wyman-Changeux model for signal amplification by receptor complexes with calculations of noise to study their interconnectedness. Importantly, we calculate the signal-to-noise ratio (SNR), describing the balance of beneficial and detrimental effects of clustering for the cell. Interestingly, we find that there is no advantage for the cell to build receptor complexes for noisy input stimuli in the absence of intrinsic signaling noise. We set up a Master equation, based on a van Kampen's expansion in the reaction volume, in order to derive the relevant dynamics, we apply a small noise approximation which we verify via numerical simulation [2]. We find that when we consider only intrinsic noise there is no penalty for the cell to make larger and larger complexes, and the SNR increases with increasing complex size. However, including intrinsic noise introduces a complex size-independent noise floor. Hence, an increase in complex size is beneficial until the amplified and hence size-dependent extrinsic noise increases beyond the noise floor. An optimal complex size for complexes with more than one receptor arises in line with estimates of the size of chemoreceptor complexes in bacteria and protein aggregates in lipid rafts of eukaryotic cells. Our model can readily be extended from single receptor-type complexes to mixed complexes of multiple receptor types. If the ligand binds specifically to one receptor type only, the receptor fraction of each type in a mixed complex should be optimal in size with specifics depending on the ligand dissociation constants only. Our work may be applicable to other receptors as well. Most well-characterized sensory receptors are believed to cluster (or to oligomerize). T-cell (white blood cell) receptors, for instance, form microclusters of 7-30 receptors [3]. Our work indicates that this number represents an optimal size for signal amplification, when size is restricted by extrinsic and intrinsic noise.

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## Worldwide spreading of economic crises

P. Argyrakis†, A. Garas, C. Rozenblat, M. Tomassini, S. Havlin

†University of Thessaloniki

We study the spreading of a crisis, such as the world crisis of the past few years and we model such events by constructing a global economic network which uses financial information about the economic relationships between the different countries of the world. We then spread this crisis from a single point of origin to the entire network by utilizing a the Susceptible-Infected-Recovered (SIR) epidemic model with a variable probability of infection. Each country can be infected by its neighbor country in the network with a certain probability. The probability of infection depends on the strength of economic relations between the pair of countries, and the strength of the target country, as it is natural for the economic dependence of one country on its trading partners. The actual data that we use involve two different sets: (1) The import-export data for all countries of the world and (2) the number of subsidiaries that each private company has established in other countries, by considering the 5000 largest world corporations. It is expected that a crisis which originates in a large country, such as the USA, has the potential to spread globally, such as the recent crisis that originated in the mortgage sector but spread practically in the entire banking industry. Surprisingly we show that in addition to the large economic powers in the world, countries with much lower GDP, such as Belgium, are also able to initiate a global crisis. Using the k-shell decomposition method to quantify the spreading power (of a node), we obtain a measure of "centrality" as a spreader of each country in the economic network. We thus rank the different countries according to the shell they belong to, and find the 12 most central countries. These countries are the most likely to spread a crisis globally. Of these 12 only six are large economies, while the other six are medium/small ones, a result that could not have been otherwise anticipated. Furthermore, we use our model to predict the crisis spreading potential of countries belonging to different shells according to the crisis magnitude. In comparing the results with the actual situation of the economies of all countries, we find a pretty good agreement between this new model and the extent of involvement of the crisis in each country.

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## Verification of the scaling relation within multifractal probability density function theory by analyzing PDFs from experiment and DNS

N. Arimitsu†, T. Arimitsu

†Yokohama National University

The probability density functions (PDFs) for energy dissipation rates, energy transfer rates and velocity differences from the experiment of fully developed turbulence in a wind tunnel and from DNS with  $4096^3$  grid points are analyzed within the framework of multifractal probability density function theory (MPDFT) to a high accuracy in order to test the validity of a new scaling relation. MPDFT is a statistical mechanical ensemble theory provided by the authors in order to analyze intermittent phenomena providing fat-tail PDFs. The new scaling relation is introduced so as to be able to analyze intermittency through any series of PDFs with arbitrary magnification  $\delta (> 1)$ . In order to extract the intermittent character of the fully developed turbulence, it is necessary to have information of hierarchical structure of the system. This is realized by producing a series of PDFs for responsible singular quantities with different lengths  $\ell_n = \ell_0 \delta^{-n}$ , ( $n = 0, 1, 2, \dots$ ) that characterize the sizes of eddies or the regions in which the physical quantities are coarse-grained. The value for  $\delta$  is chosen freely by observers. Therefore, the choice of  $\delta$  should not affect the theoretical estimation of the values for the fundamental quantities, i.e., observables characterizing the turbulent system under consideration. The A&A model within the framework of MPDFT itself tells us that this requirement is satisfied if the scaling relation has the form  $(\ln 2)/(1-q) \ln \delta = 1/\alpha_- - 1/\alpha_+$ . The validity of the generalized MPDFT is verified successfully through the precise analyses of several series of PDFs with different values of  $\delta$ . In the course of the verification, it is revealed that the system of fully developed turbulence has much wider area representing scaling behaviors than the inertial range. We will investigate in a high accuracy the validity of the new scaling relation, which MPDFT claimed as a physically required relation for arbitrary value of  $\delta$ . The theoretical PDF formula for physical quantities within MPDFT is provided with the minimal numbers of adjustable parameters. We assume that the theoretical PDF is given by two partial PDFs, one for the tail part and the other for center part. This is realized with the help of the Tsallis-type trial functions. By means of the behavior of extracted parameters, we can check the validity of the new scaling relation, and obtain the information about how to separate the coherent turbulence motion and the fluctuations around the motion.

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## On a new interpretation of turbulence via the scaling relation of A&A model within MPDFT

T. Arimitsu†, N. Arimitsu

†University of Tsukuba

Multifractal Probability Density Function Theory (MPDFT) [1-5] is a statistical mechanical ensemble theory for analyzing instability of systems providing fat-tail PDFs under the assumption that the singularities due to the instability distribute themselves multifractal way in real physical space. The degree of singularity for those quantities which are responsible for intermittent phenomena is specified by the singularity exponent  $\alpha$  that appears in the scale transformation of the Navier-Stokes equation as an arbitrary parameter taking real values. The A&A model are written down with three parameters which are fixed by the condition of energy conservation, the definition of  $\mu$  and the scaling relation that relates between the entropy index  $q$ , appeared in the definition of Rény entropy or Tsallis entropy, and  $\alpha_{\pm}$  the zeros of the multifractal spectrum  $f(\alpha)$ , i.e.,  $f(\alpha_{\pm}) = 0$ . The multifractal spectrum is uniquely related to the distribution function for  $\alpha$ . In order to extract the intermittent character of the fully developed turbulence due to the instability, it is necessary to have information of hierarchical structure of the system. This is realized by producing a series of PDFs for responsible singular quantities with different length  $\ell_n = \ell_0 \delta_n$  with  $\delta_n = \delta^{-n}$  ( $n = 0, 1, 2, \dots$ ) and  $\delta > 1$  that characterize the sizes of regions in which the physical quantities are coarse-grained. The value for  $\delta$  is chosen freely by observers. Therefore, the choice of  $\delta$  should not affect the theoretical estimation of the values for the fundamental quantities characterizing the turbulent system under consideration. The framework of MPDFT itself provides us with a scaling relation,  $(\ln 2)/(1-q) \ln \delta = 1/\alpha_- - 1/\alpha_+$ , as a proper relations to satisfy above requirement [4,5]. We will present a conjecture that the scaling relation given above provides us with a new interpretation of turbulence, i.e., the system of fully developed turbulence consists of the accumulation of the  $\delta$ -scale Cantor sets characterized by  $\delta^{\infty}$  periodic orbits of a non-linear dynamical systems with different values of  $\delta$ . The  $\delta^{\infty}$  periodic orbits appear in a  $\delta$  periodic window in which there are infinite  $\delta^k$  periodic windows ( $k = 2, 3, 4, \dots$ ), one fitting inside another. Each  $\delta^k$  periodic window ( $k = 1, 2, 3, \dots$ ) starts with the  $\delta^k$  period saddle-node bifurcation ( $\delta$  times ramification) followed by the periodic doubling bifurcation and then by the chaotic region where the  $\delta^{k+1}$  periodic window nests. Observation of the system with a magnification  $\delta$  extracts the information of the  $\delta$ -scale Cantor sets constituting the turbulence.

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### Electromagnetic field energy within concentric infinite cylinders at oblique incidence containing a dispersive and lossy metamaterial

J.T. Arruda†, A. Souto Martinez

†Faculty of Philosophy, Sciences and Letters of Ribeirão Preto - University of São Paulo

Theoretical and numerical studies of the electromagnetic scattering by two isotropic, linear, homogeneous magneto-dielectric concentric cylinders, which are irradiated by a plane and time-harmonic electromagnetic field at oblique incidence, are presented. The scattering quantities, expressed in terms of multipole moments for both TE (transverse electric) and TM (transverse magnetic) modes, are calculated based on the rigorous Kerker-Matijevic solution [1], and they follow closely the notation and development done in the classical book of Bohren and Huffman [2] for an infinitely long homogeneous nonmagnetic circular cylinder. Nevertheless, we pay special attention to the electromagnetic fields inside the magnetic inhomogeneous scatterer by means of the energy-enhancement factor, which is defined as the ratio of the effective time-averaged electromagnetic energy within the inner and outer cylinders to the corresponding energy stored in a homogeneous cylinder with the same optical properties as the surrounding medium [3]. Because of the magnetism, the relative impedance indices associated with the optical properties of the cylindrical core and shell become relevant in this framework, and they are generally smaller than the corresponding refractive indices for high permeability materials [3]. Here, both the cylindrical shell or the core are considered to be dispersive and lossy, with dispersion relations provided by an isotropic left-handed material, that is, a composite metamaterial in which both the real parts of the scalar electric permittivity and magnetic permeability are negative within a certain frequency band, typically at the microwave range [4]. Specially, for a dispersive core and dielectric shell, although the extinction efficiency decreases at the left-handed region as a function of the thickness ratio between the inner and outer cylinders, we obtain an enhancement of the corresponding effective electromagnetic energy stored in the cylindrical scatterer. As an application of this theory, using the electrodynamics approach to determine the average electromagnetic energy density in a non-Lorentz-type metamaterial [4], and an approximation provided by the Bethe-Salpeter equation, we calculate the energy-transport velocity in a two-dimensional disordered media [5] consisting of dispersive and lossy metamaterial coated cylinders.

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### Determining the stability of genetic switches: beyond protein-only models

M. Assaf†, E. Roberts, Z. Luthey-Schulten

†University of Illinois

Genetic switches allow cells to switch between distinct gene-expression states in response to environmental stimuli and/or internal signals. The ultimate stability of these states is determined by stochastic fluctuations of mRNA and proteins during gene expression (see e.g. [1]) that can give rise to spontaneous switching, even in the absence of a driving signal. When gene expression states are stable on the time scale of cellular division they can carry epigenetic information across generations, however, when they are more transient they may provide a beneficial source of heterogeneity in genetically identical populations. Previous studies of noise-driven genetic switches have shown that switching can be treated as a first-passage problem of the underlying Markov process. This was done either by using the probability generating function formalism, or by employing semi-classical approximation schemes to the chemical master equations (CMEs) or related Langevin equations (see e.g. [2]). These studies, however, focused on protein-only models and ignored the presence of mRNA and thereby the influence of transcriptional noise. Recently it has been shown that explicitly accounting for mRNA in the underlying CMEs has a dramatic impact on switching times [3,4]. A general method for accurately determining the mRNA/protein distributions and stability of feedback-based switches is of great interest, as they regulate diverse biological phenomena, such as microbial environmental adaptation, developmental pathways, and bacteriophage lysogeny [4]. In this work [5] we study the dynamics of switching in a generic-feedback on/off switch with a two-state promoter, by explicitly accounting for stochastic fluctuations of mRNA. We employ the WKB theory to treat the underlying CMEs, and obtain accurate results for the quasi-stationary distributions of mRNA and protein copy numbers and for the mean switching time, starting from either state. Our analytical results compare well with Monte Carlo simulations. Importantly, one can use the approach to study the effect of varying biological parameters on the switch stability. Furthermore, this framework is expected to be useful for studying diverse genetic circuits characterized by metastable switching, e.g., those with additional promoter states such as DNA looping or nucleosome remodeling. In particular, it can be used to help elucidate the underlying regulatory circuits responsible for phenotypical changes as a result of switching.

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## A network approach to capture both local clustering and global organization in complex signals

T. Aste<sup>†</sup>, F. Pozzi, W.M. Son, T. Di Matteo

<sup>†</sup>The University of Kent

The continuous increase in the capability of automatic data acquisition and storage is a fundamental element for our progress that is providing unprecedented potentials. However, despite the fact that huge amount of data are accumulating in many fields, often the relevant information remains hidden and information filtering is becoming a crucial bottleneck of the present times. The need for better analytical tools to handle high dimensional data is broad and demands are coming from several research fields across physics, engineering and biology. We are developing a novel framework to study and model high-dimensional complex data by combining graph theory approaches with statistical physics methods. Our approach is based on the idea of building graphs associated to the information contained in the original data-set and then reducing redundancy and interwovenness by means of a topological embedding on surfaces. There are several well-established techniques that allow reducing the complexity of the data by extracting optimal subdivision into groups of similarly behaving signals. This is, for instance, done by measuring correlations and extracting clusters by thresholding, or by using independent component analysis, or by using linkage hierarchical clustering, or by assigning cluster memberships accordingly with distance measures by k-means and k-medoids techniques. These techniques can be effective in extracting the local information content or in unfolding the hierarchical organization. However, by means of these established tools only a part of the information is extracted focusing either at local level (clustering algorithms) or at global level (hierarchical linkages). On the other hand, there are strong evidences that in complex systems localized activity coexists with global scale invariance. Researchers are therefore faced with the problem of catching simultaneously two complementary aspects: on one side there is the need to reduce the complexity and the dimensionality of the data by identifying clusters which are associated with local specialized activity; but, on the other side, there is a need of keeping the information about the emerging global organization that is responsible for cross-scale activity. In this work we have been developing a techniques that are effective in capturing both the local clustering and the global emerging dynamics overcoming such a dichotomy. In this talk I will explain, in general terms, the concepts underneath our methodology and I will show applications to biological and financial datasets.

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## Temporal evolution of a granular gas in the uniform longitudinal flow. A computing grid simulation based study

A. Astillero<sup>†</sup>, A. Santos

<sup>†</sup>Universidad de Extremadura, Spain

A granular gas is a specific kind of complex system where one of its main characteristics is the energy dissipation associated to particle collisions. As a consequence, the conservation of energy does not hold and the inner with dynamics of the granular gas is richer than the corresponding to a molecular gas. Apart from their interest in industrial and technological applications, granular gases are important at a fundamental level as physical systems intrinsically out of equilibrium and thus exhibiting a wide spectrum of complex behaviour. Although the number of grains in a fluidized granular system is of course much smaller than the number of atoms or molecules in a conventional gas, it is large enough as to make nonequilibrium statistical-mechanical concepts and tools applicable. In particular, a kinetic theory approach (based on the Boltzmann and Enskog equations suitably modified to account for inelastic collisions) has proven to be very useful. One of the most extended ways of modeling a granular gas is to consider it made of inelastic hard spheres. The inelasticity of collisions is measured by the constant coefficient of normal restitution. In a previous work, we studied the transient hydrodynamic stage of a granular gas under conditions of uniform longitudinal flow, both from an analytical and computational (Direct Simulation Monte Carlo, DSMC) points of view. The uniform longitudinal flow is characterized by a linear longitudinal velocity field, a uniform density, and a uniform temperature. In a granular gas characterized by a given value of the coefficient of normal restitution, the relevant control parameter of the problem is the reduced deformation rate (which plays the role of the Knudsen number). The relevant response parameter is a nonlinear viscosity function defined from the difference between the normal stress and the hydrostatic pressure. In that work, we referred the particle velocities to the Lagrangian frame of reference which moves with the flow mean velocity. Taking into account this frame of reference, the original problem becomes space independent. This allows us to simulate the Boltzmann equation making use of the DSMC method in an easier way. In the present work, we try to answer three questions. Firstly, we would like simulating the inhomogeneous longitudinal flow, so, which are the necessary boundary conditions to simulate it? Secondly, does an inhomogeneous initial state evolve towards a homogeneous one? Finally, how long does the relaxation towards the homogeneous state take?

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## Fractal properties revealed in genome sequences through a block entropy approach

L. Athanasopoulou<sup>†</sup>, K. Karamanos, Y. Almirantis

<sup>†</sup>Department of Theoretical Physics, Jozef Stefan Institute, Ljubljana, Slovenia

Special scaling features of Shannon's block entropy have been investigated extensively in several works and the findings are compatible with the Ebeling and Nicolis conjecture for the scaling of Shannon's block entropy for symbolic sequences generated by nonlinear dynamics [1,2]. Specifically, symbol sequences that exhibit fractal properties and long-range correlations are characterised by linearity in semilogarithmic scale. Appearance of this linearity has been shown in the case of the logistic map at the Feigenbaum accumulation point [3]. Thus, we start this work by showing that the block entropy for Cantor-like binary sequences scales in a similar way. Thereafter, performing the same analysis, we choose to study entire chromosomes at the level of alternation of regions of different functionality (in this case, the alternation of coding - noncoding segments). In order to compare our results with what is expected on the grounds of pure randomness, we construct random surrogate sequences, which we have checked that lack this type of scaling. Our results show that block entropy values of genome-generated sequences are essentially lower than their corresponding random surrogates. In our treatment we read each symbol sequence in blocks (or words) of a given length by lumping. This is done in order to test in our study the existence of the nonmonotonicity observed in the entropy scaling of Cantor-like sequences read in this way [3]. However, we found that this behavior does not persist in Cantor-like sequences if a small percentage of noise is added, and this is also the case in the genomic symbol sequences studied herein. This observation is compatible with the highly noisy way that DNA sequences evolve during the long evolutionary history of genomes. Resemblance has been found between scaling properties of genomic sequences and of sequences which are fractal - for instance, by their construction. It has been shown in previous work parallel to this investigation that the sizes of noncoding spacers follow power-law-like distributions in most chromosomes of eukaryotic organisms from different taxa [4]. Prompted by this fact we have developed a simple evolutionary model based on well-known molecular events (segmental duplications followed by elimination of most of the duplicated genes) which reproduces the same linearity in log-log plots. Symbolic sequences produced by this evolutionary model exhibit the same scaling as found in genomic sequences [4]. All these findings converge to the conclusion that "segmental duplication-gene elimination" dynamics may have contributed to the observed long-range behaviour in the coding-noncoding alternation in genomes.

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## Vlasov equation for lattice systems in long-range coupling

R. Bachelard<sup>†</sup>, T. Dauxois, G. De Ninno, S. Ruffo, F. Staniscia

<sup>†</sup>University of Nova Gorica

We consider Hamiltonian systems of particles on a lattice, interacting through a long-range potential. This potential is typically of the form  $1/r^s$ , with  $s$  smaller than the dimension of the system, and  $r$  the (fixed) distance between particles. We show that in the thermodynamic limit, their dynamics is described by a Vlasov equation. Nevertheless, at odds with the well-studied mean-field case, the distribution function is now three-dimensional (for a 1D system), since it also depends on the position on the lattice. More specifically, there exists a (continuous) set of two-dimensional Vlasov equation, for each position on the lattice, and the long-range potential couples all these equations. This result is of particular importance for the non-equilibrium behaviour of these systems. Indeed, long-range systems are known to be trapped in long-lasting regimes, often called "quasi-stationary states", whose lifetime diverge with the number of particles in interaction: one consequence is that they may be the only regimes accessible experimentally. In the case of mean-field systems, this phenomenon has been shown to originate in the existence of a Vlasov equation in the thermodynamics limit, when  $N$  goes to infinity, and the relaxation to equilibrium is then only due to finite-size effects. Thus, the Vlasov equation explains the presence of quasi-stationary regimes in lattice systems as well. Focusing on the Hamiltonian Mean-Field model with periodic boundary conditions, the stability analysis reveals the importance of the Fourier modes associated to the lattice. These modes are actually the eigenmodes of the linearized dynamics, whose growth rates are derived. It appears that the mean-field mode is always more unstable than the higher Fourier modes, and that it eventually dominates the dynamics. This result highlights the relevance of the mean-field framework to describe long-range lattices, since typically only the latter will be present at a macroscopic level. This work also completes some previous results: canonical calculations, performed at equilibrium, had already suggested the equivalence between  $1/r^s$  potentials and the mean-field models, while out-of-equilibrium simulations showed that quasi-stationary regimes for a mean-field interaction are still present in the (long-range) lattice models.

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### Effects of temperature on charge transport in semiconductor superlattice in an electric and a tilted magnetic field.

A. Balanov<sup>†</sup>, A.O. Selskii, A.A. Koronovskii, A.E. Khramov, K.N. Alekseev, M.T. Greenaway, T.M. Fromhold, N.N. Khvastunov

<sup>†</sup>Loughborough University, UK.

Semiconductor superlattices (SLs) are nanostructures formed from several alternating layers of different semiconductor materials [1]. This periodic structure leads to the formation of energy minibands that enable electrons, in the presence of an electric field, to demonstrate a number of interesting quantum-mechanical phenomena such as the formation of Wannier-Stark ladders, sequential and resonant tunnelling, Bragg reflections, and Bloch oscillations. Thus, SLs are of great interest both for fundamental research and applied science [2]. Due to the high mobility of miniband electrons and very high frequency of the Bloch and charge-domain oscillations, SLs are nowadays considered as prospective elements for sub-THz and THz electronic devices. Recently, it has been shown that a tilted magnetic field applied to a SL provides an effective method to control the electrical properties of this structure [3]. Nonlinear interaction between Bloch oscillations and cyclotron motion of the electrons induces intricate patterns in semiclassical electron dynamics, which, depending on the ratio between cyclotron and Bloch frequencies, can either accelerate or decelerate the drift of electrons through the SL. On resonance, when the Bloch and cyclotron frequencies are commensurate, the electrons demonstrate a unique type of quantum chaos, which does not obey Kolmogorov-Arnold-Moser theory. This type of chaos is characterised by the formation of "web-like" structures, known in literature as "stochastic webs", in the phase space of the miniband electrons. The appearance of these webs abruptly delocalizes the electrons, significantly increasing their drift velocity [3]. This dramatically affects the collective electron behavior by inducing multiple propagating charge domains and GHz-THz current oscillations with frequencies and amplitudes much higher than with no tilted field [4]. In the present work, we study how an increase of temperature influences the drift velocity of the electrons and its effect on the electric current through a SL subject to an electric and tilted magnetic field. We show that the thermal distribution of the electrons can, counterintuitively, enhance the contribution of Bloch-cyclotron resonances to the drift velocity. In particular, the increase of temperature leads to an enhancement of resonant maxima in the drift velocity - electric field curve. This results in a shift of the generation threshold and of the main frequency of charge-domain oscillations.

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### Love thy neighbor. A firm based multi-country model with international technology spillovers and growth regimes

G. Ballot<sup>†</sup>, E. Taymaz

<sup>†</sup>ERMES, Université Paris II

The empirical literature provides several stylized facts on long run economic growth. First, cultural and geographical proximity matters for economic growth. Certain cultural and regional groups of countries achieve higher growth performance in certain time periods. Second, although there is persistence in growth performance, there are substantial relocations among the club of high performers, and club membership. In this paper, in order to we study the determinants of growth performance and endogenous creation of growth clubs we use the agent based methodology to build one of the first models which goes from the level of the firms to the level of a country economy, and takes into account 15 countries. The interactions between the firms, both at the country level and between countries yield the stylized facts mentioned on growth and allow for new analysis of the mechanisms at work. The methodological contribution of the paper is therefore a new tool for the analysis of growth convergence/divergence between countries, a main topic in macroeconomic theory, and does not rely on the assumption of a permanent technologically leading country, which underlies most of the existing models. We focus in this paper on the role of the intensities of technological spillovers between firms belonging to neighbor countries as an important factor. Increased external spillovers increase the technological similarities between countries. This in turn has some non trivial results on the link between international spillovers and growth, due to the complex interactions and learning. First opening the frontiers to spillovers raises growth. However we find that very high spillovers lower performance. The main economic result of the paper is then the evidence of an optimum level of spillover. This intermediate level keeps alive the diversity of dominant technologies and a higher proportion of smooth transitions between the main technological revolutions while high levels of spillovers lead to more locks-in into a technological paradigm which prevent learning and growth. The results suggest a finer look at the international rules for the dissemination of technology and the methodology allows for the integration of intellectual property rights and innovation policies. Keywords. Agent-based simulation, industrial dynamics, technological spillovers, endogenous growth, multi-country modeling.

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## Equation of state of dense fluids in the high pressure - high temperature region

V. Bardik†, E.V. Pavlov, A.V. Kcleshonok, D.A. Nerukh, I.V. Zhyganiuk

†Kiev Taras Shevchenko National University

In our study we consider the problem of the statistical-mechanic foundation of the equation of state (EoS) of fluids within the framework of the generalized approach [1]. A variant of the thermodynamic perturbation theory based on scaling transformation of the partition function has been applied to the functional expansion of the free energy. We develop a new variant of the thermodynamic perturbation theory in which a reference thermodynamic state ( $P_0, V_0, T$ ) is given instead of a reference system. Our approach involves an assumption that a functional form of the perturbed potential is identical to the potential of a reference system, thus we consider the deviation of the potential of the more compressed system from the potential of the less compressed system as the perturbation, and the scale transformation of the dynamical variable is equivalent to variation of a molecular size. The various modifications of EoS have been obtained for a number of simple model potentials on the basis of the free energy functional series at a certain choice of small expansion parameters. An analysis of the experimental data on isothermal compression has shown that the application of the proposed EoS to many molecular fluids gives good results upon extrapolation in the high pressure region or in the range of thermodynamic variables where the isothermal compressibility is low. The experimental data processing carried out for some substances has revealed the necessity of modification of some potential models in the high pressure region [2, 3]. The functional form of a repulsive potential is discussed on the base of results obtained by the processing of the high frequency asymptotics of the depolarised light scattering and temperature dependences for one-particle contributions to the selfdiffusion coefficient. The results yielded on the basis of these approaches correlate well. An analysis of the theoretical results is in a close agreement with the experimental data on selfdiffusion in liquid argon and molecular dynamics simulation. The special attention is paid to the high pressure EoS for water. Here we started from an improved version of the Stillinger-David polarization potential [4]. The effective potential, taking into account the influence of H-bonds has been constructed. The processing of PVT-data by the proposed EoS gives fairly good agreement at comparison with experimental data.

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## Weak Ergodicity breaking for anomalous diffusion of single molecules in live cells

E. Barkai†, Metzler, Burov

†Physics Department Bar Ilan University

Time averages of the mean square displacement of single molecules (e.g. mRNA) in the live cell exhibit anomalous diffusion and ergodicity breaking: the time averages are random hence the measurements are irreproducible. We address this issue within the framework of the continuous time random walk [1,2] and discuss the relation of the theory with recent experiments. For example the experiments of the group of Yuval Garini (PRL **103**, 018102 (2009)) on sub-diffusion of telomeres in the nucleus of a mammalian cell. We then show the relation between this type of weak ergodicity breaking and non-stationary behavior. In particular we show how aging correlation functions describing the ensemble averages (i.e. correlation functions which depend on two measurement times and not only on the time difference as is common in stationary processes) can be used to quantify the deviations from ergodic behavior when a time average over a single trajectory is performed. Then we address the problem of random walks in quenched environments, presenting a solution of the diffusion front of the quenched trap model [3]. The main difficulty here is that the random walk is correlated with the disorder and hence so far escaped solution (while the annealed version of the model was solved a long time ago). Our approach is based on a time transformation which generalizes the well known sub-bordination method. We show that the random walk in quenched environments can be mapped on a Brownian process stopped at an operational time  $\sum(n_x)^\alpha$  where  $n_x$  is the number of visits to lattice site  $x$  and  $\alpha$  is related to the disorder. With this new type of Brownian motion we handle the intricate correlation and solve the quenched trap model at-least in certain limits. For example in the limit of strong disorder (zero temperature) we recover the renormalization group solution of C. Monthus. Our method is an alternative to the renormalization group capable of dealing with any strength of disorder.

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## Percolation in networks composed of connectivity and dependency links

A. Bashan†

†Bar Ilan University

Many systems can be efficiently modeled using a network structure where the system entities are the network nodes and the relations between the entities are the network links. However, many systems are also characterized by small subgroups in which the entities belonging to a group strongly depend on each other. We coin the relation between each two nodes in such a group as dependency links. Networks composed from both connectivity and dependency links were found to be more vulnerable compared to classical networks with only connectivity links. Their percolation transition is usually of a first order compared to the second-order transition found in classical networks. Here, we analyze both analytically and numerically the general case of a network with different sizes of dependency clusters. We study networks with three different types of dependency groups: (a) fixed size  $s$  dependency groups, (b) normally distributed sizes of dependency groups and (c) Poisson distributed sizes of dependency groups. We find that, for random networks with an average degree  $k$  that are divided into dependency groups (clusters) of size  $s$ , the fraction of nodes belonging to the giant component  $P_\infty$  is given by  $P_\infty = p^{s-1} [1 - \exp(-kpP_\infty)]^s$ , where  $1 - p$  is the initial fraction of removed nodes. Our result for  $s = 1$  (a node depends only on itself) coincides with the known Erdős-Rényi (ER) equation,  $P_\infty = 1 - \exp(-kpP_\infty)$ , for a network without dependency relations. We also show that for  $s \geq 2$  a process of cascading failures occurs and the percolation transition is of first order. For normally distributed dependency groups with an average size  $\langle s \rangle$  and width  $\sigma$ , we find that the system becomes more stable (smaller  $p_c$ ) for a broader size distribution. When  $\sigma \rightarrow 0$ , the results are the same as the case of fixed size dependency groups with  $\langle s \rangle = s$ . We also analyze both analytically and numerically the case of a Poisson distribution of dependency cluster sizes and obtain analytical equation for  $P_\infty(avs)$ :  $P_\infty = f_{k,p}(P_\infty)e^{(\langle s \rangle - 1)(pf_{k,p}(P_\infty) - 1)}$ , where  $f_{k,p}(P_\infty) \equiv 1 - \exp(-kpP_\infty)$  and  $\langle s \rangle$  is the mean value of the size of dependency clusters.

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## Statistical mechanics of the vacuum

C. Beck†

†School of Mathematical Sciences, Queen Mary, University of London

Superstatistical techniques [1,2] can be very useful to analyse scattering data in high energy physics. The general idea of superstatistics is that there is a distribution of inverse temperatures, and suitable time-scale separation. For scattering processes, this usually means that for each recorded event there was a different temperature. The superposition of all these different temperatures can yield differential cross sections that exhibit power laws rather than exponential behaviour. But generalized statistical mechanics models can also enter at a much more fundamental level, when discussing structure and dynamics of the vacuum. In the so-called chaotic string approach [3,4] one postulates the existence of a fundamental entropic field, responsible for entropy production at a microscopic level, which is evolving in a chaotic way. In this approach the vacuum is regarded as a kind of selfsimilar network of spontaneous momentum fluctuations that are coupled with running standard model coupling constants at each energy scale. The entire vacuum consists of a superposition of all these different dynamical processes. There are two relevant observables for this model, the self energy and the interaction energy of the entropic field. These can be used to define generalized thermodynamic potentials. Numerical evidence is presented that the above thermodynamic potentials distinguish observed coupling parameters and masses of the standard model, in the sense that they correspond to states of minimum vacuum energy of the entropic field. Thus these types of models of the vacuum, taken as an additional sector, provide theoretical arguments why certain values of coupling parameters, mixing angles and masses are realized in the standard model, others are not.

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### Weak ergodicity breaking in anomalous diffusion models

G. Bel†

†Ben-Gurion University of the Negev

Recent advances in technology have enabled studying the dynamics of single molecules. Many experiments tracking single molecules have reported anomalous diffusion. What new insights can be revealed by these experiments which cannot be seen in bulk measurements? In order to extract new and useful information from those experiments, it is important to study the properties of anomalous diffusion models and to find characteristics which can distinguish between the different models. Continuous time random walk (CTRW) models are widely used to model diffusion in condensed matter. There are two classes of such models, distinguished by the convergence or divergence of the mean waiting time. Systems with a finite average sojourn time are ergodic and thus Boltzmann-Gibbs statistics can be applied. We investigated the statistical properties of CTRW models with an infinite average sojourn time; in particular, the occupation time probability density function is obtained. It is shown that in the non-ergodic phase the distribution of the occupation time of the particle on a given lattice point exhibits a bimodal U or trimodal W shape, related to the arcsine law. The key points are as follows: (a) In a CTRW with a finite or infinite mean waiting time, the distribution of the number of visits on a lattice point is determined by the probability that a member of an ensemble of particles in equilibrium occupies the lattice point. (b) The asymmetry parameter of the probability distribution function of occupation times is related to the Boltzmann probability and to the partition function. (c) The ensemble average is given by Boltzmann-Gibbs statistics for either finite or infinite mean sojourn time, when detailed balance conditions hold. (d) A non-ergodic generalization of the Boltzmann-Gibbs statistical mechanics for systems with an infinite mean sojourn time was found. Unlike in ergodic systems even at the infinite long time limit, the occupation times (and therefore the physical observables) are random quantities. We have also studied the concept of weak ergodicity breaking in the context of deterministic dynamics. We showed that weak ergodicity breaking describes a system whose dynamics is governed by a nonlinear map which generates sub-diffusion deterministically. I will also discuss another model for anomalous diffusion due to coupling between stochastic processes which can lead to ergodic or non-ergodic behavior for different coupling functions. In both models the single molecule properties provide an insight into the dynamical mechanism, which could not be obtained from ensemble measurements.

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### Non-Markov kinetic equation for quantum plasma with exchange interaction

V. Belyi†, Yu. A. Kukharenko

†IZMIRAN, Russian Academy of Sciences

The nonlinear kinetic equation taking dynamical screening due to plasma polarization into account for weakly coupled plasma was derived first time [1] by Balescu using Prigogine's diagram techniques and little bit later by Lenard who solved the Bogoliubov equation for the pair correlation function in the plasma approximation. These results were generalized to the quantum case by Balescu and Guernsey [2]. These kinetic equations are fundamental for plasma physics and are comparable with the Boltzmann equation for diluted gas. In the quantum Balescu kinetic equations the exchange interaction of particles was retained only in the distribution functions. But it is also necessary to take into account the exchange interaction in the scattering amplitude and in the dielectric function. Moreover, the Balescu equation (both classical and quantum) involves the polarization of the system only in the collision integral, while the thermodynamics corresponds to the ideal gas; the dissipative and non-dissipative phenomena are not treated on the equal footing. This discrepancy can be avoided if non-Markov effects are considered [3]. This way, we obtained a nonlinear kinetic equation which generalizes the Balescu equation for spatially uniform and spatially non-uniform weakly non-ideal polarizable plasma. This equation, which included the dynamical screening of the interaction potential, described correctly the conservation of the total energy in a nontrivial way. Starting from the quantum BBGKY-hierarchy for the statistical operators, we have solved, in the so-called plasma approximation, the equation for the quantum pair correlation function the non-Markov correction being included. The solution of this equation can be expressed in terms of the resolvent of the linear Hartree-Fock equation. As a result, we obtain a quantum non-Markov kinetic equation, which involves both the dynamical screening of the interaction potential and the exchange interaction in a non-trivial way [4]. In particular, this equation contains the dielectric function which exactly describes the exchange scattering in plasma. The quantum kinetic equation derived satisfies the law of total conservation with regard for the polarization and the exchange interaction. The similar non-trivial result was found for electric field fluctuations in the systems of the charged particles with exchange interaction.

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## The US market stays prone to systemic collapses

E. Ben-Jacob<sup>†</sup>, D.Y. Kenett

<sup>†</sup>Tel Aviv University

Following the recent period of unprecedented efforts to cure the market, it is widely believed that the financial crisis is under control and recovery is on its way. Could it be, as the analysis presented here suggests, that this is an illusion, and while the remedies alleviate the symptoms they do not cure the problem and can even render the market more prone to systemic collapse? Many economists link the current financial crisis to the collapse of the subprime bubble at the end of 2007. Whereas we show that the crisis began already at the end of 2001 and it might have been a direct consequence of the US hasty and drastic interest cuts and other remedies used to overcome the fallout effect of the "dot com" bubble collapse. Traditional financial variables such as national debts, unemployment, exchange rate and trading balance fall short of revealing early roots of the crisis. To reveal the early symptoms, we investigated the time dynamics of the S&P500 in terms of the Index cohesive force (ICF) - the balance between the raw stock correlations (Figure 1B) that include the Index effect and the bare stock correlations (or partial correlations) after subtraction of the Index effect (1,2). A strong cohesive force is likely to be a signature of stiff market with high propensity to systemic collapse, and as such is a symptom of an avalanche-prone market. By the same token, healthy market is associated with relatively high and diverse bare stock correlations, hence a weak cohesive force (weak Index dominance). Employing this analysis revealed (3) a fast dramatic transition at the end of 2001, from a healthy into abnormal market behavior (signified by a very strong cohesive force), that has lasted ever since. The anomalous dominance of the Index and the market dangerous stiffness during this period led to the emergence of market "seizure" behavior - bursts of very high stock correlations that usually coincide with local minima in the Index. The dramatic differences between the healthy and seizure market states are best manifested in the 3-dimension scatter plot whose axes are the averaged Stocks-Index correlations and the averaged stocks raw and bare correlations. Each dot in this space corresponds to a specific 22-day trading window. The plot makes transparent the fact that the market state was not determined by the Index trend (positive or negative): The avalanche-prone state started in the midst of a decline in the Index and continued unchanged as the trend changed several times.

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## Intermediate-state flux structures in mesoscopic type-I superconductors - effect of sample geometry and applied current

G. Berdiyrov<sup>†</sup>, G.R. Berdiyrov, A.D. Hernandez-Nieves, F.M. Peeters and D. Dominguez

<sup>†</sup>University of Antwerp, Department of Physics

We study the dynamic properties of intermediate state (IS) flux structures in type-I superconductors, which recently received considerable interest [1-3] as one of the rare systems where competing interactions lead to spatially modulated structures. Together with ordered tubular and laminar structures much more complex patterns have been observed in experiment [1]. Experiments [2, 3] have also revealed that an applied electric current further increases the complexity of the problem: (i) topologically constrained laminar patterns can split into highly mobile tubular structures, the size and the shape of them depends on the applied current; (ii) at higher currents the IS pattern evolves into equally spaced superconducting walls in the direction of the driving force. In spite of these experimental works, no theoretical reports are available in the literature that describe the complex mechanism of these topological transformations. Here, we implement the time-dependent Ginzburg-Landau (GL) theory to study the non-linear dynamics of the IS flux structures in current-carrying type-I mesoscopic superconductors. Among other problems of pattern, we address the following questions of fundamental interest: (i) what is the structure of the IS when the system is brought in non-equilibrium by an external current? (ii) how does the annihilation process takes place between the normal domains that contain flux of opposite polarity? Numerical results show that, depending on the applied current, the flux can penetrate the sample in the form of either tubes or stripes (laminae), the shape and the size of them are strongly dependent on the material properties, the sample dimensions and the edge imperfections. We also found that the annihilations process of normal domains containing opposite flux takes place in a discrete way releasing single-quantum vortices, which results in measurable traces in the voltage vs. time characteristics of the sample. We also studied the effect of sample geometry on the IS flux structures in mesoscopic type-I superconductors [4]. In addition to the well-established tubular and laminar structures, the strong confinement leads to the formation of a phase of singly quantized vortices, which is typical for type-II superconductors, and a ring of a normal domain at thermodynamic equilibrium. The stability region and the formation process of these IS patterns are strongly influenced by the geometry of the sample. In samples where the magnetic field is parallel to surfaces (e.g., cubes and disks) the normal domains are mostly located parallel to the surface, whereas in spheres and cones radial distributions of laminar structures are obtained.

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### Steepest-entropy-ascent non-equilibrium dynamics is thermodynamically irreversible but mathematically reversible

G.P. Beretta<sup>†</sup>

<sup>†</sup>Università di Brescia, Italy

We discuss the main general features of a maximum-entropy-production/steepest-entropy-ascent nonlinear evolution equation proposed in the 1980's [1] by the present author in the framework of a fully quantum theory of irreversibility and nonequilibrium thermodynamics for a single isolated or adiabatic particle, qubit, or qudit, and for composite systems. The same equation has been rediscovered recently by various authors [2]. The nonlinear equation generates a dynamical group, not just a semigroup like for Kossakowski-Sudarshan-Gorini-Lindblad quantum master equation. It is mathematically reversible in the sense that it has smooth unique solutions both forward and backwards in time for any arbitrary nonequilibrium state. Its geometrical construction [3] leads to very intriguing and general mathematical features. It provides a deterministic description of irreversible conservative relaxation towards thermodynamic equilibrium from an arbitrary initial nonequilibrium density operator. In the context of general (equilibrium and nonequilibrium) quantum thermodynamics (or statistical mechanics), it satisfies a very restrictive stability requirement equivalent to the Hatsopoulos-Keenan statement of the second law of thermodynamics, which therefore emerges as a general theorem of steepest-entropy-ascent dynamics. Except for fully characterized families of limit cycles and of stable and unstable equilibrium states, the entropy functional is strictly increasing in forward time and strictly decreasing in backward time. Steepest entropy ascent entails a generalization of Onsager's reciprocity theorem valid in the entire nonequilibrium domain (also this result has been rediscovered recently by others [4]). It even entails an intriguing time-entropy uncertainty relation [5], analogous to the well-known time-energy uncertainty relation valid under the standard unitary Hamiltonian dynamics. From the point of view of the long-standing discussions about the problem of accounting for the arrow of time and the origin of irreversibility within statistical mechanics and its time symmetric underlying dynamics, our nonlinear steepest-entropy-ascent dynamical law implements at the single particle level the maximum entropy production principle. It also provides a clear demonstration that to describe a largely irreversible physical evolution we do not need to assume a mathematically irreversible dynamics.

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### A model of binary opinion, static and dynamical phase transition and dynamical macroscopic crossover.

S. Biswas<sup>†</sup>

<sup>†</sup>Department of Physics, University of Calcutta

A new model of binary opinion (model I) is proposed in one dimension in which the dynamics is dependent on the size of the neighbouring domains. If the neighbouring domains have opposite opinions, then the opinion of the domain with the larger size is followed. An individual also changes his/her opinion when both the neighboring domains have an opinion which opposes his/her original opinion, i.e., the individual is sandwiched between two domains of same polarity. Starting from a random configuration, the system evolves to a homogeneous state. The dynamical evolution shows a scaling behavior with the persistence exponent  $\theta \simeq 0.235$  and dynamic exponent  $z \simeq 1.02 \pm 0.02$  [1]. The model is generalised introducing a size sensitivity parameter 'p' to modify the dynamics such that a spin can sense domain sizes up to  $R = pL/2$  in a one-dimensional system of size L. For the cutoff factor  $p \rightarrow 0$ , the dynamics is Ising like and the domains grow with time  $t$  diffusively as  $t^{1/z}$  with  $z = 2$ , while for  $p = 1$ , the original model I showed ballistic dynamics with  $z \simeq 1$ . A ballistic to diffusive crossover is obtained for intermediate values of 'p'. For intermediate values of  $p$ , the domain growth, magnetization and persistence show model I like behaviour up to a macroscopic crossover time  $t_1 \sim pL/2$ . Beyond  $t_1$ , characteristic power law variations of the dynamic quantities are no longer observed. The total time to reach equilibrium is found to be  $t = apL + b(1-p)^3L^2$ , from which we conclude that the later time behaviour is diffusive. We also consider the case when a random but quenched value of  $p$  is used for each spin for which ballistic behaviour is once again obtained [2]. Introducing disorder to Model I through a parameter called rigidity coefficient  $\rho$ , the transition to a heterogeneous state at  $\rho = 0+$  is obtained. Close to  $\rho = 0$ , the equilibrium values of the dynamic variables show power-law scaling behavior with  $\rho$ . The effect of having both quenched and annealed disorder in the system is also considered. The model (Model I) can be mapped to a system of random walkers with a tendency to walk towards their nearer neighbour. Introducing a parameter  $\epsilon$  controlling this tendency we show that there exists three different dynamical behaviour as  $\epsilon$  is varied. A dynamical phase transition is observed at the point  $\epsilon = 0.5$  [3].

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## Monte Carlo simulation of spin-glass properties of amorphous magnets with random anisotropy

A.V. Bondarev<sup>†</sup>, V.V. Ozherelyev, I.L. Bataronov, Yu.V. Barmin

<sup>†</sup>Voronezh State Technical University

The molecular dynamics models of atomic structure of the Re-Tb amorphous alloys and of pure amorphous Tb were constructed. Each model contained 100 000 atoms in a cubic cell with periodic boundary conditions. Interatomic interaction was described by a model polynomial potential [1]. Using the Monte Carlo method in the frame of the Heisenberg model, simulation of magnetic properties of these models were carried out. The Hamiltonian contains the following terms [2]: exchange interaction between the nearest neighbouring Tb atoms  $J$ , local uniaxial random anisotropy  $D$ , and the applied magnetic field. Dependence of the exchange integral on the interatomic distance  $r$  was fitted by a decreasing linear function. The temperature dependencies of spontaneous magnetization, Edwards-Anderson order parameter and magnetic susceptibility were calculated with different values of the  $D/J$  ratio. At  $D/J > 8$  and  $x > 13$  at. Tb, the transition to the spin-glass state was observed [3]. The temperatures of the spin-glass transition were determined as positions of maximums on the temperature dependence of magnetic susceptibility. The magnetic phase diagrams for the pure amorphous Tb and for the Re-Tb amorphous alloys were constructed. With increasing concentration of Tb atoms the transition temperature linearly increases, which is in a good agreement with the experimental results. We also studied the behaviour of the model of pure amorphous Tb under the external magnetic field and calculated the magnetization curves, hysteresis loops, remanent magnetization, coercive field at different temperatures and different values of the  $D/J$  ratio. With high values of  $D/J$  these materials show high coercivity at low temperatures and do not reach magnetic saturation even under very high magnetic fields up to  $1000kOe$ . The FC and ZFC temperature dependencies of magnetization were also calculated. The magnetization relaxation after switching-off the external magnetic field at different values of  $D/J$  was studied. The spin-spin correlation functions were also calculated. These results qualitatively agree with the experimental results obtained for amorphous alloys based on rare-earth metals. This work was supported by Russian Foundation for Basic Research (Grant N 09-02-97503) and by Russian Ministry of Science and Education (Grant N RNP 2.1.1/4406).

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## Generalized Reaction-Diffusion: a microscopic approach

J.P. Boon<sup>†</sup>

<sup>†</sup>Université de Bruxelles, Belgium

The random walk is the classical paradigm for the microscopic mechanism underlying diffusive processes as demonstrated in 1905 by Einstein who showed how the diffusion equation follows from the mean field formulation of the microscopic random walk [1]. When one considers systems where the diffusing particles are subject to a reactive process, diffusion and reaction are coupled and reaction-diffusion (R-D) phenomena are described at the macroscopic level by R-D equations. For instance, for an annihilation process such as  $A \rightarrow 0$ , the classical R-D equation reads [2]  $\frac{\partial}{\partial t} f(r; t) = D \frac{\partial^2}{\partial r^2} f(r; t) - k f(r; t)$  where  $D$  denotes the diffusion coefficient and  $k$  the reactive rate. The classical equation yields a steady state solution showing exponential decay in space  $f(r) = f(0) \exp(-\sqrt{\frac{k}{D}}|r|)$ . However there are many systems observed in nature where it seems logical to use the language of reaction-diffusion, but where non-classical distributions are observed: the steady state spatial distribution is non-exponential e.g. when the particles encounter obstacles (labyrinthine geometry) or because the reactive process is hindered or enhanced by concentration effects. So a general description of R-D phenomena requires a generalization for both diffusion and reaction. Following the philosophy of the generalization of diffusion processes [3], we develop a microscopic approach by generalizing Einstein's master equation including a reactive term and we show how the mean field formulation leads to the nonlinear R-D equation with non-classical solutions. For the  $n$ -th order annihilation reaction  $A + A + \dots + A \rightarrow 0$ , the generalized reaction-diffusion equation (with no drift) reads  $\frac{\partial}{\partial t} f(r; t) = \frac{\partial}{\partial r} K D \frac{\partial}{\partial r} f^\alpha(r; t) - \kappa k f^n(r; t)$  where  $K$  and  $\kappa$  are dimensional parameters. We discuss scaling and non-scaling formulations and the corresponding range of values of the nonlinear exponents  $\alpha$  and  $n$ . We obtain typical steady state solutions  $f(r) = f(0)(1 + C(D, k) \frac{r}{\nu})^{-\nu}$  where  $\nu = 2/(n - \alpha)$ . Redefining the exponent as  $\nu = 1/(q - 1)$ , the solution takes the canonical form of a  $q$ -exponential. For  $q > 1$ , we find long range power law behavior showing the relative dominance of (sub)diffusion over reaction effects; we discuss under which conditions the converse is found, leading to finite support of  $f(r)$  indicating that diffusion is slow and extinction is fast.

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## Multiplicative noise, moment scaling, and fast convolution

G. Borometti<sup>†</sup>, D. Delpini, S. Cazzaniga

<sup>†</sup>Scuola Normale Superiore, Pisa and INFN, Pavia Division, Italy

Many different physical phenomena exhibit a complex behavior characterized by long-range correlations, long-time memory, scale invariance, and the emergence of non Gaussian distributions associated to their statistical description. Deviations from the Maxwell-Boltzmann statistics were usually considered as a clear mark of an out of equilibrium system, but in the last years it has been recognized that Normality is not the most general paradigm describing the equilibrium state. Indeed, in terms of a microscopic description provided by the Langevin equation, power-law tails stem naturally assuming the damping coefficient to have a stochastic nature. From a macroscopic point of view, the superposition of an additive Gaussian noise with a multiplicative one leads to a Fokker-Planck equation with linear drift and quadratic diffusion coefficients. In this talk we provide a full description of these processes in terms of their moments, allowing for a quite general time dependence for both coefficients. In particular, we characterize analytically how they thermalize to the stationary state and we highlight the existence of a direct, simple relationship between the function regulating the time dependence and the scaling of the process over time. This class of processes has also been proven to be very successful when describing the time evolution of stochastic volatility in the context of financial time series modeling. It is also extremely convenient from a computational point of view. Indeed the quadratic diffusion can be formally manipulated by means of the Lamperti transform to reduce it to a suitable form allowing the use of fast algorithms for computing integrals in function space. In this respect it is known that one of the relevant problems when dealing with stochastic processes consists in reconstructing conditional probability density functions. One of the most widely exploited techniques is probably the Monte Carlo one, but it suffers some drawbacks, not the last being the poor estimate of tail regions. In this talk we present numerical evidences of the advantages provided by the Fast Convolution Algorithm proposed by Eydeland applied to the class of processes under consideration. If any time will remain, we draw some perspectives concerning pricing of financial derivatives.

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## A Density Functional Approach to Adsorption of Mixtures on Surfaces Modified with End-Grafted Polymers

M. Borowko<sup>†</sup>, S. Sokolowski, T. Staszewski

<sup>†</sup>M. Curie-Skłodowska University, Department for the Modelling of Physico-Chemical Processes, Lublin, Poland

Our research is motivated by numerous practical applications of adsorption on surfaces modified with end-grafted chains, such as stabilization of colloids, adhesion, chromatography, drug delivery, etc. We study adsorption of binary mixtures consisting of spherical molecules or short chains on the polymer brushes. The chain molecules are modeled as freely jointed spheres. All segments of grafted polymers but the binding segment interact with the solid surface via the hard-wall potential. However, free molecules are attracted by the wall (Lennard-Jones (9-3) potential). The Lennard-Jones (12-6) potential is used for modeling interactions between all spherical species. The approach is based on the theory proposed by Yu and Wu [1] and extended to the polymer brushes in our previous papers [2-5]. The excess free energy due to hard-sphere interactions is calculated according to the fundamental measure theory, the chain connectivity is described using the first-order perturbation theory of associating fluids and the contribution resulting from attractive interactions is obtained within the mean field approximation. The thermodynamic potential of the system is calculated and minimized to obtain the density profiles of components. We show how selected factors affect adsorption, the structure of surface layer and the selectivity of the system. We study an influence of such parameters as: the grafting density, the strengths of molecular interactions, the length of grafted chains and the sizes of adsorbed molecules. The segment density profiles of the brush and density profiles of mixture components are calculated and carefully analysed. The mechanism of adsorption process depends upon relations between parameters characterizing the system. Depending on the strength of adsorbate-adsorbent interactions and the grafting density one observes primary, secondary or ternary adsorption. The type of adsorption considerably depends on the size and shape of adsorbed molecules. A competition between entropic repulsion of the grafted chains and attractive forces exerted by the wall and the bonded polymers plays a fundamental role in this process. The structure of the surface layer depends strongly upon details of the model assumed and the thermodynamic state of the system. We investigate adsorption from gas and liquid mixtures. In particular, we present theoretical excess adsorption isotherms from binary solutions. For certain sets of parameters adsorption azeotropy is observed. The results are qualitatively compared with experimental data measured for the systems used in liquid adsorption chromatography with chemically bonded phases.

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## Noise-assisted tumor-immune cells reaction

T. Bose<sup>†</sup>, S. Trimper

<sup>†</sup>Martin-Luther-University Halle, Germany

Tumor growth has become an important issue in medicine, biology and physics. The understanding of cancer growth mechanisms is necessary to develop relevant strategies against the disease. In the past, deterministic models have been proposed for interacting tumor and immune cells which are analyzed by performing stability analysis [1,2]. Further, stochastic models were presented addressing the inclusion of noise. This class of models has been investigated in terms of the related Fokker-Planck equation for white [3] and for colored noise [4]. These random models are characterized by considering only one cell type whereas the interaction between the tumor cells and the immune system is hidden in the model parameters. In the present work it will be demonstrated that immune cell - tumor cell reactions can be induced by stochastic forces. The basic model describes the time evolution of three different cell types: (i) tumor cells the density of which is denoted by  $x(t)$ , (ii) effector cells with density  $y(t)$  which can eliminate the tumor cells and (iii) tumor detecting cells  $z(t)$ . The last kind of cells is only able to recognize tumor cells but they have not the ability to kill them. In the noise-free case the appropriate mathematical model offers no stable tumor-free state. Due to the inclusion of randomness the growth and death rates of the immune and tumor cells, respectively, are altered immediately. Moreover, a birth term for the tumor detecting cells with the density  $z$  is induced by noise correlations. Here we consider distinct noise sources which couple up to the different cell types, i.e. the model contains multiplicative noise. An additional feature is that the auto-correlation functions possess a finite correlation time and moreover, non-zero cross-correlation functions are taken into account. The system is investigated by means of an approximated Fokker-Planck equation for the multivariate case which allows getting the evolution equations for the mean values of the above introduced densities. As a result we find a stable tumor-free state which is solely induced by the randomness. In particular, we detect that the strength of the noise correlation function related to the effector cells is the decisive quantity which guarantees the stability of the tumor-free state. Likewise the impact of the other noise parameters is discussed.

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## Limited Resources in an asymmetric exclusion process: A model of protein biosynthesis

C. Brackley<sup>†</sup>, M.C. Romano, M. Thiel, C. Grebogi

<sup>†</sup>University of Aberdeen

The totally asymmetric simple exclusion process (TASEP) has been applied to many systems, including messenger RNA translation in biology. The uni-directional hopping of particles on a 1D lattice is used as a model for the movement of ribosomes (molecular machines) along RNA strands - the molecules which contain the genetic code for the amino acid sequences of proteins. The TASEP is a well studied model in non-equilibrium statistical mechanics, and exhibits rich dynamics including multiple phase transitions. Depending on the relative size of the particle entry and exit rates and the internal hopping rates the system can be in either a low density (LD), high density (HD) or maximal current phase, or an LD/HD coexistence or shock phase (SP). I will discuss a recent extension to the TASEP, in which the hopping rate for particles depends on the availability of external resources. In the context of translation, these resources are the tRNA molecules that carry the amino acids to the ribosomes. We show that introducing variable hopping rates gives rise to a regime in which the particle current is limited by the availability of resources. That is to say, the current becomes constrained when the rate of resource use approaches the rate of resource replenishment; we call this the limited resources or LR regime, and it occurs within the phases found in the original TASEP model. When biologically realistic parameters for translation are used the onset of the LR regime is sharp, and the locations on the phase plane of the transitions between LD or HD and MC phases are shifted. The maximum current which the lattice can support is severely reduced. We use Monte Carlo simulations and mean field models to examine homogeneous lattices, and lattices with slow defect sites. We also consider inhomogeneous lattices where the hopping rate for different sites depends on the availability of different resources, and we treat different such lattices in contact with the same pool of resources. In this case the mean field model breaks down, but we can still predict analytically when particle queues will occur. Finally we discuss how this model can be combined with other models to give a more realistic representation of mRNA translation, and also how it can be used for other applications such as molecular transport in cells.

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## Random Matrix Analysis of Financial Data

Z. Burda†

†Jagiellonian University

We discuss various applications of the concept of free random variables and random matrices to multivariate analysis in quantitative finance. This type of analysis is very useful for determining cross-correlations and risk factors in a stock market from historical data and thus if finds a direct application to portfolio selection which is a central problem in quantitative finance. Portfolio theory is nowadays widely used by financial institutions that recently also started to apply methods of random matrices to achieve efficient diversification. We briefly explain the link between portfolio selection and random matrices and sketch the main ideas behind free probability calculus which is a very powerful tool to determine eigenvalue densities of large matrices. Free probability theory is an extension of classical probability theory to the case of non-commuting variables. Such non-commuting variables can be naturally represented as large matrices. Freeness in free probability is a counterpart of independence in classical probability. Two given matrices are free if they are independent and if they have no angular correlations. In the limit of infinite matrix size one can explicitly derive laws of free addition and multiplication which allow one to determine the eigenvalue density of a free sum or a free product of matrices. They are very useful in solving problems in multivariate analysis and nowadays are applied in wireless telecommunications, information theory, financial engineering and many other areas of research. Laws of addition and multiplication for free random matrices can be derived using  $1/N$  expansion known from statistical mechanics and methods of summation for planar Feynman diagrams. We show how to use these relations to derive exact relations between the eigenvalue density of historical estimators of covariance matrix including EWMA (that is exponentially weighted moving average) and the genuine covariance for a class of linear models of multidimensional time series, including VARMA (that is vector autoregressive moving average) models. We shall also derive lagged correlation function for uncorrelated assets. This last result can serve as a reference (null model) when one considers lagged correlation matrices in presence of temporal correlations.

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## Maximal entropy random walk

Z. Burda†

†Jagiellonian University

We discuss random walk on graphs defined by the maximal entropy principle applied to a set of trajectories between given endpoints as contrasted to ordinary random walk which maximises entropy locally. The two types of random walk processes are identical on regular graphs but in general they behave differently. In particular maximal entropy random walk on an almost regular grid with weak dilution, obtained by removing edges at random, exhibits localisation. More precisely, the probability of finding the particle in the stationary state is localised in a small part of the graph. For low dimensional lattices the localisation region, which corresponds to the largest region free of defects, has a spherical shape of radius proportional to the logarithm of the linear extension of the lattice. This region is known as Lifshitz sphere and has the same origin as the region of localised states in theory of random operators. Maximal entropy random walk provides a purely classical example of localisation but it bears resemblance to localisation known from quantum physics. We derive explicit equations for the stochastic matrix for the underlying Markov chain and its stationary probability distribution. Both these objects are expressed in terms of the Frobenius-Perron eigenvector and its eigenvalue of the adjacency matrix. We discover an interesting relation of the corresponding equations to the problem of finding a ground-state of a related Schroedinger equation. We discuss the heat-kernel and stochastic quantisation for this new type of random walk. Since all trajectories of the same length and of given endpoints generated by the maximal entropy random walk are equiprobable this type of random walk is a good candidate to define a quantum propagator of free particle in a curved background (represented by random graph) in the Feynman formulation of quantum mechanics as a sum of equiprobable trajectories. We shortly discuss this issue. Maximal entropy random walk on a lattice with weak dilution is also expected to exhibit two types of dynamics: a fast and a slow one - which are characteristic for ageing phenomena. The fast one corresponds to thermalisation within a local region free of defects while the slow one to jumps between the spheres before the particle eventually gets stuck in the largest one which corresponds to the largest Lifshitz sphere. This may serve as a model of entropy barriers.

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## Data collapse, scaling functions and new analytical solutions of generalized growth models

B. Cabella<sup>†</sup>, F. Ribeiro, A.S. Martinez

<sup>†</sup>University of São Paulo

Growth models are useful when one tries to understand, describe or predict the behavior of a wide range of time dependent processes in physics, chemistry, demography, economy, ecology, epidemiology, just to cite a few disciplines. The simplest way to deal with population growth is to consider that their individuals do not explicitly interact with external ones. This is represented by the so-called one-species population dynamics models. In this study, data collapse, scaling functions, phase transition and critical exponents are used in population growth models. These statistical physics concepts are widely used in many other research areas. Nowadays, the idea of data collapse has been extended to scaling functions, which is one particular aspect of the scaling hypothesis. The scaling hypothesis also deals with the scaling laws at critical transition values (such as in the van der Waals mean field theory) and renormalization group theory, but this latter aspect is out of the scope here. From the analytical solutions obtained from the models that we address, we call attention to the following items. For each model, the time  $t$  of the population evolution is proportional to the inverse of the intrinsic growth rate. This gives rise to a dimensionless characteristic time and can be defined as the system independent variable  $\tau$ . Furthermore, we demonstrate how to obtain the scaling function that depends on the combination of quantities such that the models become independent from the initial condition and parameters. A data collapse, from the scaling function, occurs even in the Tsoularis-Wallace model, where no explicit analytical solution is known. We also deal with models which consider an extrinsic growth rate, defined as an addition or removal of individuals proportionally to the population size. It can also be seen as the interaction factor when considering multispecies models. Furthermore, in cancer growth mathematical models, the extrinsic growth rate can be associated with treatment and the knowledge of how the transition between survival and extinction occurs is of utmost importance. The steady state (asymptotic) solution is interpreted as the order parameter. Non-trivial transitions between extinction and survival phases are found and depend on the extrinsic growth rate with well defined critical exponents.

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## Ranking and clustering countries and their products; a network analysis

G. Caldarelli<sup>†</sup>, M. Cristelli, A. Gabrielli, L. Pietronero, A. Scala, A. Tacchella

<sup>†</sup>Istituto dei Sistemi Complessi CNR

Networks emerged in the recent years as the main mathematical tool for the description of complex systems. In particular, the mathematical framework of graph theory made possible to extract relevant information from different biological and social systems. In this paper we use some concepts of network theory to address the problem of economic complexity. Such activity is in the track of a long-standing interaction between economics and physical sciences and it follows a recent analysis done on the network of trades between nations [1,2]. The authors of the above papers address the problem of competitiveness and robustness of different countries in the global economy by studying the differences in the Gross Domestic Product and assuming that the development of a country is related to different "capabilities". While countries cannot directly trade capabilities, it is the specific combination of those capabilities that results in different products traded. More capabilities are supposed to bring higher returns and the accumulation of new capabilities provides an exponentially growing advantage. Therefore the origin of the differences in the wealth of countries can be inferred by the record of trading activities analyzed as the expressions of the capabilities of the countries. Here we applied methods of graph theory to the analysis of the economic productions of countries. The information is available in the form of a rectangular matrix  $M$  of size  $N_c$  times  $N_p$ , giving the different production of the  $N_p$  goods for each country in the  $N_c$  list. This can be transformed with some linear algebra into a country-country and product-product network. In these latter forms, by using complex-networks analysis, we can attain an effective filtering of this information. We introduce here a new community finding algorithm that identifies cliques of countries connected by their common production. As an unexpected result, this analysis shows that neighboring countries tend to compete over the same markets. We also show that a classification of goods based on such community detection provides a bottom up taxonomy that can be used to determine countries activity. By defining a measure of success inspired by Google PageRank algorithm [3], we can see that this CountryRank quantity is a good proxy of the country GDP (eventually optimized with a Biased Random Walk procedure). Analysis seems to suggest that a careful tuning of products produced internationally might have a large positive effect for undeveloped countries.

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### On the decay exponents of mode coupling theory

F. Caltagirone†, U. Ferraria, L. Leuzzi, T. Rizzo, G. Parisi

†"Sapienza" Universit di Roma

An important prediction of Mode-Coupling-Theory (MCT) is the relationship between the decay exponents in the  $\beta$  regime:

$$\frac{\Gamma^2(1-a)}{\Gamma(1-2a)} = \frac{\Gamma^2(1+b)}{\Gamma(1+2b)} = \lambda$$

In the original structural glass context this relationship follows from the MCT equations that are obtained making rather uncontrolled approximations. As a consequence it is usually assumed that the relationship between the exponents is correct while  $\lambda$  has to be treated like a tunable parameter. On the other hand it is known that in some mean-field spin-glass models the dynamics is precisely described by MCT. In that context  $\lambda$  can be computed exactly, but again its computation becomes difficult when we consider more complex models including finite-dimensional ones. What was unknown up to now was the physical meaning of  $\lambda$ , *i.e.* its connection to physical observables. Recently, it has been shown that  $\lambda$  is related to the static replicated Gibbs free energy which, in its simplest form reads

$$G(Q) = \frac{\tau}{2} \sum_{\alpha\beta} (Q_{\alpha\beta})^2 + \frac{w_1}{3!} \sum_{\alpha\beta\gamma} Q_{\alpha\beta} Q_{\beta\gamma} Q_{\gamma\alpha} + \frac{w_2}{3!} \sum_{\alpha\beta} (Q_{\alpha\beta})^3 + \dots$$

through the following simple formula  $\lambda = \frac{w_2}{w_1}$ . The coefficients  $w_1$  and  $w_2$  can also be related to physical observables which are indeed measurable in real experiments or numerical simulations. We have tested through analytical computations the connection between the MCT exponents  $a$  and  $b$  and the cubic cumulants  $w_1$  and  $w_2$  in many different mean field models. We have compared our results from static computations with analytical results derived in a purely dynamical framework, when available, and with results from numerical simulations otherwise. When dynamical exact results exist they coincide with ours, as in the case of spherical  $p$ -spin or the SK model and, when only simulations are available, there is still good agreement despite the dynamical exponents are quite difficult to determine from numerical data. Our current work is focused on verifying this relation in non-fully-connected models and structural glasses, but computations in this case are more involved and still in progress.

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### A theory for Morphogenesis in the Deterministic Abelian Sandpile

S. Caracciolo†, G. Paoletti, A. Sportiello

†Dip. Fisica dell'Universit  di Milano and INFN

Since the appearance of the master-piece by D'Arcy Thompson, there have been many attempts to understand the complexity and variety of shapes appearing in Nature at macroscopic scales, in terms of the fundamental laws which govern the dynamics at microscopic level. Because of the second law of thermodynamics, the necessary Self-Organization can be studied only in non-equilibrium statistical mechanics. In the context of a continuous evolution in a differential manifold, the definition of a shape implies a boundary and thus a discontinuity. This explains why catastrophe theory, the mathematical treatment of continuous action producing a discontinuous result, has been developed in strict connection to the problem of Morphogenesis. More quantitative results have been obtained by the introduction of stochasticity, as for example in the diffusion-limited aggregation, where self-similar patterns with fractal scaling dimension emerge which suggest a relation with scaling studies in non-equilibrium. Cellular automata, that is, dynamical systems with discretized time, space and internal states, were originally introduced by Ulam and von Neumann in the 1940s, and then commonly used as a simplified description of phenomena like crystal growth, Navier-Stokes equations and transport processes. They often exhibit intriguing patterns, and, in this regular discrete setting, shapes refer to sharply bounded regions in which periodic patterns appear. Despite very simple local evolution rules, very complex structures can be generated. The well-known Conway's Game of Life can perform computations and can even emulate an universal Turing machine. The Abelian Sandpile Model, seen as a deterministic lattice automaton on periodic graphs, generates complex patterns displaying morphogenesis, allometry, and fractal self-similarity. On the plane, these patterns include patches, periodic in both dimensions, and strings, defect-lines over patches, periodic in one dimension. Both structures show combinatorial conservation laws, reminiscent of physical conservations for 'vacua' and 'propagators' in Integrable Systems. We classify completely their constituents in terms of their principal periodic vector  $k$ , that we call momentum. We derive a simple relation between the momentum of a string and its density of particles,  $E$ , which is reminiscent of a dispersion relation,  $E = |k|^2$ . Strings interact: they can merge and split and within these processes momentum is conserve. We reveal the role of the modular a group  $SL(2, Z)$  behind these laws. Here we classify completely these structures, by analysing a suitable discrete-valued Heat Equation.

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## Quasiparticle fluctuation power spectra in layered superconductors

A. Carbone<sup>†</sup>, V. Andreoli

<sup>†</sup>Physics Department - Politecnico di Torino

Several phenomena occurring in superconductors are intimately related to the existence of naturally or artificially formed layered structures. Cuprate superconductors have a perovskite layered structure with the superconductivity taking place mostly at the Cu<sub>2</sub>O planes, while the other layers, doped through chemical processes, or irradiation by photons or application of electric fields, act as charge reservoir resulting in a periodically varying quasiparticle density similarly to the modulation-doped semiconductor heterostructures. In low-temperature superconductors, when the current increases above  $I_c$ , the voltage increases in a series of rather regular steps corresponding to the emergence of resistive stripes, with the phase of the superconducting order parameter increasing at different rate on the stripe. Such structures are attractive also because of their potential of developing new devices such as field effect transistors, switches based on the normal-superconductive transition driven by electric fields at complex oxide heterostructure, highly sensitive detectors where the onset of resistive layers due to photon irradiation results in a voltage step at the contact ends. The model is based on a quasiparticle continuity equation accounting for the inhomogeneity due to naturally or artificially formed superconductive stripes or insulating layers leading to a nonuniform quasiparticle density distribution through the superconductor. Nonequilibrium distributions of quasiparticle and phonons are generated when the layered structure is perturbed by an external source of energy: by injecting quasiparticle directly into the material or by exposing the film to photons or phonons. The study of noise provides a great opportunity for analysing the quasiparticles dynamics in layered superconductors. Fluctuation phenomena have been extensively investigated in conventional and unconventional superconductors both theoretically and experimentally. Multiple noise peaks have been observed during the resistive transition in cuprates. The low-temperature peaks have been related to vortex motion. The origin of peaks observed at higher temperature and in the normal state are not completely understood. The intensity of noise in the normal state of high- $T_c$  superconductors is orders of magnitude higher than in metals with comparable values of the resistance. Such noise in excess has been related to hopping and trapping-detrapping of carriers in the interlayer spacings and greatly influenced by the number of cells in the sample.

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## Self-similarity of higher-order moving averages

A. Carbone<sup>†</sup>, S. Arianos, C. Turk

<sup>†</sup>Physics Department - Politecnico di Torino

Long-range correlations are found in several phenomena, observed for example in biological and physiological, economical and financial, solar, geophysical, environmental and solid state systems. The correlation degree is quantified in terms of scaling exponents by using measures suitably defined for such systems. The accuracy, velocity and simplicity in connection with local and causal operability of such estimators are crucial especially when real-time applications are concerned. For nonstationary time series, the detrended fluctuation analysis (DFA), the rescaled range analysis (R/S) and the detrending moving average (DMA) analysis can be adopted to explore long-range autocorrelations and multifractal features. Several experiments have been carried out to compare the performance of those methods. The DMA operates in continuous way over the signal with the advantage that a division of the time series in boxes is not needed compared to DFA and R/S. We introduce higher-order moving average polynomials for long-range correlated series. The self-similarity (scaling properties) of the moving average polynomials are systematically analysed and quantified in terms of the Hurst exponent  $H$  by using the Detrending Moving Average method. Further, we provide a proof that asymptotically the exponents  $H$  of the fractional Brownian motion and of the Detrending Moving Average agree. The asymptotic values are compared with the results obtained by the simulations. The asymptotic behavior of  $\sigma_{DMA}^2$  yields a closed-form approximation of the scaling law of the form:  $\sigma_{DMA}^2 = C_H n^{2H}$  at large  $n$ . The motivation of working out the asymptotic expression is to prove that the self-similarity index of the time series is kept unchanged after implementing each step of the algorithm. Finally, we compare the analytical results of the asymptotic behavior, with those obtained by numerical estimation. The theoretical values of  $C_H$ , are compared with those obtained by the intercepts of the curves. Since the fractional Brownian motions have been generated by the RMD algorithm, in the calculation of  $C_H$  it must be kept in mind that  $\sigma^2 = (1 - 2^{2H-2})/2^{2H\nu} \sigma_{Gauss}^2$ ,  $\nu$  being the number of steps and  $\sigma_{Gauss}^2$  the variance defined for the RMD algorithm.

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## Anomalous dynamics in DNA hairpin closing times

E. Carlon<sup>†</sup>, A. Ferrantini

<sup>†</sup>Theoretical Physics, KULeuven

We investigate by means of Monte Carlo simulations the zipping and unzipping dynamics of two polymers connected at one end and subject to an attractive interaction between complementary monomers. The simulations are performed on a lattice and the polymers follow Rouse dynamics (where the binding/unbinding between monomers fulfill detailed balance). In zipping, the polymers are quenched from a high temperature equilibrium configuration to a low temperature state, such that the two strands zip up by closing up a 'Y'-fork. In unzipping, the polymers are brought from a low temperature double-stranded configuration to high temperatures, such that the two strands separate. Simulations show that the unzipping time,  $\tau_u$ , scales as a function of the polymer length as  $\tau_u \sim L$ , while the zipping is characterized by anomalous dynamics with a scaling  $\tau_z \sim L^a$  with  $a = 1.37(2)$ . We find that this anomalous exponent is robust against variations of parameters and temperature. This exponent is in good agreement with simulation results and theoretical predictions for the scaling of the translocation time of a forced polymer passing through a narrow pore. Although the geometrical setups are different, zipping and translocation share thus the same type of anomalous dynamics. This analogy will be discussed during the talk. Systems where this dynamics could be experimentally investigated include DNA (or RNA) hairpins: our results imply an anomalous dynamics for the hairpins' closing times, but not for the opening times. The kinetic of hairpin formation has been studied more than a decade ago by means of correlated fluorescence spectroscopy in DNA sequences which contain self-complementary end nucleotides. In the closed (zipped) state the sequences form a double-helical stem ending with a loop. Experiments showed that the closing rate depends strongly on the loop length, but the dependence of the rates on the stem length has not yet been experimentally investigated. We expect that a non-trivial anomalous exponent could be observed in the closing dynamics of sequences with varying stem length. Another aspect which will be discussed in the talk is the zipping/unzipping dynamics at the critical point. This dynamics turns out to be characterized by an anomalous exponent as well.

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## Critical phenomena in heterogeneous k-core percolation

D. Cellai<sup>†</sup>, A. Lawlor, K. A. Dawson, J. P. Gleeson

<sup>†</sup>Department of Mathematics and Statistics, University of Limerick, Ireland

The k-core architecture of complex networks often gives important insights into the topological properties of real networks. A k-core is defined as the network subset which survives after a culling process consisting in removing recursively all the vertices (and adjacent edges) with less than k neighbours. This procedure is repeated until no vertex can be extracted any more. The process can have two outcomes: either all the vertices have been removed or some are left. In the latter case, the nodes left constitute the k-core. Particularly interesting is the outcome that a k-core analysis can give to understand in the resilience of networks under random damage. A few years ago, an analytical formalism has been developed to calculate k-cores in randomly damaged networks described by the configuration model, i.e. a random network defined only by its degree distribution [1,2]. Later, theoretical analysis has developed further and has been focusing on resilience of different topologies and related dynamical processes [3]. A recent work by G. J. Baxter et al. [4] has extended the concept of k-core to mixtures of vertices types, proposing, for configuration model type of networks, an analytical formalism to deal with heterogeneous k-cores. In this model, each vertex has a given internal rule which prescribes the maximum number of edges that the vertex must be connected to in order to be extracted in the culling process. In our work, we focus on binary mixtures of vertices types and study some relevant cases. We first show that the formalism by Baxter et al. can be simplified, under quite general conditions (namely a minimum value of k and a degree distribution with finite second moment), by virtue of the uniqueness of the heterogeneous k-core in infinite networks. Then we study some relevant phase diagrams presenting new critical phenomena. In particular, we identify a new scaling scenario and calculate critical exponents which are different from the ones of known percolation transitions. The same critical phenomena can be found on a regular lattice as well. As a result, we are able to describe a percolation-type model which supports a smooth interpolation between classical percolation and a genuine first order phase transition. This result gives insights into the percolation problem and constitutes a step towards a more effective infrastructure network protection.

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### Simple model of magnetization processes in rare-earth tetraborides

H. Cencarikova<sup>†</sup>, P. Farkasovsky

<sup>†</sup>Institute of Experimental Physics, Slovak Academy of Sciences

We present a simple model for a description of magnetization processes in rare-earth tetraborides. The model is based on the coexistence of two subsystems, and namely, the spin subsystem described by the Ising model and the electronic subsystem described by the Falicov-Kimball model [1] on the Shastry-Sutherland lattice. Moreover, both subsystems are coupled by the anisotropic spin-dependent interaction of the Ising type. To examine the magnetization curve corresponding to our model Hamiltonian we have used a well-controlled numerical method based on the modification of the small cluster exact diagonalization calculations [2]. Using this method we have performed exhaustive numerical studies of the model for a wide range of model parameters, selected on the base of experimental measurements [3]. We have found, that the switching on of the spin-dependent interaction ( $J_z$ ) between the electron and spin subsystems and taking into account the electron hopping on the nearest ( $t$ ) and next-nearest ( $t'$ ) lattice sites of the Shastry-Sutherland lattice leads to a stabilization of new magnetization plateaus. Besides the Ising magnetization plateau at  $m^{sp}/m_s^{sp} = 1/3$  we have found three new relevant magnetization plateaus located at  $m^{sp}/m_s^{sp} = 1/2$ ,  $1/5$  and  $1/7$  of the saturated spin magnetization  $m_s^{sp}$ . The ground-states corresponding to magnetization plateaus have the same spin structure consisting of parallel antiferromagnetic bands separated by ferromagnetic stripes. For  $m^{sp}/m_s^{sp} = 1/3$  our results are identical with ones obtained within the Ising [3] as well as easy-axis Heisenberg [4] model on the Shastry-Sutherland lattice. The accordance between our and the easy-axis Heisenberg solution [4] is found also for  $m^{sp}/m_s^{sp} = 1/2$ . In this case both approaches predict the sequence of parallel antiferromagnetic and ferromagnetic stripes. For  $m^{sp}/m_s^{sp} = 1/5$  our results postulate a new type of spin ordering. In addition, we have found, that our model is able to describe the magnetization plateaus also at  $m^{sp}/m_s^{sp} = 1/7$ ,  $1/9$  and  $1/11$  (in accordance with experimental measurements in  $TmB_4$  [3]) but the stability regions of these phases are much narrow in comparison to ones of  $1/2$ ,  $1/3$  and  $1/5$  plateau phases. Finally, the transitions from the low temperature ordered phase to the high-temperature disordered phase are analysed through the behaviour of the specific heat and the energy distribution at the transition point by the canonical Monte-Carlo method.

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### Diffusion scaling analysis based on renewal theory: an application to turbulent intermittency

R. Cesari<sup>†</sup>, P. Paradisi, P. Allegrini, D. Contini, A. Donato

<sup>†</sup>Istituto di Scienze dell'Atmosfera e del Clima (ISAC-CNR), Lecce Unit, Italy

A turbulent fluid flow is characterized by bursting and intermittent behavior [1]. This is associated with the complex dynamics of turbulent energy production and dissipation, which can be described in terms of coherent motions (e.g., eddies, vortices, filaments, ...). Coherent motions are metastable flow structures emerging from the noisy background as a signature of the self-organization determined by the fluid dynamics. These flow structures are generated at random times and remain stable for relatively long life-times. After this life-time, they become suddenly unstable and a rapid decay occurs. The generation and annihilation episodes occur on very short times with respect to the long life-time of the coherent, self-organized flow structures, so that they can be treated as quasi-instantaneous events. The emergence and evolution of intermittent structures are associated with a power-law decay of correlation functions and of Inter-Event Time (IET) distribution. The power exponent of the IET distribution can be defined to be a measure of the system's complexity. Self-organized criticality and non-extensive thermodynamics are the most common theoretical frameworks used to describe turbulence. However, in recent times, renewal theory [2], assuming that the events are statistically independent, was proposed to analyse atmospheric time series [3]. The authors estimated the system's complexity by directly evaluating the power-law decay of the IET distribution, which is a crucial parameter under the renewal assumption. The IET distribution was also used for a statistical analysis being able to check the renewal assumption. However, the results of this analysis and the estimation of the system's complexity are limited by the presence of added noise in the time series, as it affects the shape of IET distribution. In the present work we introduce a novel statistical analysis that overcomes this limitation and we show an application of the analysis to turbulence data in the atmospheric boundary layer. Firstly, the experimental time signals are processed to extract sequences of events associated with the birth and death of coherent intermittent structures. Then, we perform the proposed analysis, which is based on the estimation of the diffusion scaling of random walks derived from the experimental event sequences [4,5]. This analysis allows to estimate indirectly the system's complexity, the renewal assumption and the level of noise in the time series.

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## Semi-classical analysis of Klein tunneling through a smooth barrier

L. Chalmers<sup>†</sup>, V. Zalipaev, F. Kusmartsev, C. Linton

<sup>†</sup>Loughborough University Department of Physics

Analytical and numerical studies of electron-hole transport properties of nanostructures made from single layer graphene have been presented. This novel two-dimensional material which was originally thought to be thermodynamically unstable, has received great attention, culminating in a Nobel Prize being awarded to its discoverers. Interest is spurred by its extraordinary electronic transport properties. Charge carriers behave as relativistic fermions which are described by the Dirac Hamiltonian and possess a property of perfect transmission through any rectangular potential barrier at normal incidence. This is the much discussed Klein paradox. In the present paper we study the transmission and reflectance from a one-dimensional smooth finite-slope potential barrier of arbitrary shape. The results build upon the square and trapezoid barriers previously studied. The smooth barrier case represents a more realistic electrostatic potential across a nanostructure graphene sheet which normally arises due to the interaction with the substrate or gate electrodes. The analytical formulism we have derived is a semi-classical approximation based upon the WKB-technique. The solution describes oscillatory functions in the classically allowed regions and exponentially decaying or growing functions in the classically forbidden regions. This system can be considered analogous to the double potential well. Singularities arise in the regions of the turning points of the barrier, leading to a discontinuity in the approximation. The solutions in the classically allowed and prohibited domains are therefore matched using a boundary layer method. Between the two classically prohibited turning point singularities, localised modes of complex resonances exist which are governed by the Bohr-Sommerfeld quantization. We show that in the continuous spectrum scattering problem, when the energy of the incident wave is equal to the real part of the complex resonance, complete transmission is achieved. The imaginary part of the complex resonance is related to the lifetime. Numerical verification is obtained via the finite element method realised in the framework of COMSOL Multiphysics. Resonances of conductance obtained numerically are found to be in good agreement with those predicted by the semi-classical approximation.

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## Order and control parameters in driven, dissipating systems- parallels from avalanching systems, fluid turbulence and ecology

S. Chapman<sup>†</sup>, G. Rowlands, A. Clarke, E.J. Murphy, N.W. Watkins

<sup>†</sup>CFSA, Physics, University of Warwick

Formal dimensional analysis (the Buckingham Pi theorem [1]) can be used to obtain order and control parameters for systems that are otherwise intractable. If there is a known constraint or conservation principle, then relationships between these parameters can also be inferred. We will consider three driven, dissipating out of equilibrium systems (i) avalanche models that can exhibit self-organized criticality (SOC); (ii) ideal Kolmogorov fluid turbulence and (iii) simple ecological models for food webs based on resource flow. For a generic finite-sized avalanching system the constraint is conservation of 'sand' and the control parameter  $R_A$  that we identify [2] is essentially the driving rate, normalised to the rates of transport and dissipation respectively. A mean field approach has previously identified the self organized critical (SOC) state to be in the limit of slowly driven, interaction dominated transport (SDIDT), thus  $R_A - > 0$  is the SDIDT limit. Our result implies that critical behaviour can persist for an intermediate range of the drive, provided the drive is spatially local and the pile is sufficiently large. Intermediate, local driving simply corresponds to a spatial coarse-graining of the pile. For SOC, strong driving ultimately brings the system away from criticality to laminar flow [3]. In fluid turbulence, the constraint is that the rate of energy transfer is on average constant and the analogous control parameter is the Reynolds number  $R_E$ . In fluid turbulence, increasing the drive, i.e.  $R_E$  brings the system away from laminar flow towards disorder. This is in the opposite sense to SOC. Parallels between  $R_E$  and  $R_A$  that we identify here thus succinctly identify the similarities, and differences, between SOC and turbulence. In simple models for food webs in a dynamically balanced steady state, the constraint is that the uptake of resource is dynamically balanced by the rate at which it is utilized by all organisms within the ecosystem. We find the analogous control parameter for diversity and abundance of ecosystems is the rate of uptake of resource to an ecosystem (productivity and habitat size) normalized to the typical metabolic rate. Our result explicitly relates this to parameters for energy flow and utilization (abundance, body size), foodweb structure (body size distribution and trophic level) and spatial distribution (characteristic lengthscale for foraging) and diversity. The fundamental expression that we obtain yields well known macroecological scaling laws. Our results explain the ubiquity of macroecological patterns, and suggests a method to identify ecosystems that are not in dynamic balance and are on the point of, or are already showing, rapid change or collapse in diversity or abundance.

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## Anomalous diffusion and characterizing the fluctuating fields of finite sized turbulence in astrophysical plasmas

S.C. Chapman<sup>†</sup>, W.-C. Mueller, J. Pratt, N.W. Watkins, A. Busse, Yu-Xi Chau

<sup>†</sup>CFSA, Physics, University of Warwick

The solar wind has Reynolds number  $\sim 10^5$  and the field and flow are observed in-situ over timescales spanning several decades-providing a natural laboratory for finite range magnetohydrodynamic (MHD) turbulence studies. The solar wind exhibits fluctuations over a broad range of timescales characteristic of MHD turbulence evolving in the presence of structures of coronal origin. In-situ spacecraft observations of plasma parameters are at minute (or below) resolution for intervals spanning the solar cycle and provide a large number of samples for statistical studies. The magnetic field power spectrum typically shows an inertial range of turbulence over several orders of magnitude with approximately Kolmogorov power law and at lower frequencies, an  $\sim 1/f'$  energy containing range believed to be of direct coronal origin and at higher frequencies, a kinetic range of turbulence. At high latitudes, in uninterrupted streams of fast solar wind flow, and with recent high cadence observations of coronal structures there is the opportunity to study evolving finite range turbulence which can also inform our understanding of turbulence in boundary layers [1]. These observations provide evidence for an inertial range of MHD turbulence in the multifractal scaling of the structure functions, non-Gaussian statistics of fluctuations and generalized similarity of the finite range turbulence that is robust to changes in parameters of the bulk field and flow. However these observations typically only provide timeseries of magnetic field and flow velocity at a single point in the flow. On the other hand, three dimensional direct numerical simulation of MHD turbulence [2] give full spatial and temporal information and in particular allow the diffusive dynamics of tracer particles to be tracked in detail, but are restricted- by currently available computing power- to realizations of the turbulence that are also finite in space and time. We will use higher order methods (structure functions) to characterize the universal properties of finite sized anisotropic MHD turbulence as seen in the solar wind in the time domain. We can compare these to the results of the direct numerical simulations of MHD turbulence. We propose a new method for characterizing tracer-particle dynamics in the direct numerical simulations by analysing the convex hull [3] of groups of tracer particles in a flow. We explore the convex hull as a descriptor of anomalous diffusion related to statistical properties of the turbulent fluctuations. This relates to the statistical properties of the fluctuations in the field and flow in a fundamental manner as the tracer particles capture information about the random-walking of field lines in a frozen-in MHD flow.

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## $so(p, q)$ Toda Lattice

S. Charalambides

<sup>†</sup>University of Cyprus

The Toda lattice was considered first by Morikazu Toda [2]. The original Toda lattice can be viewed as a discrete version of the Korteweg–de Vries equation. It is called a lattice as in atomic lattice since interatomic interaction was studied. This system also appears in Cosmology. It appears also in the work of Seiberg and Witten on supersymmetric Yang–Mills theories and it has applications in analog computing and numerical computation of eigenvalues. But the Toda lattice is mainly a theoretical mathematical model which is important due to the rich mathematical structure encoded in it. There are various generalizations of the system e.g. the full symmetric Toda lattice where the tri-diagonal Jacobi matrix is replaced by a full symmetric matrix, the full Kostant Toda lattice studied by Kostant [4] and the various Bogoyavlenski Toda lattices associated with simple complex Lie algebras [5]. We use the ideas of Kostant and Bogoyavlenski, see [3], to define a Toda lattice associated with the real Lie algebra  $so(p, q)$ . More precisely we define an integrable hamiltonian system associated with the real Lie algebra  $so(2m, 2n + 1)$ . As usual there exists a periodic and a non-periodic version and we investigate both of them. We construct, using the root space, a Lax pair representation and the associated Poisson tensors. We prove Liouville integrability and examine the multi-hamiltonian structure. The system is bi-hamiltonian with a linear and a quadratic Poisson bracket. The quadratic Poisson bracket is similar to the Adler Toda bracket with some differences on the signs. The system is a projection of a canonical  $A_n$  type Toda lattice via a Flaschka type transformation. This method of Flaschka was used to prove the integrability of the original Toda lattice in [1]. We begin by giving a description of the basis of  $so(2m, 2n + 1)$ , its Cartan subalgebra, its roots and its root space. We end-up with a new set of polynomial equations in the variables  $(a, b)$ . One can write the equations in Lax pair form  $(L(t), B(t))$ , which can be described in terms of the root system. The same equations can be obtained using the projection from the  $A_n$  Toda lattice. We also compute the Casimirs associated with both Poisson brackets and prove the involution of the invariants.

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### Inhomogeneous superconductivity: effect of disorder on Carbon nanotubes, on organic and HTC superconductors

S. Charfi-Kaddour<sup>†</sup>, S. Haddad, B. Bellafi

<sup>†</sup>Faculté des Sciences de Tunis

The inhomogeneous superconductivity can result for the disorder induced by the chemical substitution, the fast cooling rate, the pressure, X-ray irradiation, etc.. Various experimental studies have given evidence for the presence of a spatially inhomogeneous phase coexistence of superconducting and non-superconducting states in low dimensional organic superconductors and in high  $T_c$  superconductors. The superconducting clusters, which have a mesoscopic size, are found to be strongly dependent on the amount of disorder in the sample regardless its origin. However, it has been reported that the behavior of the superconducting transition temperature shows clear deviation from the Abrikosov-Gor'kov law expected for dirty superconductors. Based on the time dependent Ginzburg-Landau theory, we derive a model to account for this striking behavior in organic superconductors considering different type of disorder. We show that our results are quantitatively in agreement with the experimental data (1-2). We find a universal behavior in which the key parameter is the length scale of the non superconducting domain over the superconducting coherent length. A comparison is made with experiments done on organic quasi-1D and quasi-2D superconductors ( ( TMTSF)<sub>2</sub>ClO<sub>4</sub>, Kappa-(BEDT-TTF)<sub>2</sub>X.) and on cuprates. We study the interplay between disorder and superconductivity in a rope of metallic carbon nanotubes. Based on the time-dependent Ginzburg-Landau theory, we derive the superconducting transition temperature  $T_c$  taking into account the critical superconducting fluctuations which are expected to be substantially strong in such low-dimensional systems. Our results indicate that, contrary to what is expected,  $T_c$  increases by increasing the amount of disorder (3). We argue that this behavior is due to the dynamics of the tubes which reduces the drastic effect of the local disorder on superconductivity by enhancing the intertube Josephson tunneling. We also found that  $T_c$  is enhanced as the effective dimensionality of the rope increases by increasing the number  $N$  of the tubes forming the rope. However,  $T_c$  tends to saturate for large values of  $N$ , expressing the establishment of a bulk three-dimensional superconducting order. Our results are in a good agreement with experimental data obtained by M. Ferrier (4).

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### Recent results on long-range interacting systems: the example of the HMF model

P.-H. Chavanis<sup>†</sup>

<sup>†</sup>Laboratoire de Physique Theorique de Toulouse

Systems with long-range interaction are numerous in nature. Some examples include self-gravitating systems (galaxies, globular clusters), two-dimensional turbulence (jets and vortices), chemotaxis of biological populations (clusters of bacteria) etc. A toy model of systems with long-range interactions has been introduced. This is the so-called Hamiltonian Mean Field (HMF) model. It can be viewed as a collection of  $N$  particles moving on a circle and interacting via a cosine binary potential. Interestingly, this model exhibits several features common to more realistic systems with long-range interactions and its simplicity allows one to study their properties in great detail. In this talk, we review recent results on long-range interacting systems obtained in the framework of the HMF model. We first discuss out-of-equilibrium phase transitions predicted from Lynden-Bell's theory of violent relaxation. This leads to a rich phase diagram exhibiting a tricritical point separating first and second order phase transitions as well as a critical point at which two second order phase transitions appear (second order azeotropy). This is associated with a phenomenon of phase re-entrance. We compare these theoretical predictions with direct  $N$ -body numerical simulations and find a good agreement [1,2]. We mention the relevance of polytropic (Tsallis) distributions in case of incomplete relaxation [3]. We study the Vlasov dynamical stability of these distributions for various values of the polytropic index and describe the corresponding phase diagrams. We also develop a theory of dynamical phase transitions between a homogeneous (non magnetized) phase and an inhomogeneous (magnetized) phase [4,5]. For the HMF model, we find that the collisional evolution can be fitted by polytropic distributions with a time dependent index  $q(t)$ . The phase transition takes place when the index reaches the critical value at which the distribution becomes Vlasov unstable. For systems with long-range interactions in contact with a heat bath, such as the Brownian Mean Field (BMF) model, the evolution of the distribution is governed by the mean field Kramers equation and the time at which the phase transition occurs can be obtained analytically [5]. We report the existence of a "pulse" corresponding to the system being successively non-magnetized, magnetized and non-magnetized again.

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## Non Gaussian corrections and finite size effects in a model of autocatalytic reactions

C. Cianci<sup>†</sup>, F. Di Patti, D. Fanelli

<sup>†</sup>University of Florence

The cell is a structural and functional unit, the building block of any living systems. Cells are constituted by a tiny membrane, made of lipid bilayer, which encloses a finite volume and protects the genetic material stored inside. Modern cells with their complex machineries result from evolution of ancient supposedly minimalistic cell entity, the protocell. It is customarily believed that autocatalytic reactions might have been at play in primordial protocell. The shared view is that protocell's volume might have been occupied by interacting families of replicators, organized in autocatalytic cycles. The study of the dynamical evolution of interacting species of homologous quantities defines the field of population dynamics. Population is indeed a technical term which is referred to various, completely distinct fields of applications. The classical deterministic approach to population dynamics relies on characterizing quantitatively the densities of species through a system of ordinary differential equations which incorporate the specific interactions being at play. As opposed to this formulation, a different (stochastic) level of modeling can be invoked which instead focuses on the individual-based description. This amounts to characterizing the microscopic dynamics via explicit rules governing the interactions among individuals and with the surrounding environment. The stochasticity is intrinsic to the systems and stems from the microscopic finiteness of the investigated medium. Remarkably, inherent demographic perturbations might induce regular behaviours at the macroscopic level, emerging as a spontaneous collective self-organized phenomenon[1,2]. In this paper, we investigate the stochastic dynamics of a complex network of autocatalytic reactions, within a spatially bounded domain, so to mimic a primordial cell back at the origin of life. The role of stochastic fluctuations is elucidated through the use of the van Kampen system-size expansion and shown to induce regular oscillations in time of in the concentration amount[3]. Corrections beyond the Gaussian approximation are analytically computed within the van Kampen operative ansatz. An extended Fokker-Planck equation is obtained and the moments of the multivariate non Gaussian distribution of fluctuations quantified. The theory predictions are challenged versus direct stochastic simulations and shown to return an excellent agreement. Possible implications of our findings as concerns protocells origin and evolution are addressed[4].

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## Traffic jams and protein synthesis: from a TASEP-based model to experiments

L. Ciandrini<sup>†</sup>, I. Stansfield, P. Grulich, R.J. Allen, A. Parmeggiani, M.C. Romano

<sup>†</sup>ICSMB, Physics department, University of Aberdeen

One of the most important steps of protein synthesis is the translation of the messenger RNA (mRNA). During this process, macromolecules known as ribosomes translate the genetic information encoded by the triplets of nucleotides (codons) of the mRNA, and assemble proteins amino acid by amino acid. Several ribosomes may simultaneously translate the same messenger. Like goods waiting on a production line, ribosomes can queue on the track influencing the protein production rate. We use a general approach to study mRNA translation by means of statistical-mechanical models based on the Totally Asymmetric Simple Exclusion Process (TASEP), which considers ribosomes as a one-dimensional driven lattice gas. To achieve a better understanding of the underlying biological processes and to compare the theoretical predictions with experimental results, we use a description lying between minimal TASEP-like models and more detailed mechanistic models [1], which are analytically hard to treat. We include in the model the two fundamental steps [2] of the ribosome's biochemical cycle: the interaction with transfer RNAs which transport the building-blocks of the growing protein (the amino acids), and translocation along the mRNA. In this framework it is possible to characterize analytically different phases of the system (high density, low density or maximal current phase). Crucially, we show that the transitions between these different phases occur at different parameter values than the equivalent transitions in a standard TASEP [3]. We observe that a mean-field description is inadequate when the internal stepping dynamics is slow compared to the translocation of particles. In this regime the step-cycle induces non-localised traffic jams [4]. An important goal of such model is to predict the effects of differences in transfer RNAs availability, which influences the translation of each different mRNA sequence. By applying this model to the more than 6,000 genes of baker's yeast we are able to study the characteristics of the whole transcriptome of *S.cerevisiae* and predict the 'traffic effects' of ribosomes in real (biological) sequences. We also investigate the competition of several lattices sharing a common reservoir of particles and we explain how a mixed population of lattices (i.e. lattices with different properties) behave differently by changing the amount of resources [5]. The competition for the resources in the cell might be crucial in some biological conditions, e.g., to react to stress conditions.

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## Coated solitons

M. Clerc†, M. Garcia-Ñustes, Y. Zarate

†University of Chile

During the last years, emerging macroscopic particle type solutions or localized states in macroscopic extended dissipative systems have been observed in different fields, such as: domains in magnetic materials, chiral bubbles in liquid crystals, current filaments in gas discharge, spots in chemical reactions, localized states in fluid surface waves, oscillons in granular media, isolated states in thermal convection, solitary waves in nonlinear optics, among others. Hence, one can infer the universality of the localized states dynamics. Although these states are spatially extended, they exhibit properties typically associated with particles. Consequently one can characterize them with a family of continuous parameters such as position, amplitude and width. This is exactly the type of description used in more fundamental physical theories like Quantum Mechanics and Particle Physics. However, localized states emerging in extended dissipative systems are characterized by being made of a large number of atoms or molecules (of the order of Avogadro's number) that behave coherently. The paradigmatic example of macroscopic localized state, are solitons reported in the context of fluid dynamics, nonlinear optics and Hamiltonian systems. The solitons arise from a robust balance between dispersion and nonlinearity. The generalization of this concept to dissipative and out of equilibrium systems has led to several studies in the last decades, in particular to the definition of localized structures intended as patterns appearing in a restricted region of space [1]. We have recently observed a new kind of dissipative soliton-type solutions in parametric systems, particularly in a vertically driven pendulums chain [2,3], in a water channel driven vertically [4], in a forced magnetic wire with a magnetic field composed of a uniform and an oscillatory part [2,3], and an optical parametric oscillator, which are characterized by having a shell-type phase structure, which controls the type of interactions between solitons. All these systems, in the quasi-reversible limit, are well described in the context of the parametric driven nonlinear Schrodinger equation. This model exhibits the same behavior for dissipative solitons, that is, solitons have a shell-like structure in the phase. Using different approaches, we have characterized the complex dynamics of phase fronts and its bifurcation diagram.

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## Nanoscale self-organization using standing surface acoustic waves

N. Combe†, C. Taillan, J. Morillo

†CEMES-CNRS, Toulouse Cedex, France

Experimentally, the thin films growth consists in a low energy deposition of atoms or molecules on a substrate and is generally used to create nanostructures (for instance quantum wells, wires or dots for semi-conductors or magnetic nanoparticles for metals) that can be organized using their self-assembling properties. Unfortunately, the control of the spatial or size distribution of these structures remains indirect and/or difficult: for instance, experimentalists take advantages of the elastic properties of materials (Stranski-Krastanov growth mode [1], buried dislocations network [2], pre-patterned substrate [3]). Our study aims to explore an alternative approach that could potentially improve this control. We propose to use a standing acoustic elastic-wave to self-organize the growth on a crystalline substrate. The study of the diffusion of an adatom on a substrate submitted to a standing surface acoustic wave consists in the first stage of our study. We perform both molecular dynamic simulations [4] and analytical calculations using a Langevin approach [5]. From MD simulations, we show that the standing surface acoustic wave encourages the presence of the adatom in the vicinity of the maxima of (transversal) displacements of the substrate. The longitudinal strain field of the wave modulates both absorption energy and activation energy barrier which results in a time-dependent force acting on the adatom. The effect of such force is analyzed within a Langevin approach. Using a multiple scale method, we show that the motion of the adatom on time scales large compared to the wave frequency can be described by a time-independent effective potential. Minima of this effective potential coincide with maxima of the (transversal) displacements of the substrate induced by the wave. The effective potential scales as the square of the amplitude of the force acting on the adatom. We show that, in the range of applicability of the effective potential derivation, the structuring effect induced by the wave exhibited in MD simulations also scales as the square of this force. Going beyond our main result (the structuring effect induced by a standing elastic-wave) and in agreement with our preliminary MD results on the nucleation, we expect the standing acoustic wave to be able to organize the nucleation of solid germs during atomic deposition experiments, leading to a controlled self-organization of nanostructures.

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## Entropy analysis of word-length series of natural language texts: Effects of text language and genre

V. Constantoudis<sup>†</sup>, M. Kalimeri, C. Papadimitriou, K. Karamanos, F.K. Diakonos, H. Papageorgiou

<sup>†</sup>Institute of Microelectronics, NCSR Demokritos

The output of many physical, biological and social processes is recorded in the form of time series. Most times, these time series are neither totally random nor completely periodic but exhibit correlations in a limited range which can be quantified by the properly defined entropy. The entropy can be calculated either directly from the time series itself or after its transformation to a sequence of symbols. In the case of discrete time series (time series with discrete variable values) and especially when the range of variable values is limited, this transformation is straightforward and the time series itself can be considered a symbolic sequence with alphabet the range of variable values appeared in time series. A text written in natural language can be also represented as a time series. The "time" here is the position of the word in the text and the recorded variable is a word property such as the length or the frequency of its appearance in the text, or the rank according to its frequency of appearance, or the sum of the Unicode values of the letters of a word [1-3]. In all cases, the variable takes on discrete values and thus the time series is actually a symbolic sequence with alphabet depending on the used word property. The degree of randomness in a symbolic sequence can be characterized by the block entropies which generalize the Shannon entropy to n-grams i.e. blocks with n symbols [4]. In our previous work [5], we investigated the Shannon and KS entropies of binary symbolic sequences generated by the texts. In this work, we employ the word-length series representation of natural language texts and estimate the short-block (unigram, bigram and trigram) entropies defined in this representation. First we show that these are sensitive to both text language (Greek or English) and genre (news (politics, economy, sports) or literature). We attribute this sensitivity to changes in the probability distribution of the lengths of single words and emphasize the crucial role of the uniformity of probabilities of having words with length between five and ten. Furthermore, through comparison with the entropies of shuffled data we conclude that the entropy sensitivity to text language and text can be explained by differences in single word-length distributions. Last but not least, a slight difference found always between shuffled and real data entropies can be considered that reveals the impact of word-length correlations on block entropies.

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## Sidewall roughness and dimensional variability of features on nanoscale: the case of contact holes in integrated circuits

V. Constantoudis<sup>†</sup>, M.K. Vijaya-Kumar, E. Gogolides, R. Gronheid and A. Vaglio Pret

<sup>†</sup>Institute of Microelectronics, NCSR Demokritos

With the continuous scaling down of the dimensions of fabricated structures, sidewall roughness becomes an increasing fraction of the pattern size and starts to affect their electrical performance. Up to now, the majority of related works have been focused on the sidewall roughness of transistor gates which has attracted a lot of interest in semiconductor industry. Fewer works have been devoted to the study of the sidewall roughness of holes, which in the manufacturing of integrated circuits are used for the opening of contacts in transistors or vias between different metal layers. The roughness of contact holes (called Contact Edge Roughness, CER) has recently attracted more attention since it was shown that it may affect the Source/Drain contact resistance and the saturation current, and may cause time-dependent dielectric breakdown [1,2]. Besides CER, a great concern is the variability (nonuniformity) of the diameter of contact holes (called critical dimension, CD) in integrated circuits which may contain billions of these. In addition to the other contributions to CD nonuniformity, the aim of the paper is to reveal the effects of roughness through modeling and experimental results. To this end, first we implement two methodologies for the evaluation and modeling of CER respectively. The first uses as input top down Scanning Electron Microscope images and outputs roughness and dimensional parameters (RMS, CD variation, correlation length, fractal dimension) as well as functions (power spectrum and correlation function) [3]. The modeling algorithm operates inversely and starts with the roughness and dimensional parameters and outputs contact edges whose roughness is characterized by the input parameters. Through the modeling algorithm, we show that the reduction of the nominal (input) diameter of a large collection of contact holes with similar roughness leads to a decrease of the average RMS value and increase of CD variation revealing thus the contribution of roughness to contact hole nonuniformity. In order to verify experimentally this contribution, a large number of contact holes were exposed in the same resist through Extreme Ultraviolet Lithography but at different illumination doses resulting in different CD values. By applying the CER evaluation methodology, we find that the decrease of dose and hence of CD is associated with the reduction of RMS and increase of CD variation in agreement with modeling predictions. Putting this finding together with the similar one in transistor gate roughness, we may conclude that roughness on nanoscale structures is mainly manifested through the increased variability of feature dimensions.

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## Network analysis of rough surfaces

V. Constantoudis†, N. Karasmati

†Institute of Microelectronics, NCSR Demokritos

The measurement and characterization of a rough surface is a long standing problem. First attempts have started by mechanical engineers motivated by the need for better understanding and control of the role of roughness in tribology [1]. A paradigmatic shift took place when fractal theory and scaling concepts were introduced utilizing the self-similar (or more exactly self-affine) symmetry that most surface in nature and laboratory exhibit [2]. However, with the advent of Scanning Probe Microscopies and the digital measurement of a large amount of surfaces on micro and nanoscale, it was realized that the fractal description of roughness is mostly limited to a specific range of scales [3]. This limitation can be crucial when surface structures with interesting properties like periodicity appear at scales outside this range. Furthermore, conventional tools for periodicity detection and evaluation suffer from their own limitations. For example, Fourier transform due to the kernel of harmonic waves it implies is not only sensitive to the degree of periodicity but also to the shape of the repeated structure. The above limitations motivate us to seek alternative methods for the characterization of surface roughness. To this end, the present work tries to employ the recent advances in complex network theory and critically examine the benefits of their application in roughness characterization. Similar attempts have been made in time series analysis [4,5]. The key idea here is to transform a surface to a network by considering the measured points of the surface as the nodes of the network and the link between two nodes inversely proportional to the difference of their heights. Obviously, the generated networks are undirected but weighted. However, for simplicity reasons we get weighted networks through thresholding of the initial networks. Our study consists of four steps. First, we examine the properties of networks generated by a random and uncorrelated surface and it is found that they possess the characteristics of random graphs with Poisson distribution which can be calculated analytically. Then, we generate self-affine surfaces with controlled roughness determined by three parameters (RMS, correlation length and fractal dimension), transform these to networks and calculate their properties to reveal the effects of roughness parameters. Thirdly, we add in the modeling self-affine fractal surfaces a periodic component with controlled amplitude and periodicity and quantify the changes it causes to network properties. Finally, we get the networks of real surfaces and compare their properties with the results of the conventional tools of roughness analysis.

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## Time fluctuations of vegetation patterns and early warning signals of desertification transition in semi-arid ecosystems

R. Corrado†, A.M. Cherubini, C. Pennetta

†Dottorato in Scienze dei Cambiamenti Climatici, Dipartimento di Fisica, Università del Salento, Lecce, Italy

The identification of early warning signals of regime shifts in ecosystems is a crucial issue since these transitions can cause severe losses of ecological and economic resources. In particular, environmental stresses induced either by an excess of anthropic load or by an increased frequency of meteorological extremes, can give rise in arid or semi-arid ecosystems to a desertification transition. Many recent studies highlighted the interest in vegetation patchiness analyses as a tool to provide indicators of desertification risk [1-3]. Recently, Kefi et al. [1] investigated the influence of external stress on the spatial organization of vegetation, by combining modelling and field data from three grazed Mediterranean arid ecosystems in Spain, Greece and Morocco. The model, based on a stochastic cellular automaton, was accounting for several ecological mechanisms. The ecological landscape was then determined by a certain number of parameters. In particular, we mention the mortality parameter,  $m$ , associated with the external stress (grazing pressure) and the facilitation parameter,  $f$ , controlling the strength of local positive interactions among plants. The analysis of both field data and numerical simulations showed that far from transition, the vegetation patch-size distribution follows a power law. However, at increasing  $m$  (or at decreasing  $f$ ) it appears an exponential cut-off. Therefore, Kefi et al. proposed that non power-law patch-size distributions may be warning signals for the onset of desertification. In any cases, this study and many others in the literature devoted to desertification risk, mainly focus on the analysis of the spatial fluctuations of the vegetation pattern, while only few authors [3] investigated time fluctuations. Therefore we have studied time fluctuations properties of some global and local quantities associated with steady states of the cellular automaton of Kefi et al. Precisely, we have calculated and analyzed as a function of  $m$ ,  $f$  and  $N$  (system size), the probability density function (PDF) and the first three moments of the time fluctuation distribution of the total vegetation mass fraction,  $\rho^+$ , and of the biggest (emergent) cluster size  $s_B$ . For what concerns the time fluctuations properties of  $\rho^+$  as a function of  $m$ , our results show a strong increase of the variance, which diverges at a critical value  $m_c$ , associated with the desertification transition. A significant non Gaussianity of the PDF also emerges, characterized by a negative skewness. Particularly interesting are time fluctuation properties of  $s_B$  whose PDF achieves at increasing values of  $m$ , for  $m$  close  $m^*$ , a value well below  $m_c$ , the universal form of the Bramwell- Holdsworth-Pinton distribution [4,5].

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## Exploring the universe of protein structures beyond the Protein Data Bank

P. Cossio†, A. Trovato, F. Pietrucci, F. Seno, A. Maritan, A. Laio

†SISSA

Protein structure and biological function are determined by their sequence, but proteins of different sequence or function can share the same structure. Indeed, folds are evolutionarily more conserved than sequences and the same fold can house proteins performing different biological functions. Thus a fundamental question concerns the extension of the library of protein folds: are the observed structures a small fraction of the whole fold universe? Currently it is believed that the atlas of existing protein structures is faithfully represented in the Protein Data Bank [1]. However, whether this atlas covers the full universe of all possible protein structures is still a highly debated issue. By using a state-of-the-art enhanced sampling technique [2] we performed an exhaustive exploration of the conformational space of a 60 amino acid polypeptide chain described with an accurate all-atom interaction potential. We generated a database of around 30,000 compact folds with at least 30% of secondary structure and a small radius of gyration, corresponding to local minima of the potential energy. This ensemble plausibly represents the universe of protein folds of similar length: in fact, all the known folds existing in nature of similar size are represented in the set with good accuracy. Confirming that the observed protein folds are selected based on geometry and symmetry and not on the chemistry of the amino acid sequence. However, we discover that the known folds form a rather small subset, which cannot be reproduced by choosing random structures in the database. Rather, natural and possible folds differ by the contact order [3], on average significantly smaller in the former. We find that evolution selects protein structures under the guidance of a simple principle: reducing the entanglement in the bundle formed by the protein in its folded state, making bundles with shorter loops preferable. This suggests the presence of an evolutionary bias, possibly related to kinetic accessibility, towards structures with shorter loops between contacting residues. Beside their conceptual relevance, the new structures open a range of practical applications such as the development of accurate structure prediction strategies, the optimization of force fields, and the identification and design of novel folds.

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## There is more than a power law in Zipf

M. Cristelli†, M. Batty, L. Pietronero

†Department of Physics, "Sapienza" University of Rome, Italy; The Institute of Complex Systems, CNR, Rome, Italy.

The largest cities, the most frequently used words, the GDP of the richest countries, the income of the most wealthy billionaires, and the revenues of the largest firms can be all described in terms of a single law: Zipf's Law [1-2]. Zipf's Law is a rank-size rule which captures the relation between the frequency or size of a very broad class of quantities with respect to their rank in a set of objects, events, or occurrences. It is a popular, longstanding and widely discussed phenomenon which has given rise to many more than a thousand papers [3]. The common consensus is that Zipf's Law (as a rank-size relation) corresponds essentially to a power law distribution appropriate for describing scaling analogous to many data characteristic of well-known examples such as Pareto's Law, Benford's Law, Fat Tails in Economics, Critical Phenomena, Fractals, etc. We argue here that the implications of Zipf's Law are considerably broader than any of the cases discussed hitherto, requiring an additional fundamental property of the sample distribution in question, and thus providing much more information about the system than a simple power law. We discuss this new property which we call coherence and which corresponds to a screening or competition between various elements of the set [4]. A spectacular consequence of this feature is that a subset of the Zipfian set may not show any Zipfian scaling characteristics whatsoever. Our thesis is remarkably easy to demonstrate. Consider the income of 20 people whose distribution satisfies Zipf's Law. If we consider a subsample of the first 10 persons (the richest), then this subsample will certainly satisfy the same Zipf's Law. However when we consider the second group of 10 persons (the "poorest" among the rich), the incomes of the first two persons is 1/11 and 1/12, while the ratio of the second to the first is 11/12, very different from the first two incomes in the richest set whose ratio is 1/2. There is little evidence in the literature that the importance of this point has been grasped, or if it has, it has been widely ignored. We propose an analysis to extract new and useful information from this novel property [4]. The basic idea behind such a new concept can be exemplified using the distribution of city sizes in the US. Suppose that at a certain point, we extract 'New York City' from our distribution. After such an event in a random sampling, there is still a probability that 'another New York City' could be drawn from the distribution. In reality of course, such an event cannot happen because the largest cities screen one another with respect to their growth dynamics.

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## Nonlinear Fokker-Planck equations, entropies and applications

E.M.F. Curado†

†Centro Brasileiro de Pesquisas Físicas - CBPF, Brazil

A general type of nonlinear Fokker-Planck equation can be derived directly from a master equation, by introducing generalized transition rates, or from the Langevin equation with a special kind of noise. The H theorem can be demonstrated for systems that follow these classes of nonlinear Fokker-Planck equations, in the presence of an external potential. For that, a relation involving terms of Fokker-Planck equations and general entropic forms may be obtained. At equilibrium, this relation is equivalent to the maximum-entropy principle. As a consequence, families of Fokker-Planck equations are related to a single type of entropy, and so, the correspondence between well-known entropic forms and their associated Fokker-Planck equations can be established. It can be shown that the Boltzmann-Gibbs entropy, apart from its connection with the standard - linear Fokker-Planck equation - may be also related to a family of nonlinear Fokker-Planck equations. Through numerical integration, the time evolution of the solution of nonlinear Fokker-Planck equations related to the Boltzmann-Gibbs and other entropic forms are analyzed. The time behavior in both stages, in a time much smaller than the one required for reaching the stationary state, as well as towards the relaxation to the stationary state, are analyzed. In the former case, by using the concept of classes of nonlinear Fokker-Planck equations, a rich variety of physical behavior may be found, with some curious situations, like for example an anomalous diffusion within the class related to the Boltzmann-Gibbs entropy. In addition to that, the relaxation towards the stationary state may present a behavior different from most of the systems studied in the literature. As an example where this formalism could be useful, we show that the overdamped motion of interacting particles at  $T = 0$ , where  $T$  is the temperature of the thermal bath connected to the system, can be associated with a nonlinear Fokker-Planck equation. Then, we can associate with this nonlinear Fokker-Planck equation an entropic form, satisfying the H-theorem. For  $T \neq 0$  it is also possible to associate another nonlinear Fokker-Planck equation, find its stationary solution exactly, and associate a curious entropic form with this nonlinear Fokker-Planck equation, also satisfying the H-theorem. For sufficiently high values of  $T$ , the distribution of particles becomes Gaussian, so that the classical Boltzmann-Gibbs behavior is recovered. For intermediate temperatures of the thermal bath, the system displays a mixed behavior that follows a novel type of thermostatics, where the entropy is given by a linear combination of Tsallis and Boltzmann-Gibbs entropies.

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## Renormalized kinetic theory of classical fluids in and out of equilibrium

J. Daligault†

†Theoretical Division, Los Alamos National Laboratory

We elaborate the foundations of a unifying theory for the construction of renormalized kinetic equations to describe the dynamics of classical systems of particles in or out of equilibrium. A closed, self-consistent set of evolution equations is derived for the single-particle phase-space distribution function  $f$ , the two-body correlation function  $C$ , the retarded and advanced density response functions  $\chi^{R,A}$  to an external potential  $\phi$ , and the associated memory functions  $\Sigma^{R,A,C}$ . The basis of the theory is an effective action functional  $\Omega[\phi]$  of external potentials  $\phi$  that contains all information about the dynamical properties of the system. In particular, its functional derivatives generate successively the single-particle phase-space density  $f$  and all the correlation and density response functions, which are coupled through an infinite hierarchy of evolution equations. Traditional renormalization techniques (involving Legendre transform and vertex functions) are then used to perform the closure of the hierarchy through memory functions. The latter satisfy functional equations that can be used to devise systematic approximations. The present formulation can be equally regarded as (i) a generalization to dynamical problems of the density functional theory of fluids in equilibrium and (ii) as the classical mechanical counterpart of the theory of non-equilibrium Green's functions in quantum field theory. It unifies and encompasses previous results for classical Hamiltonian systems with any initial conditions. For equilibrium states, the theory reduces to the equilibrium memory function approach used in the kinetic theory of fluids in thermal equilibrium. For non-equilibrium fluids, popular closures of the BBGKY hierarchy (e.g. Landau, Boltzmann, Lenard-Balescu-Guernsey) are simply recovered and we discuss the correspondence with the seminal approaches of Martin-Siggia-Rose (1973) and of Rose (1978). Important topical applications include the systematic generalization of the Boltzmann equation to dense gases and liquids, the kinetic theory of strongly coupled plasmas and Hamiltonian systems in general, the proper macroscopic description of fluids beyond the Navier-Stokes equations, and the physics of plasma turbulence.

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## Intermediate Toda Systems

P. Damianou<sup>†</sup>, H. Sabourin, P. Vanhaecke

<sup>†</sup>University of Cyprus

We define some new, evidently integrable Toda type systems associated with a simple complex Lie algebra. More specifically, we construct a large family of Hamiltonian systems which interpolate between the classical Kostant-Toda lattice and the full Kostant-Toda lattice and we discuss their integrability. There is one such system for every nilpotent ideal in a Borel subalgebra. We mainly focus on the case of the Lie algebra being of type A, B or C. The classical Toda lattice was generalized in several directions. One can define a Toda type system for each simple Lie algebra. The finite, non-periodic Toda lattice corresponds to a root system of type  $A_n$ . This generalization is due to Bogoyavlensky [1]. These systems were studied extensively by Kostant [2] where the solution of the system was connected intimately with the representation theory of simple Lie groups. There is also a generalization due to Deift, Li, Nanda and Tomei [3] who showed that the system remains integrable when the Jacobi tri-diagonal matrix is replaced by a full symmetric matrix. The resulting system is called the full symmetric Toda lattice. There is also a similar system called the full Kostant-Toda lattice whose integrability was studied by Flaschka, Ercolani and Singer in [4]. Our systems are defined by a suitable subset of the set of all positive roots which we call adapted. Thus, for each subset which is adapted we obtain a corresponding Hamiltonian system and the problem is to study this system and determine whether it is integrable. We conjecture that in fact it is integrable. We prove this claim for a particular class of such systems. In addition we show that these Hamiltonian systems are associated to a nilpotent ideal of a Borel subalgebra of a semi-simple Lie algebra. The ad-nilpotent ideals of a Borel subalgebra have been classified in [5]. Since for particular (extreme) choices of the ideal one finds the classical Kostant-Toda lattice or the full Kostant-Toda lattice, we call these Hamiltonian systems the intermediate Toda lattices. We study in more detail the case of systems which we call height-2 (since it is defined by a subset consisting of roots of height at most 2). For the simple Lie algebras of type A, B, C we show that there are enough functions in involution, obtained by the method of chopping, and we indicate with some examples that they are in fact functionally independent. In the case of  $C_n$  we find a Casimir which is not obtained by the method of chopping.

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## Percolation threshold of a random network of two dimensional sticks

M. Daoud<sup>†</sup>, R. Bouamrane and K. Senouci

<sup>†</sup>Laboratoire des études de physique des matériaux (USTO), Algeria.

Electrical properties in disordered system are an active area of physics research for decades. During the last years, manufacturing companies have been substituting more economical plastic composites for metals. In particular, the electrical industry has developed several techniques to incorporate conductive fillers, such as aluminium fibres, carbon nano-tube, silicon wire and other into polymer (insulating) medium. As a result there has been emergent interest in low cost large-area construct of thin film transistors on flexible substrates such as plastic for use in several applications. The understandings of the electrical properties of composites are usually used to fabricate thin film transistor. This comprehension is based on the relation that the electrical conduction in such system is a percolate process. The simplest simulation of the above composites is a two dimensional sample of conducting sticks (or straight fibre) embedded randomly in insulating matrix. The first analysis of such a system is the study of Pike and Seager. Their work was concerned only with the percolation threshold of macroscopically isotropic case of randomly distributed stick with random orientation. Whereas they found that the choice of the length  $L$  depends on the number of sticks  $N$ . Moreover, Balberg and al. investigated theoretically the percolation thresholds in two dimensional (2D) anisotropic systems of high aspect ratio of fibre by using a Monte Carlo method. They found that the percolation threshold increased with increasing anisotropy. In another work, Fangming and al. studied the percolation conductivity of a stick network as a function of alignment as well as concentration. They showed that the conductivity exhibits power-law behaviour. Recently, Ashkan and Ant studied the geometry-dependent resistivity scaling in single-walled carbon nano-tube film. They investigated the effect of four parameters: namely, tube-tube contact resistance to the nano-tube resistance ratio, nano-tube density, nano-tube length, and, nano-tube alignment on the CNT film resistivity and its scaling with device width. In this paper, the focal point is on determining the effective conductivity of composites with conductive sticks embedded in insulating matrix as function of the number and the stick's length and their percolation threshold by using a numerical simulation. We then explain these simulation results by simple physical and geometrical arguments, as well as by evaluating and characterizing the performance of thin film composites.

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## On the stabilization of $N$ -layer Hele-Shaw flows

P. Daripa†

†Department of Mathematics, Texas A&M University, College Station, Texas, USA

Stability theory plays a major role from fundamental science to applied sciences. It is useful in the design of many processes and engineering instruments as well as in explaining many phenomena. For example, design of efficient chemical enhanced oil recovery processes often requires injection of fluids with different properties, some laden with chemicals such as polymer, surfactant etc, in succession. The appropriate choice of displacing fluids and the sequence in which these should be injected are often dictated by several needs and one of them is that the front sweeping the oil be as stable as possible to interfacial disturbances. Motivated by this industrial problem, we review some of the author's and his collaborator's recent and ongoing works on the extension of two-layer Saffman-Taylor formula to  $N$ -layer case. The Saffman-Taylor formula gives the growth rate of interfacial disturbances in a Hele-Shaw cell when the displacing fluid is less viscous than the displaced one. In this talk, we will present our results on the generalization of this formula to multi-layer flows involving many interfaces. The generalization is in the form of upper bounds on the growth rates of interfacial disturbances. This is discussed in case of constant viscosity layers. The upper bound provides a way to assess cumulative effects of many layers and many interfaces on the growth rates of unstable waves. As an application of the generalized Saffman-Taylor formula, we will derive an infinite family of necessary conditions for suppressing instability of two-layer flows by introducing arbitrary number of constant viscosity fluid layers in between. The weakest and strongest necessary conditions from this infinite family will be identified. The important role that this condition plays in stabilization of hydrodynamic instabilities and enhanced oil recovery will be discussed.

We will also present results for the variable viscosity case where the viscosity profile could be either monotonic or non-monotonic. The effect of diffusion will be investigated theoretically and numerically. This talk will be based on some unpublished ongoing results and recent publications ([1,2,3,4,5]).

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## Upscaling of transport in correlated non Gaussian velocity fields: consequences for modeling mixing and reactions in porous media

P. de Anna†, T. Le Borgne, A. Tartakovsky, M. Dentz, P. Davy

†Geosciences Rennes, CNRS, Université de Rennes, France

Natural flow fields in porous media display a complex spatio-temporal organization due to heterogeneous geological structures at different scales. This multiscale disorder implies anomalous dispersion, mixing and reaction kinetics (Berkowitz et al. RG 2006, Tartakovsky PRE 2010). In this context, classical continuum models based on Fickian mixing misrepresent reactive transport. Alternative upscaled Langevin models (e.g. Tartakovsky et al. PRL 2008) describe the effects of heterogeneity as a random force, represented through a Gaussian white noise acceleration. This approach has been demonstrated to better describe different aspects of mixing and reactions with respect to classical models (e.g. advection-dispersion-reaction equations). However the Gaussian white noise representation for accelerations may be a strong assumption for porous media. Using two dimensional pore scale SPH numerical simulations of flow and transport, we demonstrate the non Gaussian nature and the long range temporal correlation of the Lagrangian velocity field. The main origin of these properties is the existence of very low velocity regions where solute particles can remain trapped for a long time. Another source of strong correlation is the channeling of flow in localized high velocity regions. Thus, this result questions the applicability of classical Langevin approaches for modeling mixing and reaction kinetics. In order to define an effective upscaled model, we adopt a upscaled model that takes into account the statistical properties of the pore scale Lagrangian velocity field. Analyzing the pore scale statistical properties of the flow, we show the spatial Markovian, and temporal non Markovian, nature of the Lagrangian velocity field. Therefore, an upscaled model can be defined as a correlated Continuous Time Random Walk (Le Borgne et al. PRL 2008) in two dimension. This account for both non Gaussian velocity distribution and long range temporal correlation property. The key feature of this model is the definition of a transition probability density for Lagrangian velocities across a characteristic correlation distance. We quantify this transition probability density from pore scale simulations and use it in the effective random walk model. In this framework, we investigate the ability of this effective model to represent correctly dispersion, mixing and reaction kinetics.

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## Renormalisation group determination of the order of the DNA denaturation transition

F. De los Santos<sup>†</sup>, M.A. Muñoz, J.M. Romero-Enrique

<sup>†</sup>University of Granada

The DNA thermal melting transition also called denaturation, coiling, or unzipping occurs when, above a certain critical temperature, the double-stranded DNA molecule unravels into two separate coils, while for smaller temperatures pre-melting stage only localized openings or bubbles exist. This phase transition is of importance for DNA duplication and transcription, and many studies have scrutinized its nature whether first or second order, trying to pin down the relevant traits of the rich phenomenology experimentally observed. Many of these and other relevant issues have been investigated by employing the Peyrard-Bishop-Dauxois model [1], whose phenomenology has been profusely analyzed by means of various analytical and numerical techniques: transfer integral calculations, Monte Carlo simulations, molecular dynamics, and Langevin dynamics. The results have been found to properly describe experiments on the melting transition, pre-melting bubbles, etc. Here, we reconsider the DNA thermal denaturation problem, analyzing the Peyrard-Bishop-Dauxois model by means of a different, simplified Langevin approach [2]. This strategy allows us to: (i) Reproduce numerically in a relatively easy way the stationary bubble probability distribution and other statistical properties for both homogeneous and heterogeneous DNA sequences, confirming the tendency for creation of thermal openings around AT-rich regions. According to our observations mutations modify the statistics of bubbles only in a local way. However, nonstrictly-local effects due to the merging of bubbles could induce large openings in locally GC-rich regions. (ii) Establish an analogy with well-known equilibrium wetting problems, deeper than previously thought, permitting us to infer results about the order of the denaturation transition. In particular, the nature of the thermal-denaturation transition of homogeneous DNA is determined from a renormalisation group analysis of the Peyrard-Bishop-Dauxois model [3]. Our approach shows that the transition is continuous for the average base-pair separation. However, since the range of universal critical behavior appears to be very narrow, numerically observed denaturation transitions may look first-order. This would explain the difficulties in determining the order of the transition that have been reported in the literature.

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## Non-bilinear spin- $S$ Ising model mapping onto a spin-1/2 Ising model

S.M. De Souza<sup>†</sup>, O.R. Santos

<sup>†</sup>Departamento de Ciências Exatas, Universidade Federal de Lavras

One of the topics of great interest in statistical physics and mathematical physics is that of exact solvable models. This is the case, for the example, of the spin-1/2 Ising model without magnetic field, solved first in 1944 by Onsager; since then, the Ising model has been widely investigated by several approaches. Nowadays, on the other hand, Ising models with spin-1/2 or higher with external magnetic fields are still challenging issues. Further exact solutions were obtained only for a very limited number of cases, mainly for the honeycomb lattices [1]. Some exact results have been obtained for restricted parameters, investigated by Mi and Yang [2] with a non-one-to-one transformation. Therefore the non-bilinear spin- $S$  Ising model that satisfies such transformation should exhibit frustrated states. Particular solutions of half-odd-integer spin Ising models could be obtained by the method proposed by Joseph [3] in which any spin- $S$  could be projected onto a spin-1/2 model. More recently, we have obtained a set of rigorous direct non-one-to-one mappings of half-odd-integer spin models onto spin-1/2 models [4], which we call the spin- $S$  projection with symmetric degeneracy: half of the spin momenta is projected onto spin-down states, whereas the other half is projected onto spin-up states. For integer spins, however, this mapping cannot be performed symmetrically, an issue which will be considered in this work. By using a non-symmetric projection we will discuss the mapping for a general spin- $S$  polynomial onto a spin-1/2 one. We propose the mapping for spin- $S$  polynomial of degree  $2S$  onto spin-1/2 with spin momenta  $+1(-1)$ . The spin- $S$  polynomial in general can be projected onto non-symmetric degeneracy spin up and down momenta. The total number of mappings for each general spin- $S$  is given by  $2(2^{2S} - 1)$  [5]. As an application of this mapping we consider a general non-bilinear spin- $S$  Ising model which can be transformed onto spin-1/2 Ising model. A further transformation yields the partition function of the effective spin-1/2 Ising model, and by a suitable mapping this non-symmetric contribution leads us to a spin-1/2 Ising model with fixed external magnetic field. Therefore the non-bilinear spin- $S$  Ising model that satisfies this transformation is a frustrated model.

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## Nonequilibrium entropy production for open quantum systems

S. Deffner<sup>†</sup>, E. Lutz

<sup>†</sup>University of Augsburg

Nonequilibrium phenomena are ubiquitous in nature. Yet, a general framework allowing their description far from equilibrium is lacking. A defining feature of out-of-equilibrium systems is that they dissipate energy, leading to an irreversible increase of their entropy. The nonequilibrium entropy production is therefore a fundamental concept of nonequilibrium thermodynamics. Traditionally, the entropy production is used to evaluate the performance of thermodynamic devices: the maximal useful work, the availability, that can be extracted from a given system, is reduced by the presence of irreversibilities, such as friction. This loss of availability is directly related to the mean entropy production. More recently, the entropy production has been instrumental in the analysis of nonequilibrium effects in many different branches of physics, including the study of quantum impurity models and of quenched quantum many-body systems. The explicit expression of the mean quantum entropy production is in general unknown, however. A remarkable property of the entropy production is that it satisfies a fluctuation theorem, which holds arbitrarily far from equilibrium. A unified version of classical fluctuation theorems, valid for nonequilibrium initial conditions and arbitrary driving, has lately been obtained by Seifert for the total entropy change occurring in system and reservoir. In this talk, we provide generic microscopic expressions for the entropy production for open quantum systems that are weakly coupled to a heat reservoir and driven arbitrarily far from equilibrium by external parameters. The time evolution and the thermodynamic properties of weakly damped quantum systems are usually described by Markovian master equations of the Lindblad type. However, Markovian master equations are limited to slow driving as their derivation is based on the assumption of time-independent Hamiltonians. A formulation of completely positive maps for fast driven quantum systems, for which the Markovian approximation is likely to break down, appears difficult. Here, we employ a thermodynamic approach to derive the exact mean nonequilibrium quantum entropy production without relying on master equations. The obtained expressions are therefore valid for driving processes that operate arbitrarily far from equilibrium. In addition, starting from the two-point energy measurement statistics for system and reservoir, we derive a quantum extension of Seifert's fluctuation theorem. Our general formalism allows us to recover and extend a number of previously known results in a unified manner.

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## New universal results for two-dimensional percolation. Connectivities and amplitude ratios

G. Delfino<sup>†</sup>

<sup>†</sup>SISSA-Trieste

Recent progresses in conformal and integrable field theory allow to address a number of basic and so far unsolved problems in two-dimensional percolation theory. We showed in [1] that the three-point connectivity on the infinite plane, which carries the most fundamental universal information about the critical theory after the critical exponents, can be determined through a suitable implementation of the permutational symmetry of the  $q$ -state Potts field theory for  $q \rightarrow 1$ , together with an extended interpretation of a formula for the structure constants of minimal models of conformal field theory. The exact result we obtained in this way has been confirmed by high precision Monte Carlo simulations [5]. Not less interesting is the problem of the determination of the universal cluster properties in the scaling region near the percolation threshold. In this case the underlying field theory becomes massive, but remains integrable. In [2] we solved the long standing problem of computing the off-critical two-point connectivity, hence obtaining for the first time complete analytic results for the universal ratios of critical amplitudes in two-dimensional random percolation. Again the field-theoretical values compare extremely well with numerical simulations; in particular, our result for the ratio of mean cluster sizes above and below the percolation threshold puts an end to a thirty-year-long controversy about this number within the lattice community (even the order of magnitude had been in question). Universality classes of percolative critical behavior other than the random one are obtained when an interaction is present among the lattice sites. For the two-dimensional Ising model, which provides the simplest example of such a correlated percolation problem, percolative and magnetic observables are simultaneously critical in zero field, while a continuous percolative transition exists in an external field above  $T_c$ , namely in a region where there are no magnetic singularities. We showed in [4] how integrable field theory allows an exact description of this phenomenon, and determined in [3] universal properties of both Kasteleyn-Fortuin and spin Ising clusters. Predictions for the three-point connectivity of critical Potts clusters have also been obtained in [1].

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## Free Photon as a Bound State

N. Delić†, J.P. Setrajčić, S. Armačević, I.J. Setrajčić, S. Cvetković

†University of Novi Sad, Faculty of Science, Department of Physics

The Hamiltonian of free photon is not a linear operator which contradicts the principle of superposition. The paper describes in detail the equations for obtaining Pauli matrix and their correspondence with the spin matrices (without any approximation) to obtain the photon Hamiltonian. It is important to note that the Pauli matrices are obtained only to follow the principle of superposition. Graviton in the paper is used to explain the situation regarding bound state  $W$  minus bosons and electron neutrinos to form an electron. At the end, since massless neutrinos are needed to form a massless photon, a composite photon is not possible; the only possible explanation of this phenomenon can be explained by the interaction of graviton, which provides this link. The Hamiltonian for free photon is not a linear operator which contradicts the principle of superposition Klein and Gordon skirted this problem solving the eigen problem of square of Hamiltonian. Since the square of Hamiltonian is a linear operator. This approach has given satisfactory description of the single photon behaving. Up to now it is considered that this approach gives real picture of photon. Here it will be demonstrated that Kline -Gordon picture of photon is incomplete [1]. Here we shall try to examine the single photon behavior [2] by means of linearized Hamiltonian. Linearization procedure is analogous to the procedure which was used by Dirac in the analysis of relativistic electron Hamiltonian [3]. Photon can rotate around its axis and this rotation is characterized by the spin vector, photon Hamiltonian which is given, can be expressed by means of spin operators. The obtained form of photon Hamiltonian, which includes the operators of translation moment and spin suggest that a free photon has wealthy internal dynamics that consists of mutual action of its translation and spin characteristics. This "internal life" will be examined further in the paper. In connection with this fact is the hypothesis that the photon is a bound state of neutrino and antineutrino. This hypothesis has been sent to CEWQO Symposium 2010 [4]. The hypothesis that the photon bound states of the neutrino and antineutrino has resulted in the first place that there are three types of photons, due to the fact that there are three types of neutrinos: electron, muon and tau neutrino.

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## Minimal model of financial stylized facts

D. Delpini†, G. Borometti

†University of Pavia and INFN, Pavia Division, Italy

The analysis of financial data has revealed features remarkably stable across different markets. Specifically, stock price returns for daily and intra-day frequencies exhibit a leptokurtic distribution and their volatility, stochastic as well, features a slowly decaying, multi-scale autocorrelation function. Also, the negative correlation between returns and volatilities is reflected in the exponentially decaying leverage effect. We discuss the statistical characterization of a linear Stochastic Volatility Model where the instantaneous volatility distributes at the stationary state according to an Inverse Gamma distribution [1]. Our choice is mainly motivated by past analysis showing that the Inverse Gamma law provides the best agreement with the empirical distribution of volatility proxies [2][3]. Here it emerges as a limit case from the more general dynamics of a multiplicative diffusion process with quadratic diffusion coefficient, which was extensively studied in terms of its moments both at finite times and at the stationary state [3]. For the moments of the distribution of price returns we detail the derivation of a general expansion depending on the starting time of the volatility process. This result clarifies the role played by the Inverse Gamma law in the emergence of fat tails and more precisely it shows that the higher order moments of the return process diverge, for every time lag, as long as the volatility has reached its stationary state. This signals that the probability of extreme returns for the considered model scales in agreement with a power law decay, and the corresponding tail index turns out to be constrained by the shape parameter of the volatility. At the same time, the explicit expressions of the skewness and the kurtosis reveal that a Gaussian-like limit in the central region of the distribution is recovered for large time horizons. We also present analytical results for the return-volatility correlation and the volatility autocorrelation functions and we comment on the consequences of the Inverse Gamma assumption on the corresponding time scales. Then we develop a systematic methodology for estimating the parameters, and we describe the empirical analysis for the case of the Standard & Poor 500 index daily returns. Finally we discuss the results of this analysis, which confirms the ability of the model to capture many of the established stylized facts as well as the scaling properties of empirical distributions over different time horizons.

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## Collective behaviour of cell assemblies: Understanding the microscopic interactions through the derivation of macroscopic laws

C. Deroulers<sup>†</sup>, M. Badoual

<sup>†</sup>Universit Paris Diderot-Paris 7, Laboratoire IMNC

As in condensed matter physics, it is common when one studies assemblies of living cells that the macroscopic behaviour cannot be derived trivially from the microscopic interactions. In contrast, cell-cell interactions are often not well-known, which makes the problem even more difficult. Only macroscopic or mesoscopic data in a simple (e.g. in vitro) situation is at hand. Still, it is not enough to postulate a macroscopic model which explains these idealised data, because it can't be used with confidence in a realistic (in vivo) situation. Knowledge of the cell-cell interactions is needed to derive a more reliable model. Here, starting from a few plausible cell-cell interaction laws, we derive the corresponding macroscopic behaviour in a simple experimental setup. Comparing these predictions with experimental data allows one to exclude some of the microscopic hypotheses. Then the selected macroscopic model(s) can be used for predictions in a more realistic setup (e.g. inside a patient). Our first example tries to understand the migration of some brain tumour (glioma) cells, which is a major goal: it makes cancer cells invade tissues far from the tumor center. Thus the tumors have no sharp boundary. A surgeon cannot remove all cancer cells, and cancer recurs invariably. In a cellular automaton-like model, some biologically inspired stochastic rules mimic how the individual cells move. Such an approach can reproduce in vitro experiments of migration of a few cancer cells on different substrates (collagen or a monolayer of noncancerous cells). It enabled us to show that cells interact with each other and with noncancerous cells while migrating [1,2]. To model now a real tumour, a cellular automaton is not as convenient as partial differential equations (PDEs), because it is not suited for the study of populations of several millions of cells. From the previous experimentally validated stochastic rules, we derived approximate PDEs [3] and used it to test an hypothetic treatment strategy in patients [4]. Interestingly, the PDEs are, because of the interactions between cells, nonlinear diffusion equations, whereas it is often postulated that diffusion of cancer cells follows Fick's law as would passive Brownian particles do. In our second example, heaps of different types of cells were prepared using magnetic nanomanipulation, which is a promising technique for tissue engineering [5]. The shape of the heaps and their evolution depend on the cell interactions. Comparing data with macroscopic models built upon different interaction laws yields information about the interactions.

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## Exploding dissipative solitons

O. Descalzi<sup>†</sup>, C. Cartes, J. Cisternas, H.R. Brand

<sup>†</sup>Universidad de los Andes

Following the theoretical work of Ruelle, Takens and Newhouse it had been shown experimentally for Rayleigh-Benard convection that by increasing the Rayleigh number beyond the appearance of the second frequency chaos sets in. More recently it has been shown in a combined experimental and numerical study that electroconvection in a thin, sheared film can lead to a transition to localized chaotic convection following the Ruelle-Takens route. In this work we show the analogue of the Ruelle-Takens route for spatially localized solutions in a prototype equation, namely, the cubic-quintic complex Ginzburg-Landau (CGL) equation. In particular this equation models a passively mode-locked laser operating in a regime in which it produces dissipative exploding solitons. The CGL equation has been derived in different contexts such as binary fluids mixtures, nematic liquid crystals, or chemical reactions since it describes the dynamics of a system at the onset of an oscillatory instability. When coexistence is observed as is the case in binary fluid mixtures (convective state surrounded by conductive state) cubic-quintic CGL equations serve to qualitatively describe localized convective regions and their interactions both, in the absence and in the presence of noise. The cubic-quintic CGL equation, a dispersive-dissipative system, has stable localized solutions including pulses and holes due to its complex coefficients (which render the equation non-variational) and quintic non-linearities. Pulses and holes are either stationary, breathing, chaotic, or moving. In the anomalous dispersion regime exploding dissipative solitons have been obtained and it has been shown that the transition to exploding solitons arises after the appearance of oscillatory localized solutions with two frequencies, as the control parameter is increased. The transition between localized solutions with one and with two frequencies, respectively, is analyzed in detail. It is found to correspond to a forward Hopf bifurcation for these localized solutions as the bifurcation parameter is increased. In addition, we make use of power spectra to characterize all time-dependent states. On the basis of all informations available, we conclude that the sequence oscillatory localized solutions with one frequency, oscillatory localized solutions with two frequencies, exploding dissipative solitons can be interpreted as the analogue of the Ruelle - Takens - Newhouse route to chaos for spatially localized solutions.

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## Unification of the psychophysical laws and the additive property of the inconsistency degree in intertemporal decision making

N. Destefano<sup>†</sup>, A.S. Martinez

<sup>†</sup>University of São Paulo

Intertemporal decision making involves choices among options whose effects occur at different moments. The implications of these choices on everyday activities led to the search of its underlying principles. The consensus is that delayed rewards are discounted (or undervalued) relative to immediate rewards. The discount process may first be assigned to changes in perception (evaluation) of a reward value at different moments. However, individuals, when forming their intertemporal preferences, may estimate time intervals in a non-objective manner. Thus, the discount process in intertemporal choices comprises not only the effect of reward value perception at different moments, but also the time perception effect. One of the main difficulties in determining discount functions from experiments is the simultaneity of both effects on time discounting. The independent analysis of each factor is not allowed by standard experiments that directly measure these functions. From discount functions one can obtain other quantities validated by experiments. The impulsivity measures the strong preference for immediate rewards over delayed ones, even though the magnitude of the delayed reward is more advantageous. Also, individuals tend to prefer smaller immediate rewards in the near future (reflecting impulsivity) but tend to prefer larger later rewards in the distant future. This preference reversal over time is referred as inconsistency in intertemporal choices. In this work we unify the Weber-Fechner and Stevens psychophysical laws using the one-parameter logarithmic function from nonextensive statistical mechanics. This allows us to propose new general discount value functions. As a main result, we find that, for each association of value and time perception effects, it is possible to dissociate the functions that describe the inconsistency in two parts: one describing the value perception effect and the other the time perception one. Moreover, the total value of the inconsistency degree is obtained from the sum of these two terms. In other words, the determination of the intricate dependence between value and time perception effects in the discount function may be softened exploring their additivity in the degree of inconsistency. Since value and time perception are additive in the inconsistency degree, experiments may be designed to measure them independently. By integration of the degree of inconsistency, one obtains the impulsivity and discount functions. The standard experiments used in the context of intertemporal decision making need to be reformulated for better understanding of the governing processes.

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## Exact solutions to nonlinear Fokker-Planck equations

J.I. Deza<sup>†</sup>, R.R. Deza, H.S. Wio

<sup>†</sup>UNMdP, Argentina

Nonlinear Fokker-Planck equations [1] have found applications in various fields such as plasma physics, surface physics, astrophysics, the physics of polymer fluids and particle beams, nonlinear hydrodynamics, theory of electronic circuitry and laser arrays, engineering, biophysics, population dynamics, human movement sciences, neurophysics, psychology and marketing. In spite of the diversity of these research fields, many phenomena addressed therein have a fundamental physical mechanism in common. They arise due to cooperative interactions between the subsystems of many-body systems. These cooperative interactions result in a reduction of the large number of degrees of freedom of many-body systems and, in doing so, bind the subunits of many-body systems by means of self-organization into synergetic entities. These synergetic many-body systems admit low dimensional descriptions in terms of nonlinear Fokker-Planck equations that capture and uncover the essential dynamics underlying the observed phenomena. These phenomena range from equilibrium and nonequilibrium phase transitions and the multistability of systems to the emergence of power law and cut-off distributions and the distortion of Boltzmann distributions. In this work, exact solutions are found and discussed for a class of one- and two-variable piecewise-linear Fokker-Planck equations. The pursuit is inspired by our previous work on one-dimensional reaction-diffusion systems on a bounded domain with partially reflecting boundary conditions [2-4]. The solution method, which resorts to Laplace transformation by assimilating the spatial coordinate to a time variable, presents several advantages over the traditional one (e.g., solution matching between sub-domains is automatically accomplished by the boundary conditions), being the most important one its suitability for extension to fractional Fokker-Planck equations, when the fractional derivatives are defined in the sense of Caputo. For one-variable piecewise-linear Fokker-Planck equations, the use of partially reflecting (albedo) boundary conditions allows to explore new and more realistic scenarios. For two-variable piecewise-linear Fokker-Planck equations, Dirichlet boundary conditions are assumed for simplicity. The existence and stability of solutions to these nonlinear Fokker-Planck equations as functions of the diverse parameters is thoroughly discussed.

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## Synchronization in networks of mobile oscillators

A. Diaz-Guilera<sup>†</sup>, N. Fujiwara, J. Kurths

<sup>†</sup>Universitat de Barcelona

We present a model of synchronization in networks of autonomous agents where the topology changes due to agents motion. We introduce two timescales, one for the topological change and another one for local synchronization. If the former scale is much shorter, an approximation that averages out the effect of motion is available. Here we show, however, that the time required for synchronization achievement is larger than the prediction of the approximation in the opposite case, especially close to the continuum percolation transition point. The simulation results are confirmed by means of spectral analysis of the time-dependent Laplacian matrix. Our results show that the tradeoff between these two timescales, which have opposite effects on synchronization, should be taken into account for the design of mobile device networks. After an initial period of characterizing complex networks in terms of local and global statistical properties, attention turned to the dynamics of their interacting units. A widely studied example of such behavior is synchronization of coupled oscillators arranged into complex networks. The interplay between topology and dynamics is crucial for synchronization achievement. In most studies of such systems the network has a fixed structure, but there are also many interesting scenarios where the topology changes over time in various fields, such as power transmission system, consensus problem, mobile communication and functional brain networks. Within the general framework of time-dependent or evolving networks, we can identify the particular case of networks whose nodes represent physical agents that move around but interact with each other only when they are close enough. Examples include the coordinated motions of robots, vehicles, and groups of animals, in which cooperative dynamics emerge. Especially, there are many examples where synchronization plays a crucial role: chemotaxis, mobile ad hoc networks, and wireless sensor networks. Despite the importance of this topic, prior research on synchronization in time-dependent networks of populations of agents has concentrated so far on two special cases: (i) where the network topology changes fast and (ii) where the population is dense and arranged in a ring. In the former case, the fast-switching approximation (FSA), which averages out the effect of agent motion, is commonly used. However, for better understanding of synchronization of mobile agents and design of an efficient network, it is very important to clarify when and how FSA fails.

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## Complex dynamics of transcriptional response: how do cells get on the fast lane?

E. Domany<sup>†</sup>, A. Zeisel, W.J. Köstler, N. Molotski, J.M. Tsai, R. Krauthgamer, J. Jacob-Hirsch, G. Rechavi, Y. Soen, S. Jung, Y. Yarden

<sup>†</sup>Dept of Physics of Complex Systems, Weizmann Institute of Science

A major component of the response of cells to changing conditions is a shift of the transcriptome to a new state which may be more adequate for facing the new environment. The time dependence of this shift is of considerable interest and is often inferred from measurements of mRNA concentrations. By common wisdom, based on a "minimal model", this dynamic process is fairly simple: for each gene, the mRNA production rate,  $\lambda$ , and possibly also the mRNA degradation rate,  $\gamma$ , change to a new fixed value. The new steady state mRNA levels are then given by  $\lambda/\gamma$ , while the time it takes for the gene to get close to this desired value is governed by  $\lambda$  alone. By monitoring time-dependent changes of pre-mRNA abundance in MCF10A cells stimulated by EGF we discovered that in general, this simple picture is inaccurate. In order to bring genes to higher desired levels in a short time, cells use a strategy of production overshoot for a relatively brief interval, which is reflected by the time dependent pre-mRNA levels. Basically, cells "floor the accelerator" for a short burst, in order to get onto the fast lane. Conversion of pre to mature mRNA is a fast process; hence the short-time dynamic response of mRNA abundance is governed by the magnitude and duration of the overshoot. By formulating and solving differential equations for production of pre-mRNAs, their conversion into mRNAs and the degradation of the latter, we are able to infer production and degradation rates from the measured kinetics of the pre- and mature mRNAs. This estimation of mRNA production and degradation rates does not require transcriptional inhibition or interfering in any other way with the process. Both production and degradation rates show complex, unexpected dynamics. In particular, the transcription of genes whose mRNAs reach their peak between 4 to 8 hours after stimulation often starts in the first hour. This suggests that inference of transcriptional co-regulation might be improved by grouping genes according to their production times, rather than by their mRNA profiles. To demonstrate the universality of these results, we studied the response of murine dendritic cells to LPS stimulation and the differentiation of human embryonic stem cells in response to retinoic acid. In both systems we found genes that exhibited pronounced production overshoot of the respective pre-mRNAs.

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## Why the "explosive percolation" transition is actually continuous

S.N. Dorogovtsev<sup>†</sup>, R.A. da Costa, A.V. Goltsev, J.F.F. Mendes

<sup>†</sup>University of Aveiro

We describe the current state of the art for the explosive percolation problem and present the solution of this quest. Until recently, the percolation phase transitions were believed to be continuous, however, in 2009, a remarkably different, discontinuous phase transition was reported in a new so-called "explosive percolation" problem. Each link in this problem is established by a specific optimization process. It was reported that in this process, the percolation cluster emerges discontinuously and already contains a finite fraction of nodes at the percolation threshold. This conclusion was based on computer experiments. To emphasize this surprising discontinuity, this kind of percolation was named "explosive". Further investigations of "explosive percolation" in similar systems, also based on numerical simulations, supported this strong result but, in addition, surprisingly for abrupt transitions, revealed power-law critical distributions of cluster sizes resembling those found in continuous percolation transitions. This self-contradicting combination of discontinuity and scaling have made explosive percolation one of the challenging and urgent issues in the physics of disordered systems. We resolve this confusion and show that there is not actually any discontinuity at the "explosive percolation" threshold, contrary to the conclusions of the previous investigators. We develop the exact theory of this phenomenon. Employing strict analytical arguments for a representative class of models, we find that the "explosive percolation" transition is actually continuous though with a uniquely small critical exponent of the percolation cluster size. These transitions provide a new class of critical phenomena in irreversible systems and processes. We obtain a complete description of the scaling properties of these second order transitions. For all these models, we find the scaling functions and the full set of critical exponents, and, also, the upper critical dimensions which turn out to be remarkably low. Our theory explains the continuous nature of the explosive percolation transition and indicates the order parameter and susceptibility for this irreversible process. We also explain the results of computer experiments for models of this kind. These transitions provide a new class of critical phenomena in irreversible systems and processes.

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## Algebro-geometric approach to Yang-Baxter equation

V. Dragovic<sup>†</sup>

<sup>†</sup>GFM Univ. Lisbon/Math. Inst. SANU Belgrade

One of the central objects in mathematical physics in last 25 years is the  $R$ -matrix, or the solution  $R(t; h)$  of the quantum Yang-Baxter equation as a paradigm of modern understanding of the addition relation. Here  $t$  is so called spectral parameter and  $h$  is Planck constant. If the  $h$  dependence satisfies the quasi-classical property  $R = I + hr + O(h^2)$  the classical  $r$ -matrix  $r$  satisfies the classical Yang-Baxter equation. Classification of the solutions of the classical Yang-Baxter equation was done by Belavin and Drinfeld in 1982. The problem of classification of the quantum  $R$  matrices is still open. We present algebro-geometric approach to the quantum Yang-Baxter equation, based on Krichever's notions of vacuum vectors and vacuum curves. Having in mind Baxter's considerations leading to the discovery of the Baxter  $R$  matrix and Faddeev-Takhtadzhyan study of the vectors, Krichever suggested a sort of inverse approach, following the best traditions of the theory of the "finite-gap" integration. Krichever's method is based on the vacuum vector representation of an arbitrary  $2n \times 2n$  matrix  $L$ . Such a matrix is understood as a  $2 \times 2$  matrix with blocks of  $n \times n$  matrices. In other words,  $L$  is a linear operator in the tensor product  $C^n \otimes C^2$ . The vacuum vectors  $X; Y; U; V$  satisfy, by definition, the relation  $LX \otimes U = hY \otimes V$ . We assume additionally the following convention for a notation  $X_n = Y_n = U_2 = V_2 = 1; U_1 = u; V_1 = v$ . The vacuum vectors are parametrized by the vacuum curve, which is defined by the affine equation  $P_L(u; v) = 0$ : The polynomial  $P_L(u; v)$ , called the spectral polynomial of the matrix  $L$  is of degree  $n$  in each variable. In general position, the genus of the spectral curve is equal  $g = (n - 1)^2$  and Krichever proved that  $X$  understood as a meromorphic function on the curve is of degree  $N = g + n - 1$ . And, following ideology of "finite-gap" integration, Krichever proved a converse statement. We give classification of  $4 \times 4$  rank one solutions. We discuss geometric interpretation and possible higher-dimensional generalizations. It appeared that vacuum curves in  $4 \times 4$  case are exactly Euler-Chasles correspondences. The Yang-Baxter equation itself provides the condition of commutation of the two Euler-Chasles correspondences. The classification follows by application of the Euler theorem in general case, and by studying possible degenerations. This is practically the same picture we meet in the study of Poncelet theorem.

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### Transition state theory: a generalization to nonequilibrium systems with power-law distributions

J. Du†

†Tianjin University

Transition state theory (TST) made it possible to obtain quick estimates for reaction rates of a broad variety of processes in physics, chemistry, biology, and engineering. It has been a cornerstone or a core of reaction rate theory and profoundly influenced the development of theory of chemical dynamics. However, one key assumption to TST is that thermodynamic equilibrium must prevail throughout the entire system studied for all degrees of freedom; all effects that result from a deviation from thermal equilibrium distribution, such as Boltzmann-Gibbs (B-G) distribution with the exponential law, are neglected [1]. In the reaction rate theory, we are interested in the processes of evolution from one metastable state to another neighboring state of metastable equilibrium; the assumption thus would be quite farfetched. An open problem, which is being investigated intensively, is reaction rate theory away from equilibrium. TST is no longer valid and cannot even serve as a conceptual guide for understanding the critical factors that determine rates away from equilibrium [2]. Thereby TST must be generalized if it is employed to describe rates of reactions in the nonequilibrium systems with non-exponential or power-law distributions. We deal with the stochastic dynamics for the reaction coordinate and momentum, which is modeled by the Langevin equations and the corresponding Fokker-Planck equations, and then we study the conditions under which the stochastic dynamics will give rise to the stationary power-law distributions. In the investigation of the generalized TST for a nonequilibrium system with the power-law distributions, we consider a general many-body Hamiltonian system. It is assumed that the system far away from equilibrium has not to relax to a thermal equilibrium state with the B-G distribution, but asymptotically approaches to a nonequilibrium stationary-state with the power-law distributions. Thus, following the standard line of TST we obtain a generalization of TST rates made suitable for a nonequilibrium system with the power-law distribution for positive and reverse reaction processes. Furthermore, we derive more specific expressions of the generalized TST rate constants for a one-dimension and an n-dimension case for a nonequilibrium Hamiltonian system. Finally, we consider the potential function as a harmonic approximation at the saddle point of the high barrier and the minimum of the potential bottom, respectively, thus we receive a generalized Arrhenius rate for the nonequilibrium system with the power-law distribution.

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### Some peculiarities of escape from a metastable state for anomalous diffusion in the form of Levy flights

A. Dubkov†, A. La Cognata, B. Spagnolo

†Lobachevsky State University

The barrier crossing event for superdiffusion in the form of symmetric Levy flights and relevant noise-induced phenomena are analytically investigated. As it is well-known, we cannot apply standard Markovian theory of mean first-passage times in such a case because of non-local boundary conditions. First of all, using the backward fractional Fokker-Planck equation [1] derived from Langevin equation with the noise source having  $\alpha$ -stable probability distribution we obtain the general differential equation useful to calculate the nonlinear relaxation time for a particle moving in a fixed cubic potential. For Cauchy noise ( $\alpha = 1$ ) we obtain the closed expression in quadrature for the mean residence time in a potential well as a function of the noise intensity, the initial position and the parameters of cubic potential. A monotonic behavior of the nonlinear relaxation time as a function of the initial position of the particle is obtained in this case. For free anomalous diffusion the nonlinear relaxation time decreases monotonically with increasing the noise intensity as in the presence of the cubic potential [2]. Basing on the exact quadrature formula, further we investigate some peculiarities of the noise enhanced stability phenomenon [3] for Levy flights, putting a particle in an unstable initial position just beyond the potential barrier. The non-monotonic behavior of the lifetime of metastable state as a function of the noise intensity is demonstrated. The area in which there is an enhancement of lifetime with respect to the dynamical time is found. Because of heavy tails, the particle spends a finite time in the area of metastability even for very small intensity of driven noise. In this limit and for unstable initial positions of particle after the potential barrier but upper than potential well level we have divergent behavior of lifetime with Gaussian noise source. For Levy flights, however, we obtain for lifetime a finite non-monotonic behavior as a function of the noise intensity. Then we analyze the resonant activation phenomenon [4] for jump diffusion, i.e. diffusion induced by white non-Gaussian noise source with  $\alpha$ -stable probability distribution in the system with randomly flipping potential barrier. Basing on the set of Kolmogorov's equations for joint probability density functions we obtain the approximate results for the escape time as a function of mean rate of barrier switching.

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## Matryoshka locally resonant sonic crystal

D.P. Elford<sup>†</sup>, L. Chalmers, F.V. Kusmartsev, G.M. Swallowe

<sup>†</sup>Loughborough University

The results of numerical modelling of sonic crystals with resonant array elements are reported. The investigated resonant elements include plain slotted cylinders as well as various their combinations, in particular, Russian doll or Matryoshka configurations. The acoustic band structure and transmission characteristics of such systems have been computed with the use of finite element methods. The general concept of a locally resonant sonic crystal is proposed, which utilises acoustic resonances to form additional band gaps that are decoupled from Bragg gaps. An existence of a separate attenuation mechanism associated with the resonant elements, which increases performance in the lower frequency regime has been identified. The results show a formation of broad band gaps positioned significantly below the first Bragg frequency. For low frequency broadband attenuation a most optimal configuration is the Matryoshka sonic crystal, where each scattering unit is composed of multiple concentric slotted cylinders. This system forms numerous gaps in the lower frequency regime, below Bragg bands, whilst maintaining a reduced crystal size viable for noise barrier technology. The finding opens new perspectives for construction of sound barriers in the low frequency range usually inaccessible by traditional means including conventional sonic crystals. Recent years have seen a growing interest in the potential for the use of sonic crystals as noise barriers, with reported sound attenuation up to 20 dB and 25 dB. Such crystals usually consist of periodic arrays of a high mechanical impedance material (often as cylindrical rods) and are known to give high attenuation at selective but often rather narrow frequency bands as a consequence of multiple scattering phenomena. An advantage of sonic crystals noise barriers is that, by varying the distance between the scatterers, it is possible to attain peaks of attenuation in a selected frequency range. Further advantages of a sonic crystal barrier in comparison with more traditional solid sound barriers, are its ability to allow light to pass and, uniquely, that it does not present an obstruction to the free flow of air. The relationship between the lattice parameter and operating frequency suggest extremely large barriers will be required to attenuate lower frequency noise such as traffic noise.

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## Randomized central limit theorems

I. Eliazar<sup>†</sup>, J. Klafter

<sup>†</sup>Holon Institute of Technology

The Central Limit Theorems (CLTs) stand at very foundation of Probability Theory and Statistical Physics. The CLTs address the stochastic scaling limits of aggregates and extrema of large ensembles of independent and identically distributed (i.i.d.) random variables. In the case of aggregates the CLTs assert that the ensembles' stochastic scaling limits are governed by either Gaussian or Lévy probability laws, and in the case of extrema the CLTs assert that the ensembles' stochastic scaling limits are governed by either the Fréchet, Weibull or Gumbel Extreme Value probability laws. The underlying i.i.d. random variables represent the microscopic details of the ensemble considered, whereas the ensemble's stochastic scaling limits represent its macroscopic behavior. Hence, each CLT establish a "universality effect" emerging in the transition from the microscopic to the macroscopic levels: The statistics of the CLT scaling limits are invariant with respect to the statistics of the underlying i.i.d. random variables - and are hence universal. In particular, the CLTs for aggregates determine the universal fluctuations of random walks, and the CLTs for extrema determine the universal statistics of rare and extreme events. In this Talk we introduce the notion of "Randomized Central Limit Theorems" (RCLTs) - in which the deterministic CLT scaling schemes are replaced by stochastic scaling schemes - and explore "randomized counterparts" to the CLTs. The pioneering RCLT result is, in effect, Holtsmark's law for gravitational fields. In Holtsmark's setting stars with i.i.d random masses are scattered randomly and homogeneously across the three dimensional Euclidean space. Also, a unit-mass probe is set at an arbitrary point in space, and the gravitational force exerted (along a given axis) by the stars on the probe is measured. The random distances of the stars from the probe turn out to induce a stochastic scaling scheme of the star-masses. This stochastic scaling scheme yields Holtsmark's law - which asserts that the gravitational force exerted on the probe is invariant with respect to the statistics of the underlying i.i.d. star masses, and is thus universal. In this Talk we set forth from Holtsmark's RCLT result and establish a comprehensive and unified theory of RCLTs. We present "randomized counterparts" to the classic CLTs, show that the RCLT stochastic scaling schemes are governed by Poisson processes with power-law statistics, and show that the RCLT stochastic limits are governed by the Lévy, Fréchet and Weibull probability laws.

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## Universal statistics and control of random transport processes

I. Eliazar<sup>†</sup>, J. Klafter

<sup>†</sup>Holon Institute of Technology

Random transport processes are central in a broad range of scientific fields ranging from propagation in disordered media to search patterns of foraging animals, and from human travel to drug delivery. This Talk addresses - and affirmatively answers - the following elemental "universality question": Are there statistics which are common to all random transport processes - regardless of their underlying spatial topology and transport mechanism? We consider a transport model in which independent agents are stochastically emitted into a general space - with general topology and general transport mechanism. The agents are detected only when in a target zone - an arbitrary subset of space. The agents' "target-zone statistics" include the processes tracking (i) the number of agents present in the target zone, (ii) the first passage times of the agents to the target zone, and (iii) the residence times of the agents in the target zone. Lady Fortuna governs the statistics of the agents' random paths in space, whereas we control the statistics of the agents' emission times and velocities (with which they move along their random paths). In the context of this general setting we establish a set of "universality results" that determine the class of "emission controls" which render the agents' "target-zone statistics" invariant with respect to agents' random paths - which, in turn, depend on the underlying spatial topology and transport mechanism. All the path-invariant "target-zone statistics" are shown to possess a universal intrinsic power law structure. The universal results presented in this Talk are of fundamental importance. On the practical level the results establish a class of "emission controls" which facilitate the universal control of the "target-zone statistics" - regardless of the underlying spatial topology and transport mechanism. Such "emission controls" are resilient with respect to environmental changes. On the theoretical level, the results provide a unified and universal mechanism for the generation of an array of ubiquitous power law statistics: (i) anomalous diffusion - characterized by power-law mean square displacements; (ii) 1/f noise - characterized by power-law power spectra; (iii) Zipf's and Heaps' laws - empirically characterizing power-law word-frequencies and word-vocabularies; (iv) the Beta and Pareto laws - characterized by power-law probability densities; (v) Weibull's and Frchet's extreme-value laws, and Levy's one-sided law - characterizing, respectively, the universal linear scaling limits of minima, maxima, and sums of positive-valued i.i.d. random variables.

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## Classical randomness induced transitions in quantum walks

D. Ellinas<sup>†</sup>

<sup>†</sup>Technical University of Crete Department of Sciences Division of Mathematics

With applications in the field of quantum information processing in mind, where quantum walks are to be called as subroutines by larger computational or communicational algorithms, we investigated the role of classical noise in quantum walks (QW). Classical randomness in the form of discrete telegraphic noise or continuously distributed random variables affecting the reshuffling SU(2) matrices of quantum walks on integers are investigated. Carrying out an analytic study in terms of quantum statistical moments and generating functions over classical distributions, or at the operator level in terms of density matrix unital completely positive trace preserving (cptp) maps, we study transitions of QW performance induced by classical randomness. A marked continuous transition is found in the asymptotics of the quantum walks by tuning the telegraphic noise parameters, from the regime of classical diffusion with time rate  $O(\sqrt{t})$ , to the regime of quantum diffusion with time rate  $O(t)$ , for the second statistical moment of the position operator. A similar mathematical analysis carried out for continuous random variables reveals a complex mechanism of switching from classical to quantum behaviour for the QW, in relation to the contractive or not contractive character of the action of a stochastic averaging cptp map on coin system density matrix. The role of classical randomness in a QW is further investigated at the stochastic average level, where it is shown that randomized quantum walk when treated on the stochastic average by means of an appropriate cptp evolution map for the density matrix of its walker system, turns out to be equivalent to a classical walk quantized according to the so called epsilon-quantization method. This result emphasizes the dual role of quantization and randomization in the context of a classical walk. For the resulting evolution maps in the above considerations, identified as cptp maps with unitary Krauss generators, various unitarizations schemes are investigated by employing the unitary dilation theorem. The overall conclusion is that classical randomness may turn out to be remarkable tuning mechanism between quantum and classical regime of functioning of QW, thus an accessible tool of influence performance of quantum algorithms.

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## The probabilistic solutions of some large nonlinear stochastic dynamic systems with State-Space-Split method

G.K. Er†

†Faculty of Science and Technology, University of Macau, China

The probabilistic solutions of nonlinear stochastic dynamic systems excited by white noises are governed by Fokker-Planck (FP) equations. Hence the FP equations are widely used in various areas. However, it is difficult to obtain the exact solution of FP equations in practice. Only in some special cases, the exact solutions are obtainable for some one or two-degree-of-freedom systems. Practical problems are usually modeled as large multi-degree-of-freedom (MDOF) systems. Therefore, the equivalent linearization (EQL) method was proposed which is suitable for the approximate solution of MDOF nonlinear stochastic dynamic (NSD) systems [1]. The advantage of the EQL method is that it can be used for analyzing large NSD systems, but it is considered unsuitable when the system nonlinearity is not weak or multiplicative random excitations are present, because in either case the probability distribution of the system response is usually far from being Gaussian. Various methods were proposed to improve the solutions, but all these methods are only limited to analyzing one or two-degree-of-freedom systems. Monte Carlo simulation (MCS) can be employed for analyzing MDOF systems, but the amount of computation with it is usually unacceptable for obtaining the PDF solutions of large nonlinear stochastic dynamic systems, especially for small probability problems. The problem of numerical stability, convergence, round-off error, and the requirement for large samples may also limit the application of MCS method for analyzing large nonlinear dynamic systems. A new methodology named state-space-split (SSS) method was proposed by the author recently [2, 3]. The SSS method is combined with the exponential polynomial closure (EPC) method [4, 5] to analyze the probabilistic solutions of large NSD systems or the FPK equation in high dimensions. Therefore, the whole solution procedure is named SSS-EPC method in the statement. In this paper, the SSS-EPC method is further employed to analyze the PDF solutions of some MDOF nonlinear stochastic dynamic systems to verify the effectiveness of the SSS-EPC method in these cases. Numerical results show that the results from SSS-EPC method are close to Monte Carlo simulation for some large MDOF NSD systems with polynomial type of system nonlinearity compared to the results from equivalent linearization method.

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## Numerical convergence of the block-maxima approach to the Generalized Extreme Value distribution

D. Faranda†, V. Lucarini, G. Turchetti, S. Vaienti

†Department of Mathematics and Statistics, University of Reading

We perform an analytical and numerical study of Extreme Value distributions in discrete dynamical systems. In this setting, recent works have shown how to get a statistics of extremes in agreement with the classical Extreme Value Theory. We pursue these investigations by giving analytical expressions of Extreme Value distribution parameters for maps that have an absolutely continuous invariant measure. We compare these analytical results with numerical experiments in which we study the convergence to limiting distributions using the so called block-maxima approach, pointing out in which cases we obtain robust estimation of parameters. In regular maps for which mixing properties do not hold, we show that the fitting procedure to the classical Extreme Value distribution fails, as expected [Freitas et al, 2009]. However, we obtain an empirical distribution that can be explained starting from a different observable function for which Nicolis et al. [2006] have found analytical results. Our main finding is that a block-maxima approach for Generalised Extreme Value distribution is totally equivalent to fit an Extreme Value distribution after normalising sequences are computed. To prove this we have derived analytical expressions for normalising sequences, showing that  $V$  correspond to the parameters of fitted Generalised Extreme Value distribution. This approach works for maps that have an absolutely continuous invariant measure and retain some mixing properties that can be directly related to the exponential decay of Hitting Time Statistics. Since Generalised Extreme Value distribution approach does not require the a-priori knowledge of the measure density that is instead required by the Extreme Value approach, it is possible to use it in many numerical application [Vitolo et al. 2009]. The theory was generalised to the case of non smooth observations and therefore it holds also with non absolutely continuous invariant probability measure. In this way we introduce a new way to compute the dimension of fractal obtained using Iterated Function Systems through the parameters estimation of extrema. The numerical simulations show agreement with known results. Understanding the full extreme values behavior for singular measures it is crucial to apply proficiently this analysis to operative geophysical models since in these case we are always dealing with singular measures providing a complete tool to study extreme events in complex dynamical systems used in geophysical or financial applications.

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## Flux-Flow Oscillator (FFO) made with the Fluxon Cloning

H. Farhan-Hassan<sup>†</sup>, F.V. Kusmartsev, D.R. Gulevich, P.N. Dmitriev and V.P. Koshelets

<sup>†</sup>Loughborough University

In present paper we have developed a new device, Flux-Flow Oscillator (FFO) where flux cloning phenomena have been demonstrated. Such FFO made with the use of flux cloning circuit can in principle operate even without magnetic field, that is in a very different manner than conventional FFO developed nowadays for practical applications. We have designed such a novel device and build it up with the use of the long Josephson T-shaped junction of a linear overlap geometry made up with Nb-AlO<sub>x</sub>-Nb technology. We have theoretically described the properties of such a device and the dynamics of vortices there. These theoretical studies have been performed in the framework of a sine-Gordon model, which includes surface losses. Finally we have tested the device experimentally and demonstrated that the flux cloning can lead to a strong coherent terahertz radiation. There the shape of the spectral lines and the current-voltage characteristics have been also measured. Vortices, tornados and hurricanes, like Katrina, may arise suddenly anywhere. Their prediction has grave importance for our life. The vortex nucleation has been most studied in superconductors. A common belief written in all textbooks is that a single vortex cannot be nucleated inside a superconductor. It may only penetrate from the border or be created in a pair together with anti-vortex. This fact follows from the fundamental law of the vorticity conservation[1]. However for multi-connected weak superconductors (Josephson junction, JJ), there may arise a fluxon cloning, the phenomenon predicted theoretically in the Ref. [1,2]. In general, the fluxon cloning circuits provide the producing fluxon without applied magnetic field. It is worth noting that there are many interesting configurations for a long Josephson junction such as overlap, inline and annular geometries. However, the inline and annular structures are not suitable for application as flux flow oscillators (FFO)[3,4], therefore previous studies have used only overlap structure as FFOs operate by applying external magnetic field. In this structure, there arise Fiske resonances which are associated with strong emission of electromagnetic radiation. Recently, it was shown that the cloned vortices may be ordered to form a train of fluxon, which is eventually operating as a flux flow oscillator created without external magnetic field for annular geometry by means of T-junction [5]. In this paper we will confine our attention to study theoretically and experimentally how a fluxons cloning circuits can be used as a FFO operating without external magnetic field.

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## Simple model of magnetization processes in rare-earth tetraborides

P. Farkasovsky<sup>†</sup>, H. Cencarikova

<sup>†</sup>Institute of Experimental Physics, Slovak Academy of Sciences

We present a simple model for a description of magnetization processes in rare-earth tetraborides. The model is based on the coexistence of two subsystems, and namely, the spin subsystem described by the Ising model and the electronic subsystem described by the Falicov-Kimball model [1] on the Shastry-Sutherland lattice. Moreover, both subsystems are coupled by the anisotropic spin-dependent interaction of the Ising type. To examine the magnetization curve corresponding to our model Hamiltonian we have used a well-controlled numerical method based on the modification of the small cluster exact diagonalization calculations [2]. Using this method we have performed exhaustive numerical studies of the model for a wide range of model parameters, selected on the base of experimental measurements [3]. We have found, that the switching on of the spin-dependent interaction ( $J_z$ ) between the electron and spin subsystems and taking into account the electron hopping on the nearest ( $t$ ) and next-nearest ( $t'$ ) lattice sites of the Shastry-Sutherland lattice leads to a stabilization of new magnetization plateaus. Besides the Ising magnetization plateau at  $m^{sp}/m_s^{sp} = 1/3$  we have found three new relevant magnetization plateaus located at  $m^{sp}/m_s^{sp} = 1/2, 1/5$  and  $1/7$  of the saturated spin magnetization  $m_s^{sp}$ . The ground-states corresponding to magnetization plateaus have the same spin structure consisting of parallel antiferromagnetic bands separated by ferromagnetic stripes. For  $m^{sp}/m_s^{sp} = 1/3$  our results are identical with ones obtained within the Ising [3] as well as easy-axis Heisenberg [4] model on the Shastry-Sutherland lattice. The accordance between our and the easy-axis Heisenberg solution [4] is found also for  $m^{sp}/m_s^{sp} = 1/2$ . In this case both approaches predict the sequence of parallel antiferromagnetic and ferromagnetic stripes. For  $m^{sp}/m_s^{sp} = 1/5$  our results postulate a new type of spin ordering. In addition, we have found, that our model is able to described the magnetization plateaus also at  $m^{sp}/m_s^{sp} = 1/7, 1/9$  and  $1/11$  (in accordance with experimental measurements in  $TmB_4$  [3]) but the stability regions of these phases are much narrow in comparison to ones of  $1/2, 1/3$  and  $1/5$  plateau phases. Finally, the transitions from the low temperature ordered phase to the high-temperature disordered phase are analysed through the behaviour of the specific heat and the energy distribution at the transition point by the canonical Monte-Carlo method.

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## Random reversals of the magnetic field generated by turbulent flows

S. Fauve<sup>†</sup>

<sup>†</sup>Ecole normale supérieure

It has been known since the end of the 19th century that mechanical work can be converted to electromagnetic energy using a dynamo. This energy transformation process is based on an instability mechanism. It has been proposed by Larmor in 1919 that the magnetic field of the sun is also generated by electromagnetic induction due to the flow of the electrically conducting solar plasma. It is now believed that the magnetic field of most planets and stars results from the dynamo process. However, in these astrophysical objects, no electric wiring neither magnetic material, constrain the geometry of the current and field lines. In addition, the flow is strongly turbulent. How is it then possible to generate the large scale coherent magnetic field observed for many stars and planets by extracting work from an incoherent turbulent flow? What are the respective contributions of the mean flow and the fluctuations to the dynamo process? How do the fluctuations affect the dynamo threshold? Above threshold, is the scaling law for the mean magnetic energy density qualitatively modified by the fluctuations? I will present an experiment performed with a turbulent von Karman flow of liquid sodium (the VKS experiment), designed to answer some of the above questions and that led to the generation of a magnetic field. After a short review of the main results, I will discuss the phenomenon of field reversals, also observed in this experiment: the mean magnetic field generated by the flow has, to leading order, the shape of an axial dipole. On long time scales, several thousands of the large scale flow eddy turn-over time, this magnetic field randomly reverses its polarity. This phenomenon displays a hierarchy of time scales similar to the Earth's magnetic field: phases with a given polarity of the magnetic field are on average much longer than the duration of a reversal which is itself much longer than the hydrodynamic time scales. A striking result of the experiment is that, despite the strong level of turbulent fluctuations and the random occurrence of reversals, their geometry, i.e., the trajectory in phase space that connects the two states with opposite polarities, is very robust, such that successive reversals can be clearly superimposed. When the dipolar component of the magnetic field reverses, part of the magnetic energy is transferred to the quadrupole. Reversals also display a characteristic asymmetry: they begin with a slow decay of the dipolar component, followed by a much faster recovery and end with an overshoot of the dipole above its value in the quasi-stationary regime. All these features, together with the other regimes of the magnetic field observed in the experiment, are understood in the framework of dynamical system theory.

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## Non-commutative integrable systems

R.L. Fernandes<sup>†</sup>

<sup>†</sup>Instituto Superior Tecnico

The existence problem for global action-angle variables for a commutative integrable system goes back to the classical paper of Duistermaat (1980), where he finds the complete set of obstructions. The notion of non-commutative integrable systems goes back to A.S. Miscenko and A.T. Fomenko, in the 1980's, who proved an action-angle type theorem for them, and was studied in detail in more recent works (e.g., [1,2]). The study of local action-angle variables in the setting of Poisson manifolds can be found in [3], paving the way for a theory of global action-angle variables in the non-commutative context. Our main aim is to describe the global geometry of non-commutative integrable systems on Poisson manifolds and the existence of global action-angle variables. In order to understand the approach proposed here, we consider first the classical case of a Lagrangian fibration with non-singular fibers  $p : S \rightarrow B$ . By the classical Arnold-Liouville Theorem, the base of the fibration carries an integral flat affine structure. This means that for each  $b \in B$  there is a full rank lattice  $P_b \subset T_b^*B$ , and that the union of these lattices give a smooth tale fibration  $P \rightarrow B$ . The monodromy of this lattice is called the Hamiltonian monodromy of the Lagrangian fibration and is the first obstruction to the existence of global action angle-variables. In fact, as shown by Duistermaat, it is the obstruction for the fibration, which is a torus bundle, to be given by a torus action. The second obstruction comes from requiring this bundle to be trivial as Lagrangian bundle and this leads to the notion of a Lagrangian Chern class. There is a geometric way to understand all this. The bundle  $T^*B \rightarrow B$  can be thought of as symplectic (for the canonical symplectic structure) groupoid (in this case, an abelian group bundle) which acts symplectically on the bundle  $p : S \rightarrow B$  (groupoids act on maps, rather than on spaces; "symplectic" means that the graph of the action is a Lagrangian submanifold). This action is locally free and transitive, so its kernel is an tale subgroupoid  $P \rightarrow B$  (the monodromy lattice). Moreover, since the action is symplectic, this kernel is a Lagrangian subgroupoid. The quotient  $T^*(B)/P \rightarrow B$  is therefore a Lagrangian fibration. Now there are two issues: is this fibration trivial? Is the original fibration isomorphic (as a Lagrangian fibration) to this fibration? The obstructions to a positive answer to each of these questions are the obstructions given above and first discovered by Duistermaat. We show that this approach can be extended for non-commutative integrable systems in Poisson manifolds.

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### Coupled Ising models and interdependent binary choices: a simple model

A. Fernández del Ríof, E. Korutcheva, J. de la Rubia

†Departamento Física Fundamental (UNED)

The study of socioeconomic discrete choice problems with interaction (of social influence type) allows for the use of statistical mechanical tools and models [1,2]. The simplest possible model is the infinite range Ising model with constant field, which considers completely homogeneous populations which already gives rise to interesting socioeconomic interpretations [1]. Two such Ising models coupled directly through an additional term in the Hamiltonian have been studied. Two different coupling schemes are considered. General analytical derivations have been carried out for the nonzero field case, solutions have been numerically calculated and phase diagram sections constructed for the zero fields (no pure private deterministic utilities) case. The non-local model [3] (Weiss type coupling term) can be understood as describing the same decision making process in two groups, where individuals are subject to social influence from both their group and the other one. Examples of interest can be the study of public opinion on a given subject in two neighbouring countries, companies in two related business sectors and their production technology option... In the local model [4], a single group is considered where coupling is only through each individual, which is making two interdependent decisions. It can be used to study the interaction between different social pathologies or traits, opinion on related issued and demand contexts. Both phase diagrams are qualitatively similar with a major difference: in the non-local case, for strong enough inter choice coupling, there are no stable solutions. This and other differences can be naturally explained in a choice making context. Both models present both second order and first order phase transitions. There are regions with multiple equilibria, some physically equivalent, others metastable (yielding hysteresis possible). For both models there are regions of the phase diagrams where social and choice interaction count and regions where it does not (and these will be the same for the nonzero field models). Choice interdependence introduces a higher trend to consensus (not favouring any specific direction). It also gives rise to interesting considerations concerning metastability and hysteresis in the light of interacting groups/decisions whose perception/interdependence is reversing. Metastability regions involve (socially reinforced) situations in which the groups/options may be prevented from aligning (or dis-aligning) even when the conditions seem right.

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### Spin-glass model and inverse phase transition

U. Ferrari†, L. Leuzzi

†Dipartimento di Fisica, Università "Sapienza", and IPCF-CNR, UOS Roma, Italy

In macromolecular compounds in solution certain complex molecules or polymeric chains can fold into practically inactive conformations, displaying a negligible interaction with the surrounding system. The presence of inactive components can induce the existence of a fluid phase at a temperature below the temperature range at which the system is in a solid phase (crystalline, semi-crystalline or amorphous, depending on the degree of frustration). This corresponds to the occurrence of an *inverse transition*, else said "melting upon cooling", that is a reversible transition between a completely disordered isotropic fluid phase and a solid phase whose entropic content is greater than the entropy of the fluid. Recently it has been shown how the Blume-Emery-Griffith (BEG) spin-glass model, with quenched disorder, present such an *inverse phase transition*. This result was ruled out by both analytical calculation on its mean-field approximation and numerical simulations, with the appearing of a thermodynamic first order transition with a reentrance in the phase diagram. We propose a generalization of the mean field BEG model, introducing a  $p$ -body interaction:  $\mathcal{H} = -\sum_{i_1 < \dots < i_p} J_{i_1 \dots i_p} \sigma_{i_1} \dots \sigma_{i_p}$  the exact solution for the glassy phase is obtained by the one step replica symmetry breaking Ansatz. Different scenarios arise for both the dynamic and the thermodynamic transitions. These include (i) the usual random first order transition (Kauzmann-like) preceded by a dynamic transition, typical of mean-field glasses, (ii) a thermodynamic first order transition with phase coexistence and latent heat and (iii) a regime of inversion of static and dynamic transition lines. In the latter case a thermodynamic stable glassy phase (i.e., zero configurational entropy) is dynamically accessible from the paramagnetic phase. A detailed study of the complexity and of the stability of the static solution is performed throughout the space of external thermodynamic parameters.

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## Probing the dynamics of Andreev states in a coherent Normal/Superconducting ring

M. Ferrier<sup>†</sup>, F. Chiodi, B. Dassonneville, K. Tikhonov, P. Virtanen, T.T. Heikkilä, M. Feigelman, S. Gueron, H. Bouchiat

<sup>†</sup>Laboratoire de Physique des Solides, Univ. Paris-Sud, CNRS, Orsay Cedex, France

A dissipationless current is known to flow through a thin (subnanometer) insulating barrier between two superconductors  $S$  with a superconducting phase difference, the well-known Josephson effect. This supercurrent may also flow through a long (micrometer) non-superconducting metal wire at low temperatures, a spectacular consequence of the quantum phase coherence throughout the normal metal. The way the supercurrent responds to a dc phase difference, the current-phase relation [1], was only recently measured with a Hall probe [2]. It reflects the phase dependence of the Andreev bound states (ABS) [3], entangled electron-hole states which form in the normal metal as a consequence of the superconducting mirror-like boundary conditions. These phase dependent ABS were recently detected in a carbon nanotube between  $S$  electrodes. However little is known about the dynamics of ABS and their relaxation. In order to investigate the evolution of the current phase relation in such superconductor/normal metal/superconductor (SNS) junctions with high frequency phase driving, we have inductively coupled one NS ring to a multimode superconducting resonator. The in-phase ( $x'$ ) and out-of-phase ( $x''$ ) ac magnetic susceptibility of the ring is deduced from the dc flux dependence of the resonance frequency and quality factor of the resonances from 300 MHz up to 6 GHz. Whereas at very low temperatures we essentially measure the phase derivative of the supercurrent, at higher temperature we find a strong frequency dependence in the current response of the ring: the ABS do not follow adiabatically the phase modulation. Indeed, different behaviors are observed for frequencies below and above the inverse of the diffusion time through the normal wire. Moreover, when increasing the rf excitation amplitude beyond linear response the out of phase response exhibits a strong increase close to odd multiples of a half flux quantum corresponding to the closing of the induced minigap in the N wire. These results are compared to recent theoretical findings and stimulate future similar investigations on more exotic junctions. This experiment also illustrates a new tool to probe the fundamental time scales of phase coherent systems that are decoupled from macroscopic normal contacts and thermal baths.

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## Models of driven polymer translocation

A. Fiasconaro<sup>†</sup>, J.J. Mazo, F. Falo

<sup>†</sup>Centro Universitario de la Defensa de Zaragoza and Instituto de Ciencia de Materiales de Aragón, CSIC-Universidad de Zaragoza

Transport mechanism of molecules inside cells and/or through cell membranes is getting nowadays more and more attraction. The increasing interest in this subject is related to the intrinsic importance of understanding the basic mechanisms of the living systems, but also in the enormous improvement of technological capabilities at nanometer length scale. These, on the one hand, allow to detect and to measure the mechanism at the nanoscale, and, on the other hand, open the possibility of constructing from scratch structures (with both natural and synthetic materials) able to imitate the biological functioning [1]. In this context, the passage of biomolecules through nanopores is ubiquitous, both in biological and in nanotechnological processes. Examples of these two types are the passage of mRNA through nuclear pores or the translocation of DNA in graphene pores. Long molecule translocation is usually driven by constant fields in the pore or by chemical potential differences in both sides of the membrane. In other cases the translocation is assisted by an ATP-based molecular motor [2]. Goal of this work is to present the model of a simple motor driving the translocation of a one dimensional polymer chain with different drivings, such as oscillating, dichotomous and more specific ATP-based mechanism, i.e. a molecular motor activated by the absorption of molecules of ATP. The ATP based motor, quite different from the two others, drives with a constant force a polymer chain in one direction, while in its activated state, while it leaves the polymer diffuse freely otherwise. The activated state remains fixed in time and the waiting times arise from an exponential distribution of times. The mean activation time gives rise to the Michaelis-Menten behavior of the polymer velocity. The values of the mean translocation time and the velocity of the polymer as function of the activation frequency of the motor has been evaluated. The latter presents a very good agreement with the experiments showing a clear Michaelis-Menten dependence with the ATP concentration, and, consequently, the same kind of behavior arise for the stall force of the motor. The simple mechanism is also able to reproduce qualitatively recent experimental outcomes of the DNA packaging in bacteriophage [3,4], with a capsid simulated as a simple restriction on the space of polymer destination.

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## Large deviation functions for systems coupled to two thermostats

H. Fogedby<sup>†</sup>, A. Imparato

<sup>†</sup>Aarhus University, Denmark

There is a strong current interest in the thermodynamics and statistical mechanics of fluctuating systems in contact with heat reservoirs and driven by external agents. The current focus stems from the recent possibility of direct manipulation of nano-systems and bio-molecules. These techniques permit direct experimental access to the probability distribution functions for the work or for the heat exchanged with the environment. These methods have also opened the way to the experimental verification of the recent fluctuation theorems, which relate the probability of observing entropy-generated trajectories, with that of observing entropy-consuming trajectories. We shall here focus on the Gallavotti-Cohen fluctuation theorem [1] which establishes a simple symmetry for the large deviation function for systems arbitrarily far from thermal equilibrium. Close to equilibrium linear response theory applies and the fluctuation theorem becomes equivalent to the usual fluctuation-dissipation theorem relating response and fluctuations. Here we consider two non-equilibrium systems: A harmonically bound Brownian particle and a harmonic chain, both coupled to two distinct heat reservoirs at different temperatures. A simple example of non-equilibrium system has been introduced recently by Derrida and Brunet [2]. In this model a single particle or rod is coupled to two heat reservoirs at different temperatures. For this model the large deviation function can be determined explicitly. Here we extend this model by allowing the particle to be bound in a harmonic potential. We evaluate the large deviation function for the bound particle using the method by Derrida and Brunet, based on deriving a differential equation for the heat characteristic function. In the case of a harmonic chain coupled to two reservoirs [3] the heat transport takes place by means of the ballistic motion of phonons (lattice vibrations) and a local heat gradient cannot be maintained. Here the large deviation function for the harmonic chain is found using a path integral formulation together with a transfer matrix method. For the chain the large deviation function depends parametrically on the transmission Greens function (see also [4]). In both cases the large deviation functions are consistent with the Gallavotti-Cohen fluctuation theorem related to the entropy production at the heat sources. We support the analytical results with numerical simulations based on sampling independent trajectories.

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## Analytical and numerical studies of magnetic nanodiscs and their responses to magnetic field gradients with some applications in magnetic sensing

M. Forrester<sup>†</sup>, F.V. Kusmartsev, E. Kovacs, K. Kuerten, R. Giles

<sup>†</sup>Loughborough University

In the natural world there is an abundance of life that functions on a day-to-day basis under the influence of the Earth's magnetic field. For example, the migration of birds is governed by their sensitivity to magnetic fields through light-sensitive cells in the retina. The photochemical reaction in the retina results in a pair of reactive radicals being produced, the reaction yield of which is dictated by the orientation of the bird with respect to the geomagnetic field. This reaction yield controls the visual perception of the bird and enables its navigation [1]. Also, birds have complex structures of magnetite in their beaks (trigeminal system) that may aid their movements by enabling magnetic field detection or amplification. It has proved to be extremely difficult to understand the mechanisms of magneto-reception across the animal-kingdom, especially as humans lack sensitivity to fluctuations in magnetic field gradients themselves. But, for those involved in biomimetics, understanding the responses in nature to electromagnetic fields is of paramount importance. As such, it is one of our goals in nanomagnetism to reproduce and enhance the designs we see in nature and in doing so we create metamaterials. We give a full description of the magnetic response of nanomagnets to an applied magnetic field through a series of phase diagrams that describe the changes in the magnetic moments of  $N$  interacting magnetic discs to an applied magnetic field gradient. As a magnetic field gradient is applied to a magnetic multilayer (MML) of nanodiscs there emerges a series of reversible (RPC) and/or discontinuous phase changes (DPC's) in the magnetization that are associated with the response of the magnetic moment  $s$  (MM's) to the field. With understanding of the magnetic field strengths at which these phase changes occur, the precise targeting of the MML's response characteristics to an applied magnetic field (AMF) can be done. For many applications, such as creating magneto-receptors, the interesting physics is generally not that achieved at the saturation field and may occur around a DPC of the MM orientation. Thus, we examine the system of three interacting MML's in great detail, demonstrating a complete analytical [3] and numerical description [4] for the first time, and show the phases associated with magnetization reversals. We find that nanomagnets are extremely versatile and can even be tailored to create terahertz detectors or devices controlled by T-rays [5].

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## Critical behaviour of the generalised two-dimensional self-avoiding trails model for polymers

D. Foster<sup>†</sup>, C. Pinettes

<sup>†</sup>LPTM, Universit de Cergy-Pontoise

We present results for the two-dimensional generalised self-avoiding trail model in order to reconcile the different apparently contradictory results on the critical behaviour of the self-avoiding trails model. Our results suggest that the collapse transition in this model corresponds to a possible coexistence between two critical phases, giving the collapse transition first-order and critical properties. The understanding of the critical behaviour of self-avoiding walk models remains topical since lattice walk models form the basis of many models used to study the behaviour of biomolecules (both static and dynamic). It is therefore important to understand which results are intrinsic to the system being studied and which results may arise from competition effects between the interactions present in the model and the underlying lattice. The standard model for a polymer models the quality of the solvent through the introduction of an attractive interaction energy between non-consecutive nearest-neighbour visited sites. The quality of the solvent then changes from good solvent to bad solvent as the temperature is lowered, passing through the theta point. This model reproduces well the observed physics of a polymer in dilute solution. It may be expected that as long as the salient features of the physical system (range of interactions and excluded volume interaction) are included in the model, the critical behaviour of the system does not depend on the fine details of the model (lattice type, precise way in which the short-range interactions are included...) A number of models now show that this is not always the case, due to geometrical frustration effects between the walk and the lattice. This is particularly true of the Self-Avoiding Trail model, introduced as a competing model for polymer collapse. In this model the walk is allowed to visit sites more than once, and cross, but remains self-avoiding on the lattice sites. Two competing views on the universality of the collapse transition have emerged. In the first, the collapse transition behaviour seems to be Mean-Field[1,2] like and in the second, the behaviour seems more like the behaviour of the  $O(n = 0)$  model introduced by Blote and Nienhuis[3,4,5]. We investigate the generalisation of this model, including a bending energy and different energies for doubly visited sites where the walk crosses (crossings) and where the walk does not cross (collisions). Our results indicate that the true picture is likely to be a coexistence between two critical phases.

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## Modelling of traffic flow at an intersection with the possibility of turning

E. Foulaadvand<sup>†</sup>, S. Belbasi

<sup>†</sup>Zanjan university, Iran

We have developed a Nagel-Schreckenberg cellular automata model for describing of vehicular traffic flow at a single intersection. A set of traffic lights operating in fixed-time scheme controls the traffic flow. Open boundary condition is applied to the streets each of which conduct a uni-directional flow. Streets are single-lane and cars can turn upon reaching to the intersection with prescribed probabilities. Extensive Monte Carlo simulations are carried out to find the model flow characteristics. In particular, we investigate the flows dependence on the signalisation parameters, turning probabilities and input rates. It is shown that for each set of parameters, there exist a plateau region inside which the total outflow from the intersection remains almost constant. We also compute total waiting time of vehicles per cycle behind red lights for various control parameters. We have tried to shed some light onto the problem of traffic optimisation by extensive simulations. Total current is shown to remain constant on a plateau of green time period given to one of the directions. Three dimensional plot of total current for given input rates of vehicles in terms of signalisation parameters shows the existence of a 2D plateau region encompassed by almost flat planes of sharp decreasing currents. Besides total current, total waiting time per cycle has been computed. Our investigations reveal that in the parameter space, the minimisation of total waiting time per cycle does not fully coincide with the maximisation of total current. This arises the natural question of *what quantity should be optimised in order to acquire the most efficiency for the intersection?* In our Nagel-Schreckenberg cellular automata, not only the case of uni-cell occupation, where each cell accommodates a car, has been considered but also the more realistic case of multi cell occupation in which a car occupies more than one cell has been investigated. The corresponding results differ quantitatively and in some case qualitatively. Seeking for advanced models properly designated for modelling the behaviours of drivers at intersections is inevitable and unavoidable. We would like to end by emphasizing on the role of empirical data for adjusting the parameters of any intersection traffic model. Despite utilising multi-cell occupation can render the deceleration/acceleration value of moving cars to a realist one, the behaviour of halted cars in the queue when the light goes green is still suffering from a satisfactory modelling.

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### The time needed to board an airplane: a power law and the structure behind it

V. Frette<sup>†</sup>, P.C. Hemmer

<sup>†</sup>Stord/Haugesund College, Norway

A simple model for the boarding of an airplane is studied. Passengers enter the airplane in arbitrary order with respect to their reserved seats. Queues are formed along the aisle, as some passengers have to wait to reach the seats for which they have a reservation, while others are busy taking their reserved seats. In most cases the boarding process is much slower than for the optimal passenger order: there is only one case (permutation) where the passenger order is identical to the seat order. We characterize this dynamical system by calculating the average boarding time, when all permutations of  $N$  passengers are given equal weight. To first order, the boarding time for a case (permutation) is given by the number of sequences of monotonically increasing values (with each passenger labeled by the number of his/her seat) along the queue direction. We show that the distribution of such sequences is symmetric on  $[1, N]$ , which leads to an average boarding time  $(N + 1)/2$ . The full distribution of sequences is shown to approach the normal distribution as  $N$  increases. Furthermore, we have found an exact expression for the distribution of sequences of increasing numbers. However, there are significant corrections to the first order result for the average boarding time, due to certain correlations between ordering in the queue and the substrate (seats): this occurs for some (but not all) cases (permutations) where the sequence of the seats is partly mirrored in the passenger queue. These cases with correlations have a boarding time that is lower than the one predicted by the number of sequences of increasing numbers along the queue. Using enumeration we find, to second order, that the average boarding time behaves like  $N^\alpha$ , with  $\alpha \simeq 0.7$  (and not 1.0) as a result of these correlations. The large number of cases with reduced boarding time has been classified. We also give some indicative results on the geometry of the queue-substrate correlations, with a sorting into geometry groups. With increasing  $N$ , both the number of correlation (reduction) types and the number cases (permutations) belonging to each type increase rapidly. The total number of reductions in boarding time, as compared to the first order result, increases faster than  $N!$ , leading to the significant deviation in the value of the power law exponent.

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### The interplay between ultracold atoms, semiconductor surfaces and quantum electronic systems

M. Fromhold<sup>†</sup>, G. Sinuco-León, B. Kaczmarek, J. Shearing, T.E. Judd, R.G. Scott, T.W.A. Montgomery, A.M. Martin, P. Krüger and T.M. Fromhold

<sup>†</sup>University of Nottingham

This talk will explore how room-temperature semiconductor surfaces can manipulate atoms cooled to nK temperatures and, conversely, be probed by the atoms themselves. Quantum-mechanical reflection can shield the ultracold atoms from the disruptive influence of a nearby room-temperature surface. By considering experiments performed at MIT [1], it will be shown that inter-atomic interactions and the aspect ratio of the condensate both strongly affect the reflection process [2]. We show that Fresnel zone plates, fabricated in a solid surface, can sharply focus atomic Bose-Einstein condensates that quantum reflect from the surface or pass through the etched holes. The focusing process compresses the condensate by orders of magnitude despite inter-atomic repulsion. Crucially, the focusing dynamics are insensitive to quantum fluctuations of the atom cloud and largely preserve the condensates' coherence, suggesting applications in passive atom-optical elements, for example zone plate lenses that focus atomic matter waves and light at the same point to strengthen their interaction. We explore transmission zone-plate focusing of alkali atoms as a route to erasable and scalable lithography of quantum electronic components in two-dimensional electron gases embedded in semiconductor nanostructures. To do this, we calculate the density profile of a two-dimensional electron gas immediately below a patch of alkali atoms deposited on the surface of the nanostructure by zone-plate focusing. Our results reveal that surface-induced polarization of only a few thousand adsorbed atoms can locally deplete the electron gas. We show that, as a result, the focused deposition of alkali atoms by existing zone plates can create quantum electronic components on the 50 nm scale, comparable to that attainable by ion beam implantation but with minimal damage to either the nanostructure or electron gas. [2]. Finally, we present calculations, which predict that current through a two-dimensional electron gas (2DEG) can trap ultracold atoms  $\sim 1$  micron away with orders of magnitude less spatial noise than a more usual metal trapping wire [3]. This may enable the creation of hybrid systems, which integrate ultracold atoms with quantum electronic devices to give high sensitivity and control: for example, activating a single quantised conductance channel in the 2DEG can split a Bose-Einstein condensate (BEC) for atom interferometry. In turn, the BEC itself offers structural and functional imaging of quantum devices and transport.

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## General, combinatorial formula for the density of states: insights into the energy equipartition principle and the theory of phase transitions

A. Fronczak<sup>†</sup>

<sup>†</sup>Faculty of Physics, Warsaw University of Technology

A completely new approach to the problem of energy distribution in statistical mechanics is developed that results in a new, exact and general, combinatorial formula for the density of states (DOS) [1]. In our approach DOS is expressed by Bell polynomials that describe how equal energy portions are distributed between different (or identical) subsystems (i.e. isolated particles, pairs of interacting particles, triplets of particles etc.) of the considered system. Our formula provides a number of interesting insights into fundamental problems of equilibrium statistical physics. In particular, it formalizes the energy equipartition principle by showing what kind of systems (in terms of thermodynamic potentials) fulfils the homogeneous energy distribution. We also show that given our formalism one can investigate properties of the configuration space in the vicinity of phase transitions [2]. And, although our approach to phase transitions is fairly abstract it provides a few extremely interesting conclusions related to mechanisms leading to the occurrence of phase transitions. In particular, one can show that phase transitions go along with a very nontrivial pattern of energy distribution characterized by the well-defined classes of forbidden microstates. For example, in the critical point of the second order phase transition microstates with noninteracting particles are forbidden. In general, our approach provides convincing arguments for the hypothesis that higher-order phase transitions can not be observed in systems with only pairwise interactions. We claim that to observe higher order transitions one have to study systems, whose Hamiltonians include many-body interaction terms. A similar hypothesis has been put forward by F.W. Wu in [3]. It is also worth noting that the third order transition observed in the famous spherical model of T.H. Berlin and M. Kac [4] does not contradict the hypothesis. This is due to the fact that the spherical condition can be treated as a kind of many-body interaction. We illustrate our approach by applying it to: classical ideal gas, the non-interacting lattice gas and one-dimensional lattice gas, which is equivalent to one-dimensional Ising model [5]. In the vicinity of  $T = 0K$  the density of states describing the one-dimensional system shows a very interesting behavior, corresponding to a very specific degeneracy of energy distribution.

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## International trade network: a statistical mechanics perspective

A. Fronczak<sup>†</sup>, P. Fronczak

<sup>†</sup>Faculty of Physics, Warsaw University of Technology

In recent years, an extensive research effort has been devoted to analyzing the structure, function and dynamics of the international trade network (ITN) (see e.g. [1,2]) from a complex network perspective [3]. The knowledge of topological properties of this network and its evolution over time is not only important per se (e.g., because it enhances our descriptive knowledge of the stylized facts pertaining to the ITN), but it may be relevant to better explain macroeconomic dynamics. In particular, it has been suggested that the analysis of ITN may help to recognize the pattern of economic interdependencies responsible for propagation of crises across countries and can be used to explain the role of international trade in spurring efficiency of economic recovery after recession. Our approach to international trade is a significant step towards clarifying the mentioned macroeconomic issues. We use equilibrium statistical mechanics to study ITN. We analyze a set of year-by-year trade relationships between all countries of the world covering the time interval 1950-2000. Although the total number of countries and the overall economic conditions influencing the network change over the period, in each year ITN is shown to be a typical representative of the ensemble [4] in which every network,  $G$ , is assigned the probability  $P(G) \sim \exp[-H(G)]$ , where  $H(G)$  plays the role of network Hamiltonian given by the sum of all bilateral trade volumes multiplied by the specific field parameter conjugated to each trade flow. It is intriguing that the ensemble is characterized by a factorizable partition function which can be exactly calculated. Behind the descriptive power of our approach that is confirmed in a number of tests consisting in comparison of GDP-driven Monte Carlo simulations of the trade network with real data on ITN, it is also important to stress predictive abilities of the model. In particular, we show here that bilateral trade fulfills a simple fluctuation-response theorem [5], which states that the average relative change in trade is a sum of relative changes in GDPs of trade partners. We argue that the theorem provides valuable insights into mechanisms underlying the emergence and propagation of world-wide crises. Yearly changes in import / export volumes between countries prove that the theorem really holds. It is also worth to note that the famous gravity model of trade proven to be empirically strong through econometric analysis naturally emerges from our approach.

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### Ranking and clustering countries and their products; a network analysis

A. Gabrielli†, G. Caldarelli, M. Cristelli, L. Pietronero, A. Scala, A. Tacchella

†Istituto dei Sistemi Complessi (ISC), CNR, Rome, Italy

In this work we analyze the network of countries and products from UN data on country production following some pioneering recent studies [1,2]. By using the framework of network science and mathematics of graphs, we define [4] the country-country and product-product weighted networks. On this basis we are able to introduce a novel method of community detection based on elements similarity. More precisely it is based on the well-known "minimal spanning tree" algorithm with the introduction of additional ingredients consisting in isolating the strongly correlated parts of the spanning tree. As a result, in opposition with classical economical predictions, we find that the country clustering reveals unexpected socio-geographic links among the most competing countries. On the same footings the same community detection algorithm permits to study efficiently the products clustering. This new method therefore suggests a new bottom-up algorithm of classification of produced goods which can improve the standard methods adopted by the international institution. In the second part of the the talk we reformulate [4] in an exact algebraic and spectral context, the original "reflections method" introduced in [1,2] with the aim of correlating countries and products to get the ranking of both in the global competition frame. This new formulation allows us to clarify some problematic aspects of the original reflection method. In particular we are able to explain why the fitnesses of all countries converge in the large number of iterations to a constant value and the same happens for products. Moreover the same approach permits to reveal the spectral nature of the asymptotic ranking. In addition to that we define two alternative procedure to rank different countries and their products over the global market with the use of an appropriately generalized PageRanking methods. The first and rougher method is based on unbiased Random Walks on the bipartite network countries-products. The second one instead is characterized by the introduction of economically motivated bias in the Random Walks. These analyses are a good proxy of country GDP pro capita and therefore could be possibly used to determine both the robustness of a country economy in the global competition method and also predict their evolution in the future.

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### Fluctuations energy harvesting with nonlinear oscillators

L. Gammaitoni†, F. Cottone, I. Neri, F. Orfei, F. Travasso, H. Vocca

†NiPS Laboratory, University of Perugia

Among the renewable energy sources, kinetic energy is undoubtedly the most widely studied for applications to the micro-energy generation. At small scales (milli and micro) kinetic energy is usually available as non-equilibrium fluctuations, random vibrations or displacement noise. Fluctuations energy harvesting requires a transduction mechanism to generate electrical energy from motion. This is traditionally achieved by means of inertial generators with the mechanical component attached to an inertial frame that acts as the fixed reference. The inertial frame transmits the vibrations to an inertial mass attached to a spring, producing a relative displacement between them. In terms of energy balance, the input energy is divided into three main components: (1) part of the energy is stored into the dynamics of the mass and is usually expressed as the sum of its kinetic and potential energy: when the spring is completely extended (or compressed), the mass is at rest and all the dynamic energy is represented by the potential (elastic) energy of the spring. (2) part of the energy is dissipated during the dynamics meaning with this that it is converted from kinetic energy of a macroscopic degree of freedom into heat, i.e. the kinetic energy of many microscopic degrees of freedom. (3) finally, some of the energy is transduced into electric energy. The functioning of the vibration harvester, within this scheme, can usually be quantitatively described in terms of the dynamics of the two relevant quantities: the mass displacement  $x$  and the voltage difference  $V$ . Both quantities are function of time and obeys proper equations of motion. In general these are stochastic quantities due to the random character of practically available vibrations and the equations become stochastic differential equations, also know as Langevin equations. A common assumption is that the potential energy of the oscillator grows with the square of the bending (linear approximation). This approach presents some limitations that appears to conflict with the the main feature of being capable of harvesting energy on a broad-band of frequencies. The natural candidates to replace linear oscillators are the so called "non-linear" oscillators. In a recent years few possible candidates have been explored[1-4]. In this talk we discuss the performances of nonlinear oscillators with reference to the power generated in tested environments. The results presented here show that there are nonlinear potentials that allow better performances in terms of power generated by a vibration energy harvester, compared to the standard linear cantilever.

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## Exact maximum-likelihood method to detect patterns in real networks

D. Garlaschelli<sup>†</sup>, T. Squartini, G. Fagiolo

<sup>†</sup>Lorentz Institute for Theoretical Physics, University of Leiden, The Netherlands

In order to detect patterns in real networks, randomized graph ensembles that preserve only part of the topology of an observed network are systematically used as fundamental null models. However, their generation is still problematic. The existing approaches are either computationally demanding and beyond analytic control, or analytically accessible but highly approximate. Here we propose a solution to this long-standing problem by introducing an exact and fast method that allows to obtain expectation values and standard deviations of any topological property analytically, for any binary, weighted, directed or undirected network. Remarkably, the time required to obtain the expectation value of any property is as short as that required to compute the same property on the single original network. Our method reveals that the null behavior of various correlation properties is different from what previously believed, and highly sensitive to the particular network considered. Moreover, our approach shows that important structural properties (such as the modularity used in community detection problems) are currently based on incorrect expressions, and provides the exact quantities that should replace them. We will also consider a specific application of our method to the World Trade Web, or International Trade Network (ITN). The ITN has received renewed multidisciplinary interest due to recent advances in network theory. However, it is still unclear whether a network approach conveys additional, nontrivial information with respect to traditional international-economics analyses that describe world trade only in terms of local (first-order) properties. We employed our randomization method to assess in detail the role that local properties have in shaping higher-order patterns of the ITN in all its possible representations (binary/weighted, directed/undirected, aggregated/disaggregated) and across several years. We found that the properties of all binary projections of the network can be completely traced back to the degree sequence, which is therefore maximally informative and should become the main focus of economic models of trade. By contrast, all weighted representations of the ITN are not fully explained by weighted local properties, which are therefore of limited informativeness. This implies that a network perspective can convey original insights to the empirics of international trade.

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## Nonextensive statistical effects in the quark-gluon plasma formation at relativistic heavy ion collisions energies

G. Gervino<sup>†</sup>, A. Lavagno, D. Pigato

<sup>†</sup>Universit di Torino, Italy

The physics of high energy heavy ion collisions is directly connected to several problems in statistical mechanics and thermodynamics due to a large average number of particles involved and possible phase transition phenomena in the hot and dense fireball created during the collisions [1]. In relativistic heavy ion collisions the baryon density can reach values of a few times the saturation nuclear density and/or high temperatures. At large densities, constituent quarks are shared by neighboring baryons and should eventually become mobile over a distance larger than the typical size of one baryon. This means that quarks become deconfined and that at large densities and/or high temperatures they are the real degrees of freedom of strongly interacting matter instead of baryons. However, in energy density range reached in relativistic heavy-ion collisions, non-perturbative effects in the complex theory are not negligible. The generated plasma in the early stages of the collisions does not at all resemble a quasi-ideal gas of quarks and gluons because strongly dynamical correlations are present, including long-range interactions [1]. In the absence of a converging method to approach at finite density one often turns to (effective) model investigations. Recently, there is an increasing evidence that the nonextensive statistical mechanics can be considered as an appropriate basis to deal with physical phenomena where strong dynamical correlations, long-range interactions and microscopic memory effects take place [2]. In particular, in the last years there is a growing interest to high energy physics applications of nonextensive statistics and several authors have outlined the possibility that experimental observations in relativistic heavy ion collisions can reflect nonextensive statistical behaviors [3,4,5]. The main goal of this paper is to study how nonextensive statistical effects influence, from a phenomenological point of view, the nuclear equation of state and, as a consequence, the relative phase transition from hadronic matter to quark-gluon plasma at finite temperature and density reachable in high-energy heavy-ion collisions. Focusing our investigation to lower temperatures and higher baryon chemical potentials than the corresponding critical end point value, a mixed phase of hadrons, quarks and gluons can be formed following the Gibbs conditions for the phase equilibrium on the global conservation of baryon number, electric charge fraction and strangeness. We show that nonextensive statistical effects play a crucial role in the equation of state and in the formation of mixed phase also for small deviations from the standard Boltzmann-Gibbs statistics.

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### Statistical properties of the quantum vacuum under the influence of electromagnetic fields

A.S. Gevorkyan<sup>†</sup>, K.B. Oganessian

<sup>†</sup>Institute for Informatics and Automation Problems, NAS Armenia

In the representation of the quantum field theory the vacuum is the seat of all energetic particle and field fluctuations. In other words vacuum is characterized by physical parameters and structure that constitute an energetic medium which pervades the entire extent of the universe. If to assume that the quantum theory of field can be precisely described without perturbation, then the properties of the vacuum are analogous to the properties of an ensemble of the quantum harmonic oscillators. We considered for the first time this problem within limits of the stochastic differential equations of Maxwell-Langevin type. It has allowed us to construct the regular theory without application of the perturbation methods. In particular, for the quantum distribution, without borders and external fields the second-order partial differential equation is obtained. In the framework of this equation the equilibrium state of unperturbed Casimir vacuum is investigated in detail. It is shown that actually the free vacuum is quantized and the main parts of particles (approximately 86 percent) are concentrated on the ground state. Also it is shown that the accounting of quantum vacuum in the scheme of Maxwell electrodynamics, except for Minkowski space-time is described by two additional dimensionalities which are compactified. In other words the Casimir vacuum is a Bose condensate and satisfies conditions of medium for which the refraction indexes are unities. For the quantum distribution of vacuum under the influence of external fields the equation of Fokker-Plank type is obtained and the refraction indexes of vacuum are constructed. It is shown that in the region surrounding the region of localization external fields refraction indexes change subject to time by law  $\ln t$  on the large scale of space  $\ln r$ . These oscillations of refraction indexes create varying quantum halo which can be measured. Finally it is important to note that in standard relativistic quantum field theory, space-time is considered as a fixed arena in which the physical processes take place. This circumstance is the weak place of the theory because in a general case this assumption is not fulfilled. The developed approach allows solving this problem self-consistently. As a result of this, the obtained expressions for various parameters in a limit of thermodynamic equilibrium include effects which are usually characteristic of nonlinear and nonlocal theories.

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### New parallel algorithm for simulation of spin-glasses in the external field with consideration of relaxation effects

A.S. Gevorkyan<sup>†</sup>, H. Abajyan

<sup>†</sup>Institute for Informatics and Automation Problems, NAS of Armenia, RA

We study statistical properties of a disordered spin system under the influence of an external field taking into account relaxation effects. For the description of the system the spatial 1D Heisenberg spin-glass Hamiltonian is used. The system of spins is defined on a regular lattice where only one randomly oriented spin is located on each node of lattice which randomly interacts with its nearest-neighbor. Exact solutions which define angular configuration of a spin on nodes are obtained from equations of stationary points of Hamiltonian and the corresponding conditions for the energy local minimum. On the basis of these recurrent solutions an effective parallel algorithm is developed for simulation of stable spin-chains of an arbitrary length. It is shown that by way of an independent order of  $N^2$  numerical calculations (where  $N$  is the number of spin in the each chain) it is possible to generate ensemble of spin-chains, which is completely ergodic that is equivalent to full self-averaging of spin-chains' vector polarization. Distributions of different parameters (energy, average polarization by coordinates, and spin-spin interaction constant) of unperturbed system are calculated. In particular, it is analytically proved and by the numerically shown, that for the Heisenberg nearest-neighbor Hamiltonian model, the distribution of spin-spin interaction constants as opposed to widely used Gauss-Edwards-Anderson distribution satisfies Lévy alpha-stable distribution law. This distribution is a non-analytic function and does not have variance. In the work we have studied in detail critical properties of an ensemble depending on value of external field parameters (from amplitude and frequency) and have shown that even at weak external fields the spin-glass system is strongly frustrated. It is shown that frustrations have fractal character, they are self-similar and do not disappear at scale decreasing of area. It is shown by the numerical computation that the average polarization of spin-glass on different coordinates can have values which can lead to catastrophes in the equation of Clausius-Mossotti for dielectric constant. In other words, for some values of external field's parameter, in a system a critical phenomenon occurs which is impossible to describe by the real-valued Heisenberg spin-glass Hamiltonian. For the solution of this problem the complex-valued disordered Hamiltonian is used for the first time. Physically this type of Hamiltonian extension corresponds to accounting relaxation effects which occur in the medium under the influence of an external field. For this case also the parallel algorithm is developed. The efficiency of the algorithm is proved by the numerical calculations.

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### Self-Assembly mechanism in colloids

A. Giacometti<sup>†</sup>, R. Fantoni, F. Lado, J. Largo, G. Pastore, F. Sciortino

<sup>†</sup>Dipartimento di Scienze Molecolari e Nanosistemi, Università Cà Foscari Venezia

Motivated by recent experimental findings in chemical synthesis of colloidal particles, we draw an analogy between self-assembly processes occurring in biological systems (e.g. protein folding) and a new exciting possibility in the field of material science. We consider a self-assembly process whose elementary building blocks are decorated patchy colloids of various type, that spontaneously drive the system toward a unique and predetermined target macroscopic structure. To this aim, we discuss a simple theoretical model – the Kern-Frenkel model – describing a fluid of colloidal spherical particles with a pre-defined number and distribution of solvophobic and solvophilic regions on their surface. The solvophobic and solvophilic regions are described via a short-range square-well and a hard-sphere potentials, respectively. Computer simulations, integral equations and a simple cluster theory are presented to discuss structural and thermodynamical properties, with particular emphasis on the case of equal distribution of the two regions, the so-called Janus particle. We discuss both the case of one and two attractive caps, as a function of the fraction of covered attractive surface, thus interpolating between a square-well and a hard-sphere potential on changing the coverage. We show that integral equation theory provides quantitative predictions in the entire explored region of temperatures and densities from the square-well limit down to a significantly small value of the coverage, whose value depends upon the number of patches. Janus particles can be regarded as a particular case of the one-patch case in the limit of half-coverage. It is shown that for sufficiently small coverages, crystallization prevents the possibility of observing the evolution of the line of critical points, providing the angular analog of the disappearance of the liquid as an equilibrium phase on decreasing the range for spherical potentials. A very rich phase diagram is found, with a fluid-fluid transition competing with micellization and a fluid-solid transition, depending on the size and the number of the patches. Other interesting structures (micelles, lamellae, chains etc) are also observed at extremely low coverages. The relevance of these findings for the possible realization of self-assembling materials will be outlined.

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### On the time evolution of a population age distribution, including migration

D. Giusti<sup>†</sup>, V.G. Molinari

<sup>†</sup>ENEA- Bologna University

Aim of the present paper is to propose a kinetic equation whose subject is not a physical system composed by a large amount of particles, but the time evolution of an age distribution function of a population of men and women. As we shall see, and as it is to be expected, this equation does not drive the distribution function toward an equilibrium state as is the case for the Boltzmann equation. It is rather brought, by the "time arrow", toward the natural evolution of people's life. Once the kinetic equation has been obtained, it will be solved, and some examples of possible interest will be presented. The time evolution of interacting populations are often described using Volterra equation. This way one can only obtain time evolution of the whole number of individuals. In this work interactions are rather accounted in the usual way of transport theory, and age pyramids time evolutions are given. So far, it is our opinion that in this paper some problems concerning human demography have been worked out for the first time in the framework of kinetic theory: in this way it is possible to get more detail in the description of the human populations time evolution. We introduce the age-distribution function at a fixed time  $t$ , denoted by  $n(a, t)$ , and so defined that gives the probable number of persons which, at the time  $t$ , have age in the range about the value  $a$ . This does not mean that the given element actually contains this number of persons at the time  $t$ . This is the average number of such persons obtained when the fluctuations are disregarded. Therefore, any result we will obtain, must be understood as probable result of the evolution of the population considered. Besides, it is clear that the age-distribution function is never negative and that it must tend to zero as the age becomes very high. To determine the age distribution of a population, meant as a stably constituted set of individuals connected by bonds of reproduction, we must solve the kinetic equation. This is obtained by resorting to the integral transforms. As first step to obtain the kinetic equation, it is necessary to know some parameters connected with the behaviour of the rate of change in the age distribution function at a fixed time owing to the birth and to the mortality of the persons. Particularly attention will be given to the effect of the emigrations.

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## The dynamical behavior of spin-subband entanglement in a Rashba-Dresselhaus nanoloop

M.M. Golshan†, R. Safaiee

†Physics Department and Nanotechnology Research Institute, Shiraz University, Iran

The study of entanglement in composite systems is one of the major goals of quantum information science, where entangled states are regarded as a valuable resource for processing information in novel ways. Evidently, if the subsystems are completely disentangled, full information about each subsystem is available and vice versa. Since statistical information is contained in the entropy and since an increase in the entanglement would reduce the local information, it is logical to use the von Neumann entropy, as a measure of unavailable information (entanglement) [1]. Among the various proposals for experimental realizations of quantum information processing, solid state systems are particularly attractive since they are more easily integrated into large quantum networks. In particular, semiconductor nanostructures in which the electronic spin states are mixed with the environmentally induced states (spin-subband entanglement) are of special interest since the electronic spin provides a simple two-level quantum entity (qubit), whose entanglement dynamics is important in the development of quantum computations [2]. Spin-orbit coupling (SOC) in such nanostructures provides a way for orbital degrees of freedom to influence spin states. Two basic mechanisms of SOC are the Rashba and Dresselhaus spin-orbit couplings. It is demonstrated that the SOC's strengths strongly depend upon applied gate voltages [3]. It is then evident that in investigating the dynamical behavior of electronic spin-subband entanglement in heterostructures, these couplings would be of great importance. Choices of heterostructural materials, along with externally applied electric fields, reduce the electronic motional freedom, confining it to 1D nanowires, 2D quantum dots, quantum loops, etc. [4]. The quantum loop has provided new means for construction of quantum memories and networks [5]. In the present work, we report the spin-subband entanglement dynamics of the electron in a quasi-1D quantum loop acted upon by a strong perpendicular magnetic field, where both the Rashba and Dresselhaus SOC's are taken into account. Assuming that the system forms a pure ensemble, we calculate the time evolution of the density operator, from which the von Neumann entropy of the reduced density matrix, as a function of time, is obtained. As a result, we demonstrate that for a noncoherent spin initial state, the entanglement exhibit periodic oscillations whose amplitudes strongly depend upon the relative strength of Rashba and Dresselhaus SOC's as well as applied magnetic field. Moreover, in the long run, the entanglement undergoes the phenomenon of collapse-revivals, arising from the Dresselhaus SOC.

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## Position dependent entanglement of electronic spin-subband states in a Rashba Nanoloop

M.M. Golshan†, R. Safaiee

†Physics Department and Nanotechnology Research Institute, Shiraz University, Iran

As a fundamental concept in quantum communication, quantum computation, etc., quantum entanglement has attracted vast attention in recent years [1,2]. Combinations of electronic spin states and environmentally induced states in heterostructures are considered as outstanding candidates for realizing entanglement and, consequently, solid-state-based quantum computing. The spin-orbit interaction (SOI) in semiconductor heterostructures, due to the asymmetry of the interface potential, provides a way to couple electronic spins with its orbital degrees of freedom. In particular, nanoloop heterostructures have provided new means for constructing quantum memories, switchable flux transformers, spin interferometers and so on [3]. In the present study, we investigate the space-dependence of spin-subband entanglement dynamics for a model of electrons confined in a quasi-one-dimensional quantum loop in a strong perpendicular magnetic field with Rashba SOI. In so doing, we first present the governing Hamiltonian in the second quantization form. For simplicity, we approximate the confining (gate) effects as a parabola and then the orbital motions of the electron can be reduced to a harmonic oscillator [4]. The von Neumann entropy, a measure of entanglement, is then computed as a function of time. Using the well established material parameters for InAs nanoloops [5], our numerical computation of von Neumann entropy shows that, in general, the spin-subband entanglement varies from one point to another along the nanoloop. Taking the initial condition as a pure disentangled one, formed by subband and the state belonging to the z-component of spin, the entanglement exhibits periodic oscillations with just one local minimum (dip) in each oscillation, independent of the electronic location in the nanoloop. On the other hand, when the initial state is formed by the subband states and a coherent superposition of spin states, the entanglement still periodically oscillates either with two local minima (dips) or without them, depending on the location of the electron in the loop. We further show that for an entangled initial state formed by subband excitations and spin states, regardless of electronic position in the loop, the entanglement undergoes periodic oscillations, with maximum correlation and no local minima. The physical reasons behind such conclusions are also presented.

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## Explosive synchronization transition in complex networks

J. Gomez Gardenes<sup>†</sup>, A. Arenas, S. Gómez, Y. Moreno

<sup>†</sup>Universidad de Zaragoza

Synchronization processes describe the coherent dynamics of a large ensemble of interconnected autonomous dynamical units, such as neurons, fireflies or cardiac pacemakers. Recent results have shown that the topological features of the complex interaction backbone strongly influence both the value of the critical coupling for the onset of synchronization and the stability of the fully synchronized state [1]. However, the topological properties of the underlying interaction network do not appear to affect the order of the synchronization phase transition, whose second-order nature remains unaltered. Recently, explosive collective phenomena have attracted much attention since the discovery of an abrupt (explosive) percolation transition in random graphs [2] and scale-free networks [3]. However, several questions about the microscopic mechanisms responsible of such an explosive transition, its first-order nature and their possible existence in other physical contexts remain open. In this talk, we report the results shown in [4] where an explosive transition was discovered in the context of the synchronization of Kuramoto oscillators in scale-free networks. The microscopic mechanism responsible of such explosive transition rely on the positive correlation between the structural and the dynamical properties of the system. In particular we will show, that explosive synchronization shows up when the internal frequencies of the Kuramoto oscillators are chosen to be proportional to the number of interactions each oscillator has, i.e., the degree of the oscillator. The characteristics of the explosive transition will be extensively studied in several scale-free network models in order to show that the correlation between the heterogeneous structural patterns and the dynamical properties of the oscillators is at the roots of the explosive critical phenomena. Finally, we will study analytically the synchronization properties of a star-graph in which a similar structure-dynamics correlation is imposed. In this simple context we will be able to reproduce the results obtained in synthetic scale-free networks and prove analytically that the synchronization transition is of first order. Our findings represent the first abrupt synchronization transition in complex networks and also provide with a deeper understanding of the microscopic roots of explosive critical phenomena.

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## Self-propelled particles under shear

G. Gonnella<sup>†</sup>, D. Marenduzzo, E. Orlandini, G. Saracco

<sup>†</sup>Dipartimento di Fisica dell'Università di Bari

The study of active matter has been a growing topical field of research in recent years. A suspension of self-propelled (SP) particles, modeling for instance microbial or bacterial fluids, fish schools, or synthetic swimming microrobots, is a primary example of an active material. Differently from their passive counterparts, active and SP particles continuously burn energy from their surroundings or from internal sources, typically to move, and this drives them out of equilibrium even in steady state. Elucidating the possibly universal properties of such active and self-propelled matter has prompted physicists to consider highly simplified models, such as the Vicsek model, which has become a paradigm in this field [1]. This model exhibits a continuous transition from a disordered phase to an ordered one in which flocks of SP particles form and move coherently, with long range order even in two dimensions. In our recent work we have extended the study of active materials to the case of externally driven, sheared, suspensions [2]. We found that in the presence of shear (i) there is no critical order-disorder transition with the imposed velocity acting as a symmetry breaking field. We checked this point by analyzing the behavior of the variance of the order parameter and by applying the so-called short-time dynamics approach. (ii) Moreover, we found that coarsening of the domains is arrested so that clusters of particles assume an anisotropic shape with a well defined size decreasing with shear rate. As a definition of typical domain sizes we used the square-root of the inverse of the second moment of the structure factor. (iii) Finally, shear suppresses the "giant density fluctuations" which are observed in the quiescent limit [3]. This term refers to the observations of a non-standard scaling of the fluctuations in local number of particles as a function of the local average number of particles, in dilute systems where the local density is allowed to vary a lot, such as in the Vicsek model. In correspondence of the change of the exponent describing giant fluctuations, also the super-diffusive behavior observed in the ordered phase of the Vicsek model becomes ordinary diffusion in presence of shear. Our results lead to a series of predictions for externally driven active systems, such as bacterial fluids, bird flocks, or even inanimate active matter such as vibrated granular rods. Besides providing a model system for driven active matter, our results should be relevant to rheological experiments on suspensions of bacterial or microbial swimmers, and to the large scale behavior of social animals, such as birds flocks and insect swarms, which are known to be fragmented or dispersed by strong winds.

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### Power-law statistics of bursts generated by the nonlinear stochastic differential equation

V. Gontis<sup>†</sup>, M. Alaburda, B. Kaulakys

<sup>†</sup>Institute of Theoretical Physics and Astronomy, Vilnius University

We model financial variables by the nonlinear scaled stochastic differential equation (NSDE) reproducing the main statistical properties observed in the financial markets [1, 2]. The proposed model is based on the class of nonlinear stochastic differential equations, providing the power-law behavior of spectra and the power-law distributions of the probability density [3, 4]. These equations include special cases already considered in the theory of finance, for example, Cox-Ingersoll-Ross (CIR) process, Constant Elasticity of Variance (CEV) process, etc. Agent based herding model serves as a microscopic background for the macroscopic description of financial markets by nonlinear stochastic differential equations. In this contribution we analyze power-law statistics of bursts generated by the same class of stochastic differential equations. We investigate the class of equations numerically and find power-law statistics of bursts, exhibiting very universal behavior. Our investigation reveals links between the proposed model and models where signals consist of bursts characterized by power-law distributions of burst size, burst duration and interburst time, as in the case of avalanches in self-organized critical models and the extreme event return times in long-term memory processes [5]. The universal nature of power-law behavior suggests that it does not arise as a consequence of particular interaction but it is a characteristic signature of complexity. Such model system may represent the limiting behavior of the dynamical or deterministic complex systems, explaining the evolution of the complexity into chaotic regime. The theoretical reasoning of the observed power-law statistics of burst occurrence is in progress. One of the reasons for the appearance of the power-law statistics is the scaling property of the NSDE. Special cases of the class of equations allow explicit forms for the solution of the Fokker Planck equation. We expect that the NSDE can serve as a limiting theoretical approach in the analysis of the nonlinear complex systems with the elements of self-organization and for analysis of transition to the chaotic behavior. The approach presented may be useful for the modeling of crashes in economics, finance and social systems. Theoretical results may serve as a background for the extensive empirical analyzes of real systems.

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### Characterizing a strong first order phase transition on a 3D diluted system

A. Gordillo-Guerrero<sup>†</sup>, L.A. Fernandez, V. Martín-Mayor, J.J. Ruiz-Lorenzo

<sup>†</sup>Universidad de Extremadura, Caceres, Spain

In first order phase transitions (FOPT) energy, pressure or magnetization change abruptly at the transition point. On the contrary, in second order phase transitions (SOPT) the changes are continuous and only the second derivatives of the free energy (specific heat, susceptibility, etc.) present singularities. Depending on the internal parameters, some systems can undergo the two types of transitions. The exact values on the parameter space where the order of the transition changes are called multicritical points. Some models undergoing first order phase transitions are now well characterized. Nevertheless, these systems do not present intrinsic disorder, i.e. dilution, anisotropy, random interactions, or random fields. Theoretical and numerical studies claimed that, in 3D, transitions will always be of the second order type for strong disorder. However they also opened the possibility for the existence of FOPT for smooth disorder [1]. Previous numerical results confirmed this point for a soft FOPT [2]. The numerical study of FOPT in the presence of disorder presents inherent difficulties: exponential critical slowing down, metastability and long tailed pdfs. This restricted previous results to the second order side of the phase diagram or to very small systems [3]. These difficulties are now under relative control using non-canonical ensembles [4] and longer simulation times. We have expanded the results of [2] by studying a stronger FOPT and characterizing the details of its multicritical point. To concrete, we have studied the phase transition in the 3D eight-state Potts model with site-dilution. In this case, the spin concentration,  $p$ , will switch the order of the transition from a FOPT in the pure case ( $p = 1$ ) to a SOPT at a critical value  $p_c$ . As we expected, the dilution dramatically soften the transition. For a given system size,  $L$ , the system will perform a clear SOPT for values of the dilution growing with  $L$ . To estimate the location of  $p_c$  in the thermodynamic limit we performed a FSS study of the crossing point of the correlation length, obtaining a value clearly smaller than  $p = 1$ . We have also studied the scaling with  $L$  of latent heat and surface tension in presence of disorder. Our results have only been possible due to the massive use of different computing facilities. Apart from typical Beowulf clusters, we used a distributed computing platform based on BOINC [5]. The adaptation of the code to the new computing paradigm and the control of the simulations supposed a great challenge for us.

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## Trade-offs and noise tolerance in signal detection by genetic networks

R. Guantes<sup>†</sup>, J. Estrada, J.F. Poyatos

<sup>†</sup>Universidad Autonoma de Madrid

Signal transduction networks are commonly constituted by genetic circuits, or modules, comprised of a small number of interacting molecular elements. Recent accounts of how these modules compute biochemical information highlighted the intricate relationship between structure and function, and the capacity of these units to process different signal attributes (e.g., signal amplitude, duration, or frequency). In particular, the relevance of oscillatory stimuli (and response oscillatory codes) to understand signaling systems is increasingly being appreciated. Oscillatory inputs can be used to probe and characterize genetic networks offering several advantages: the response may be easier to discriminate from noise than by applying a step stimulus [1], and systems identification theory can be employed to validate molecular models of the network under study [2]. Alternatively, oscillations in protein abundance or protein localization [3], can act as a robust strategy to encode regulatory information, similar to the neural spiking codes. What type of constraints could circuits experience in the performance of frequency and amplitude detection, and how are they affected by molecular noise? Here, we consider a simple detection process—a signal acting on a two-component module—to analyze these issues. Using a unified theoretical framework to quantify amplitude and frequency signal detection, we show that feedback interactions impose a trade-off in the detection of amplitude and frequency signal changes [4], whose intensity depends on feedback strength. These limitations are overcome by feed-forward loop connections. This allows us to identify optimal circuit architectures for detection of either amplitude, frequency or both signal features simultaneously. When we integrate molecular noise in this framework, we find strong resilience of optimally designed circuits to noise, and uncover new mechanisms of noise tolerance in signal detection. Specifically, coherent feed-forward loops attenuate noise propagated through intermediate components, while incoherent feed-forward loops and negative feedbacks are able to filter frequency noise by shaping noise spectral properties. Our findings are supported by numerical simulations of genetic circuits and the analysis of experimental data for feed-forward loop architectures. Overall, our results emphasize the limits imposed by circuit structure on its characteristic response and the seemingly paradoxical advantage of improving signal detection with noisy circuit components.

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## Dynamical response of systems driven by Lévy-type noises

E. Gudowska-Nowak<sup>†</sup>

<sup>†</sup>M. Smoluchowski Institute of Physics, Jagiellonian University

A standard approach to studies of relaxation and noise-induced effects in stochastic dynamics assumes a Gaussian character of the noise term describing interaction of the analyzed system with its complex surroundings. An additional assumption about the existence of timescale separation between the dynamics of the measured observable and the typical timescale of the noise allows external fluctuations to be modeled as temporally uncorrelated and therefore white. However, in many natural phenomena the assumptions concerning the abovementioned properties of "gaussianity" and "whiteness" of the noise can be violated. In the ubiquity of complex systems observed in Nature, non-Gaussian fluctuations prevail and transport properties deviate from the standard theory of Brownian motion. Various facets of "anomalous diffusion" have been extensively studied over the past years by use of either continuous time random walks (CTRW) or fractional kinetic equations. Recent development of simulation techniques based on Langevin equation in random subordinated time links uniquely stochastic trajectories of anomalous diffusion with probability density functions governed by fractional Fokker-Planck equations and provides efficient tools to study CTRW and their asymptotics leading to general forms of (stable) Lévy noises. Non-equilibrium systems driven by such perturbations are known to manifest interesting physical properties and have been addressed in various realms, ranging from the description of the dynamics in plasmas and diffusion in energy space to exciton and charge transport in conformationally changing polymers and incoherent atomic radiation trapping. Following the standard way of reasoning, for systems close to equilibrium, the relaxation properties of measurable physical quantities are related, in view of the linear response theory, with fluctuation-dissipation theorem. Accordingly, the response or the generalized susceptibility, which is a function of the unperturbed equilibrium system, can be related to the correlation between spontaneous fluctuations of a given dynamical variable. Further generalizations of the fluctuation-dissipation theorem to far from equilibrium situations involve the concept of nonequilibrium steady states. In the talk, a brief review of stochastic dynamics under the influence of Lévy noise perturbations will be given focusing on the analysis of typical response functions and their scaling characteristics. As a case of special interest, we will discuss the process driven by noise composed of Lévy distributed pulses with intensity scaling according to another Lévy law.

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### The probabilistic solutions of stochastic oscillators with even nonlinearity in displacement under Poisson excitation

X.-X. Guo<sup>†</sup>, G.-K. Er

<sup>†</sup>University of Macau

Poisson white noise is a discrete random excitation model applied in engineering field in the form of a sequence of random impulses arriving at exponential-distributed time. It is a typical non-Gaussian process. Under such excitation, the system experiences free vibration between successive pulses of the input and forced vibration at the impulse, so the evaluation of the stochastic response is complex, but it is a basis for reliability analysis or other statistical analysis. The analysis on the probability density function (PDF) of the responses of nonlinear stochastic dynamic (NSD) systems has attracted many researchers in various areas since many problems in science and engineering can be modeled as NSD systems. Excited by white noises, the PDF solution of the NSD systems is governed by Fokker-Planck (FP) equation. However, solving the FP equation in high dimensions has been a challenge for almost one century until the state-space-split (SSS) method were proposed recently [1]. The SSS method can make the FP equation in high dimension reduced to the FP equations in low dimensions and the low dimensional FP equations can be solved with the exponential polynomial closure (EPC) method [2,3]. Hence the investigation on the effectiveness of EPC method in various cases of low-dimensional nonlinear stochastic oscillators is fundamental for employing the SSS-EPC procedure in analyzing the probabilistic solutions of large-scale NSD systems or the FP equation in high dimensions. With EPC method the PDF solution is approximated as an exponential function of polynomials in state variables and the solution can be obtained in closed form. The availability of the EPC method has also been demonstrated in various cases for the oscillators excited by Poisson white noises [4,5]. The effectiveness of the EPC method in analyzing the nonlinear stochastic oscillators with even nonlinearity and excited by Poisson white noises has not been investigated. In this paper, the EPC method is employed to investigate the PDF solution of nonlinear stochastic oscillators with even nonlinearity in displacement excited by Poisson white noise to further test the effectiveness of the EPC method in this case. The solution procedure is presented and numerical results from EPC and Monte Carlo simulation are compared to validate the EPC method in various cases of Poisson white noise excitations.

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### Effective permeability scaling in immiscible two-phase flow in porous media

A. Hansen<sup>†</sup>, M. Grøva

<sup>†</sup>Norwegian University of Science and Technology

Tallakstad et al. [1, 2] have recently observed that the effective permeability of two-phase flow of air and glycerol under steady-state conditions in a two-dimensional model porous medium consisting of a glass bead packing clamped between two glass plates, shows a power law dependency with respect to the capillary number, characterized by an exponent of 0.54 +/- 0.08. The capillary number is the dimensionless ratio between typical viscous forces to capillary forces at the pore scale. A second dimensionless number that characterizes the flow is the viscosity ratio between the two immiscible fluids. In the experiments, this was 10 000. Recent experiments on a three-dimensional model porous medium indicate a similar power law, but with an exponent of 0.30-0.45 [3]. We have investigated this scaling law using a model based on solving the Kirchhoff flow equations for a two-dimensional network of hour glass shaped tubes [4-6]. Hence, the capillary forces created by the menisci between the wetting and non-wetting fluids will depend on their positions in the tubes. The network is biperiodic so that a steady state flow may be obtained. In contrast to the experimental system, the model is closed so that the saturation is fixed. In the two-dimensional experimental system the fractional flow rates are fixed at 7/15 (air) and 8/15 (glycerol). For non-wetting saturation of 0.2 we find an effective permeability scaling as the square root of the capillary number both for unit viscosity ratio and for a viscosity ratio of 10 000. This is consistent with the results of Tallakstad et al. [1, 2]. A necessary condition for obtaining this law is initializing the fluid component distribution so that there are large stagnant patches of fluids held in place by capillary forces. If, on the other hand, the system is initiated with evenly distributed droplets of the two immiscible fluids, an exponent of 0.7-0.8 is found for the entire range of saturations and viscosity ratios we have studied. The power law spans up to four decades. It breaks down when all the fluid clusters are mobilized so that the fluids behave as a single component fluid. The effective permeability then approaches the single phase permeability of the network. We note that in the biperiodic, closed system that we are studying, both initializations are accessible whereas in the experiment, only the one favoring large stagnant clusters has been considered.

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## Exergy and information

P. Harremoës†

†Copenhagen Business College

We present an abstract model of energy and information exchange in order to study the validity of some of the basic properties of thermodynamics for non standard systems. The model is relevant for economyphysics, thermodynamics of selfgravitating systems like black holes, the measurement process in quantum physics, origin of life, and Maxwell's demon. The abstract model builds on the following basic principles: Energy is defined in classical mechanics where all processes are reversible. By interaction with the system an agent wants to extract energy from the system. Information about the system allows the agent to make better decisions about how to interact with the system. These basic principles are of different validity for different systems. We combine a model by Holevo (1982) about probabilistic systems with the model of classical thermodynamics as described by Lieb and Yngvason (1998). Time and our interaction is modelled as in Harremoës (1993). Each measurement on a physical system gives a probability distribution over a sample space. Two preparations that cannot be distinguished by any measurements are said to represent the same state. We allow the construction of product states where several preparations are made independently, and we allow 'joint' measurements on product states that are not products of measurements on individual states. Classical mechanical systems are reversible and the time development is determined by the Hamiltonian. In such systems energy is conserved, and can for instance be stored by a weight raised in the gravitational field. Stored energy under our control is useful since it allows us to control certain energy consuming processes in non-reversible world. One of the main results is that  $Ex = T_{eq} \cdot D(S||S_{eq})$  whenever the concepts makes sense. Here  $Ex$  denotes the (internal) exergy of a system, i.e. the average amount of energy that can be converted into mechanical energy in the environment, and  $D(S||S_{eq})$  the number of (energy relevant) bits that we know about the actual system in the state  $S$  deviates from being in an equilibrium state  $S_{eq}$  where no energy can be extracted. The temperature  $T_{eq}$  is the absolute temperature of the state  $S_{eq}$  and it tells how a bit of information can be converted to extractable energy. Other concepts like entropy can then be derived. The equation holds. The proof is via a Carnot type argument about the interaction between an abstract system and a classical statistical mechanical system like an ideal gas in a cylinder. In order for the ideas to make sense for random systems we need to look at long sequences of identically prepared system and their average exergy.

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## Dynamic scaling, data-collapse and the role of group in Barabási-Albert networks

M.K. Hassan†, M.Z. Hassan and N.I. Pavel

†Dhaka University

Many complex systems can be described as an interwoven web of large network if the constituents are regarded as nodes or vertices and the interactions between constituents as links or edges. It has been found that most natural or man-made networks are neither a regular where all the nodes have exactly the same number of links nor they are completely random where it is almost impossible to find nodes that have significantly higher or fewer links than the average links instead the degree distribution are found to obey power-law. Barabási and Albert just over a decade ago proposed a model in which they argued that most natural or man-made networks are not static rather grow by addition of new nodes which establish links to the well connected existing ones *preferentially* rather than *randomly* - known as the preferential attachment (PA) rule. In this work we investigate the role of groups by assuming that a dyad, which is the smallest and the most elementary group, arrives sequentially in the growth of the Barabási-Albert (BA) network instead of a monad. Both nodes of the dyad, already linked by an edge, are considered to establish links either (i) to a single node picked by a single attempt or (ii) to nodes picked by two independent attempts, while each attempt is made following the preferential attachment rule. We show analytically that in both cases the degree distribution exhibits power-law  $P(k) \sim k^{-\gamma}$  with  $\gamma = 4$  which we also verified numerically. We then argue that if the nodes of the BA network are characterized by the generalized degree  $q$ , i.e. the product of their degree  $k$  and the  $(\gamma - 1)$ th root of their respective birth time, instead of  $k$  alone then the distribution function  $F(q, t)$  exhibits dynamic scaling  $F(q, t \rightarrow \infty) \sim t^{-1/(\gamma-1)} \phi(q/t^{1/(\gamma-1)})$  where  $\phi(x)$  is the scaling function. This is verified by showing that a series of distinct curves obtained by plotting  $F(q, t)$  vs  $q$  for different network sizes  $N$  collapse onto a single universal curve if  $t^{1/(\gamma-1)} F(q, t)$  is plotted against  $q/t^{1/(\gamma-1)}$  instead. Finally, we show that the resulting networks fall into two universality classes depending on whether the existing nodes are picked by single attempt or by two independent attempts albeit the degree distribution are the same in both.

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## Inelastic collisions and clustering of sea ice floes - the influence of floe-size distribution on cluster formation and patterns of motion of sea ice

A. Herman<sup>†</sup>

<sup>†</sup>Institute of Oceanography, University of Gdansk, Poland

Large areas of subpolar oceans are seasonally covered with ice. Contrary to a multi-year ice pack, seasonal ice cover is usually strongly fragmented and consists of separate floes. However, both ice types are commonly modeled based on a continuum approximation; few attempts to account for the granular nature of sea ice have been made, all based on an unrealistic assumption of a constant floe size (e.g., [1]). The observed floe sizes are power-law distributed (e.g., [2]), and at low ice concentrations the ice dynamics is strongly influenced by inelastic collisions between floes. This study draws from an analogy between sea ice and other granular media composed of interacting, inelastically scattering particles. Exploring this analogy has a twofold purpose. Firstly, clustering in granular materials has been mostly studied for narrow particle-size distributions (PSD; [3]). Hence, the results of this study may be of interest to other types of granular media with wide PSDs. Secondly, the aim is to explain some phenomena observed in sea ice and hitherto unaccounted for in models. To this end, a 2D event-driven molecular-dynamics model, similar to those used for studying other granular media, is developed to simulate a set of  $N$  round particles ('floes') with power-law PSD within a rectangular domain with periodic boundaries. Between collisions, the floes' motion is determined from a momentum equation in the form of a Stokes-flow problem, i.e., with acceleration proportional to the difference between the instantaneous and the (generally floe-size dependent) equilibrium velocity under given wind conditions. The model successfully predicts cluster formation and nontrivial floe velocity patterns observed in sea ice. Particle concentration in clusters and their characteristic size are shown to depend on the exponent of the PSD. The largest particles play an important role in cluster formation, in that they influence the motion of smaller neighboring particles. The average size of particles within clusters is larger than in diluted regions. The size-dependent equilibrium velocity leads to nontrivial motion pattern: regions with spatially coherent velocity and high velocity gradients ("ice jets" and neighboring "traffic jams" with velocity close to zero). The results of this study are also noteworthy from the point of view of processes shaping the PSD in sea ice. It was suggested recently that formation of clusters is important for freezing of neighboring floes [4]. Thus, the dependence of cluster size on PSD, as shown here, suggests the existence of an interesting feedback between the PSD and clustering.

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## Watersheds, bridges and explosive percolation

H.J. Herrmann<sup>†</sup>, N. Araujo, J.S. Andrade Jr.

<sup>†</sup>ETH Zurich, Switzerland

Since the introduction of the product rule of explosive percolation by Achlioptas, D'Souza and Spencer [Science 323, 1453 (2009)] several realizations of discontinuous percolation transitions have been proposed. Despite the active research on this subject, there are still open questions about the properties of the system and the main mechanisms leading to an abrupt transition. The largest cluster model and the Gaussian model yield peaked cluster-size distributions and compact clusters, a clear picture of discontinuity which is observed in any dimension. A Hamiltonian formalism provides a clear connection between equilibrium statistical mechanics and explosive percolation, disclosing the homogenization of the clusters size as a key mechanism and also that, for random graphs, merging bonds should dominate over the redundant ones. The generalized best-of- $m$  product rule reveals that all the exponents for the size dependence of the spanning cluster, the conducting backbone, the cutting bonds, and the global conductance of the system, change continuously and significantly with  $m$ . Mixing classical percolation with the best-of-10 rule allows to observe the change of a continuous to a discontinuous transition at a tricritical point, which we characterize carefully in 2D by a new set of critical exponents and a tricritical crossover scaling. The interface of the explosive clusters is fractal having the same dimension as watersheds, optimal path cracks, and the set of percolation bridges above the threshold  $p_c$ . In fact, if the formation of the first spanning cluster is suppressed, at  $p_c$ , bridge percolation exhibits a theta-point scaling, with a novel tricritical exponent, for which values in all dimensions below the upper-critical dimension  $d_c = 6$  are calculated. Above the upper-critical dimension, the set of bridges is dense and has the dimension of the system. Similarly, when bonds are randomly removed from a spanning cluster, without breaking connectivity, a theta-point scaling is also obtained for the cutting bonds and above  $d_c$  the set of cutting bonds has the same dimension as the random walk and the optimal path. For weak disorder, optimal path cracks have dangling ends and isolated clusters also yielding novel exponents, while for strong disorder, the crack is localized in the watershed line.

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## Extraordinary magnetoresistance: Sensing the future

T. Hewett, F.V. Kusmartsev

†Loughborough University

Extraordinary magnetoresistance (EMR) was first discovered a decade ago by Solin et al. in symmetric van der Pauw disks consisting of semiconducting material containing a circular metallic inclusion. This new form of magnetoresistance gave rise to extremely large magnetoresistance values (up to 1,000,000% in a 5T applied magnetic field), orders of magnitude larger than previously discovered effects, at room temperature using exclusively non-magnetic materials in the systems construction [1]. The EMR effect has a great possibility for future applications due to the size of the effect observed at room temperature especially in the area of information storage technology where read heads of current hard disk drives utilise the GMR effect. We have modelled various systems where the EMR effect has arisen experimentally via the finite element method with our results offering an extremely good agreement with the existing experimental results. The EMR effect depends strongly on the geometry of the system and thus we have discovered a multi-branched geometry that enhances the EMR effect by up to four orders of magnitude when compared to the experimental geometries. Our model allows for a transparent understanding of the mechanism behind the effect by visualising the change in the current flow throughout the systems with the application of magnetic field. The large magnetoresistance arises as a result of the expulsion of electrical current from the metallic region, induced by the transverse applied magnetic field. Our model has allowed for further investigation into the effect of the systems conductivity, mobility and interface resistance on the EMR effect. Our results may be relevant to many strongly inhomogeneous materials consisting of a mixture of conducting and insulating components. One such group of materials are the silver chalcogenides ( $\text{Ag}_{2+\delta}\text{Se}$  and  $\text{Ag}_{2+\delta}\text{Te}$  compounds) where a large linear magnetoresistance has been discovered; remaining unsaturated even in fields of up to 5.5T [2]. The excess silver has been shown to form a network along grain boundaries with silver droplets dispersed in the semiconducting matrix in analogy with the EMR effect [3]. With the use of a model containing a random distribution of branches and droplets we have shown that indeed such a linear magnetoresistance effect can arise.

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## Vortices in superconducting nano-networks with anti-dot arrays

K. Hirata†

†National Institute for Materials Science

Superconducting thin film perforated with regularly-distributed anti-dots (holes) arrays composes a superconducting network. Vortices (magnetic flux quanta) in the network behave as electrons in atomic lattice of crystals, which is well known as a periodic Bloch wave function. Repulsive interaction between vortices, and attractive one between vortices and anti-dots resemble to those between electrons, and between electrons and atoms in crystals, although vortices can be accommodated in a giant vortex. In two dimensional (2D) electron gas system in semiconductors, Landau levels are composed under extremely magnetic field (around 1000 Tesla), which energy spectrum is well known as a Hofstadter butterfly. However, in the superconducting networks, it is expected to realize it in the lower field of around 0.1 Tesla as a spectrum of superconducting transition temperature against magnetic field, which is suggested in accordance with the similarity between 2D Schrodinger and linearized Ginzburg-Landau equation. To confirm the variety of the vortex physics similar to the solid state physics, we have fabricated such superconducting networks with triangular and square lattice of anti-dots in inter-metallic and high-Tc superconductors (HTSCs) with photo-lithography and focused ion beam, and have measured magneto-resistance of flux-flow. In these materials, we have observed matching effect at the matching field, which has been well known and studied. Additionally, the fractional matching occurred at 1/4, 1/3, 1/2, 2/3, 3/4, 6/7 of the matching field. Most of them are well explained by commensurability between Abrikosov vortex lattice and anti-dots arrays [1]. 1/3 of the matching is most remarkable among them. Another feature, which cannot be explained by the commensurability, may be related to the spectrum as in that of Hofstadter butterfly. On the other hand, when the anti-dots of nano-networks consist of randomly (irregularly)-distributed media, we found a phase transition at 1/3 of the matching field (around 1 Tesla; irradiated by heavy-ions) in the case of high-Tc superconductors [1]. In this case, the transition is a kind of field induced one. Fractional matching also predicted, but not confirmed by the experiments. Another topic in the superconducting networks is the first-order melting transition of pinned vortex solid. The first-order melting transition of vortex lattice in HTSCs is one of the most remarkable results in HTSCs. Then, what will happen in pinned vortex solid? Depending on the commensurability between Abrikosov vortex lattice and anti-dots arrays, it is suggested that vortices show a pinned solid, a floating solid, and liquid [3, 4]. Experimentally, these phases and phase transitions are suggested.

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## Graphene barrier potential

P. Holland<sup>†</sup>, F.V. Kusmartsev, V.V.Zalipaev

<sup>†</sup>Loughborough University

Graphene is a unique material which combines a low two-dimensionality with a gapless massless electronic spectrum. Due to the former, electron transport is confined to two dimensions and has a zero effective mass. This means they can penetrate any barrier providing it has sharp vertical walls. This phenomenon is known as the Klein paradox and has attracted strong attention from both the experimental and theoretical community. In spite of Klein tunnelling, the barrier plays a role of a potential well for holes. Therefore there may arise hole bound states or vice versa for the potential well. These bound states correspond to localised resonant modes existing within the potential barrier or within the potential well. In the present work we describe a novel way of looking at this problem, that is the double faces of the barrier potential for a two dimensional Dirac system. On one face this is a barrier, on the other face this is a potential well. If in the first case the penetration of the particle inside the barrier is prohibited, then in the second case it is opposite to that, the barrier may localise the carriers. We compare these two classes of solutions and find criteria where they exist. General equations for localised modes and transmission coefficient through the barrier at any conditions have been investigated properly and analytical expression have been derived. Graphene is a unique material which combines a low two-dimensionality with a gapless massless electronic spectrum. Due to the former, electron transport is confined to two dimensions and has a zero effective mass. This means they can penetrate any barrier providing it has sharp vertical walls. This phenomenon is known as the Klein paradox and has attracted strong attention from both the experimental and theoretical community. In spite of Klein tunnelling, the barrier plays a role of a potential well for holes. Therefore there may arise hole bound states or vice versa for the potential well. These bound states correspond to localised resonant modes existing within the potential barrier or within the potential well. In the present work we describe a novel way of looking at this problem, that is the double faces of the barrier potential for a two dimensional Dirac system. On one face this is a barrier, on the other face this is a potential well. If in the first case the penetration of the particle inside the barrier is prohibited, then in the second case it is opposite to that, the barrier may localise the carriers. We compare these two classes of solutions and find criteria where they exist. General equations for localised modes and transmission coefficient through the barrier at any conditions have been investigated properly and analytical expression.

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## Role of topological structure in dynamics of noisy neural networks

D. Holstein<sup>†</sup>, A.V. Goltsev, A.F. Ioffe, J.F.F. Mendes

<sup>†</sup>Department of Physics, I3N, University of Aveiro, Portugal

Different presumed topologies underlying neural networks lead to varying functionality. The main aim of our investigations was to study the role of as well the network topology as also the spontaneous activity, caused by synaptic or intracellular noise or other stochastic input from other neuronal areas, in dynamics of neural networks with complex network architecture. For this purpose we formulate a stochastic model of neural networks [1], which consists of two populations of neurons, excitatory and inhibitory. Though the proposed model uses simple rules to govern the stochastic dynamics, it demonstrates various patterns of self-organization of neural networks, phase transitions in neural activity driven by noise, hysteresis phenomena, and a rich set of dynamical phenomena: decaying and stable neural oscillations. We show that a positive role played by noise in neural dynamics is that it stimulates spontaneous neural activity [2]. We compare dynamics of neural networks with different topologies: classical random graphs and scale-free networks. For scale-free networks, as e.g. given by the static model [3], with the degree exponent smaller than 3, we find that a critical noise level for activation of a finite fraction of the network decreases with the network size and vanishes in the thermodynamic limit. It means that even weak synaptic noise results in a strong spontaneous activity of neurons. This is opposed by a constant critical noise level under increased network size for the topology of the classical random graph. We obtain a complex phase diagram of our model with various regimes of dynamical behaviour such as weak activity, synchronous [4] or oscillatory behaviour and overactivation. Our analytical calculations and simulations of networks with 10000-50000 neurons show that also the noise threshold for the onset of oscillatory behaviour is strongly reduced in the case of scale-free topology. We also demonstrated that hubs, i.e., strongly connected excitatory and inhibitory neurons, play an important role in dynamics of neural networks. They orchestrate neural dynamics. A removal of a few hubs may dramatically change dynamics of neural networks and destroy synchronization between neurons. However, if the network is damaged at random then it is necessary to remove much more neurons for destroying synchronization and global neural oscillations.

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## On the use of the Weibull distribution as a model for the distribution of earthquake interevent times

D. Hristopulos<sup>†</sup>, V. Mouslopoulou

<sup>†</sup>Technical University of Crete

One of the most interesting problems in statistical seismology is the nature of the probability distribution that governs the times between successive earthquake events, i.e., the so-called interevent time distribution. Knowledge of this distribution is important for the assessment of seismic hazard. For example, the ability to infer return times for very large earthquakes based on the statistics of less strong and more frequent events would be of great value. If individual seismic events are random, i.e., that they are spread out as uniformly as possible over time, they follow the Poisson distribution. In this case, the interevent times follow the exponential probability density function. The interevent time distribution captured the interest of physicists as well: Bak [1] proposed a universal scaling function, which was then modified by Corral [2] to include local corrections in the form of the gamma distribution. However, several recent analyses of earthquake catalogue data show that neither the Poisson model nor the scaling functions are adequate. Recorded sequences of earthquake interevent times include segments during which events are definitely clustered. There seems to be an agreement that the distribution of interevent times contains both a correlated component which is due to triggered seismic events (e.g., aftershock sequences) and uncorrelated background activity [3,4]. Various probability distributions have been proposed to describe the clustering, including power laws, the gamma distribution, mixtures of the gamma and the exponential, a theoretical model derived from analysis of the Epidemic Type Aftershock Sequence (ETAS) model, and the Weibull distribution. Several studies test the performance of these models on catalogue data with no emerging consensus. One should note that the spatial system size, the number of events in each sequence, and the cutoff magnitude ( $M_w$ ) vary significantly between studies. We propose a theoretical justification, based on ideas from fracture mechanics, for the use of the Weibull distribution as a model of earthquake interevent times. We consider a single fault system (as opposed to a large zone containing many fault systems). We then use a high quality data set to test various theoretical proposals. The data set involves 2562 relocated microearthquakes (with magnitudes 1

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## Langevin equations, random fields and applications to inverse problems in spatially distributed processes

D. Hristopulos<sup>†</sup>

<sup>†</sup>Technical University of Crete

Spatially distributed systems involve many environmental and biological systems, described by models with locally varying coefficients. An issue of practical importance in such systems is the inference of model parameters from the available, possibly sparse and scattered, observations. Efficient and accurate solutions to the inverse problem of parameter inference as well as spatial interpolation and conditional simulation are important for the monitoring and forecasting of the transport of atmospheric or groundwater contaminants, weather patterns, and ambient radioactivity levels. Covariance functions play a significant role in the inference process. Classical covariance models (e.g., exponential, Gaussian) are subject to the curse of dimensionality due to the cubic dependence of covariance matrix operations on the data size. We show that stochastic (Langevin) partial differential equations [1] driven by white or color noise can be used to obtain parametrically rich families of covariance functions. The latter can also be obtained from the Gaussian model of statistical field theory [2]. However, in spatial modeling the focus is on the short-range correlations which are important for determining the local structure and not on the long-range properties of the covariance near critical points. Covariance models derived from suitable Langevin equations are characterized by sparse structure of the precision matrix (inverse covariance), at least for lattice data. The sparseness derives from the locality of the operators in the Langevin equations and leads to explicit spectral forms. In certain cases, the correlations in real space can be derived analytically by direct integration of the spectral representation, given by the Hankel transform of the density [3]. The existence of explicit expressions for both the covariance and the precision matrix overcomes the curse of dimensionality in the numerical procedures of parameter inference, spatial interpolation and conditional simulation [4,5]. We derive such explicit expressions for uniformly-sampled time series or spatial data sampled on regular lattices, and we illustrate generalizing approaches for irregularly spaced data. We present a simplified method-of-moments (MoM) algorithm for approximate but fast parameter inference, based on the asymptotic matching of the spatial model with its continuum limit [3-5]. We then describe how parameter estimates obtained by MoM can be used in the Bayesian framework to derive posterior parameter densities using Markov Chain Monte Carlo. Finally, we discuss issues related to applications of the above framework to scattered data and our future research in this direction.

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## Relaxation and aggregation of polymer chains and their biological implications

C.-K Hu†

†Institute of Physics of Academia Sinica

Many important biological macromolecules are polymer and the function of a biological system depends on behavior of polymers of that system. In this talk, I briefly review our recent works on relaxation and aggregation of polymer chains and the implication of such behavior for understanding protein aggregation problem or non-equilibrium behavior of a biological system. Neurodegenerative diseases include Alzheimer's disease (AD), Huntington's disease (HD), Parkinson's disease (PD), frontotemporal lobar degeneration, etc. Such diseases are due to progressive loss of structure or function of neurons caused by protein aggregation. For example, AD is considered to be related to aggregation of A $\beta$ 40 and A $\beta$ 42 (protein with 42 amino acids). We have used a lattice model to study the aggregation rates of proteins and found that the probability for a protein sequence to appear in the conformation of the aggregated state can be used to determine the temperature at which proteins can aggregate most easily [1]. We have used molecular dynamics and simple models of polymer chains to study relaxation and aggregation of proteins under various conditions [2-4]; the neighboring monomers along a polymer chain are connected by rigid bonds or spring of strength  $k_s$  [2-4]. We found that the velocity distributions of monomers in a wide range of simulation time can be well described by Tsallis  $q$ -statistics [2,4,5] with  $q$  larger than or equal to 1 and a single scaling function. The value of  $q$  is related to the conformation constraining potential, the interactions with background fluid, the destruction of chain homogeneity or the value of  $k_s$ ; when  $q$  becomes 1, the velocity distribution of monomers becomes Maxwell-Boltzmann distribution. We found that when the bending-angle dependent and torsion-angle dependent interactions are zero or very small, then protein chains tend to aggregate at lower temperatures [3-4]. Such result is useful for understanding aggregation of A $\beta$ 40 and A $\beta$ 42. Our results [1-4] form good basis for further studies on protein aggregation. In the next step, we will use a more realistic model to study the influence of various mutations on protein aggregation rates. We also found that polymer chains with neighboring monomers connected by rigid bonds or strong springs show glassy behavior at low temperature and relax very slowly. Such results are useful for understanding that a biological system can maintain in non-equilibrium and viable state for very long time [4].

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## Electoral Systems and Distribution of Votes

M. Ifti†, D. Prenga, I. Buzi

†University of Tirana, Albania

We study the behaviour of the number of votes cast for different electoral subjects in the Albanian elections of the last 10 years. We report the frequency of obtaining a certain percentage (fraction) of votes versus this fraction for the parliamentary elections. In the Brazilian elections this follows a power law with exponent -1 [1]. In countries that use proportional voting with open lists, this distribution has a lognormal shape [2]. When the voting system is "first past the pole" (majority voting), this distribution has Gaussian shape [3]. The width of the curve depends on the number of "zealots" in the system. In the distribution of votes in Albania we identify two regimes. They are present in the distribution of votes when ridings are taken as electoral units, as well as when polling stations are considered as electoral units. We use the latter for it provides more "experimental points", hence better statistics. In the low percentages regime we see a power law distribution. As the binning interval gets smaller, the power law exponent at first grows, and then becomes constant when the number of intervals becomes larger than 100. This power law exponent is about -1.7. In the power law regime we find over 80% of the points, while they relate to 20% of the votes cast. Votes of the small electoral subjects are found in this regime. The other regime includes percentiles from 15% to 85%, and exhibits a Gaussian distribution, followed by a long tail. It corresponds to big electoral subjects (Democratic Party and Socialist Party). Some districts (Shkodër, Kukës, Berat, Dibër, etc.) exhibit a distribution that is entirely power law, while in others (Tiranë, Durrës, Elbasan, Korçë) the Gaussian is present. The same distribution is found for the local elections. The regime of small fractions shows a mixed distribution: power law and exponential. The regime above 15% for the councilors voting (proportional vote) exhibits a Gaussian with an exponential correction. For the mayors voting (majority vote), we find a Gaussian corrected by a lognormal term. We compare these results with the ones found for the elections in Kosovo. The different behaviours show that the distribution in the case of Albania is due to the voting system (regional proportional with closed lists).

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## Optimization and statistical estimation in image processing via Langevin equations

J. Inoue<sup>†</sup>, W. Norimatsu

<sup>†</sup>Hokkaido University

Mathematical bases of probabilistic image processing based on Bayesian statistics were now established and various different tools such as Markov chain Monte Carlo (MCMC for short) method or loopy Belief propagation on the graphical model to calculate the posterior or the marginal probability have been improved. Especially, image restoration (or sometimes it is referred to as image reconstruction), which is a procedure to recover the original image from the degraded less-informative image, is one of the most well-understood subjects in this research area. To deal with this type of massive information processing and to reveal the statistical performance of the algorithm, we usually use the MCMC based on Metropolis-type update rule to calculate the marginal distribution of posterior. For this benchmark test, one can evaluate the performance and the dynamical process of restoring image analytically for a class of mean-field infinite range model. However, one can also choose the different type of update rule. Here we shall propose a Langevin approach to solve combinatorial optimization problems or to estimate the probabilistic structures of the problems in computer science. Especially, we formulate the maximizer of posterior marginal (MPM) estimate for Bayesian probabilistic image restoration by using the Langevin equation. We also evaluate the statistical performance from the view point of statistical mechanics of information. To define the problem of image restoration, the multi-state ferromagnetic Ising model and additive white Gaussian noises are introduced as the regularization term and the model of degrading process, respectively. Then, the recursion relations with respect to each pixel are derived via the extremum condition of energy function. The long-time average of time series derived from the recursion relations gives the maximum a posteriori (MAP) estimate and it is shown that the estimate is regarded as a kind of the linear filter in signal processing. Obviously, the energy function possesses a lot of local minima and we try to construct the estimate (to find the global minima) using the fluctuation around the MAP estimate by solving stochastic differential equations — Langevin equations — with temperature decreasing (noise term reducing). We evaluate the performance both numerically and analytically. Finally, we show that it is possible to construct the MPM estimate (estimation of posterior probability density) by simulating the Langevin equation numerically. We can show that the corresponding Fokker-Plank equation leads to the posterior probability density. We compare the result with the estimate by using the conventional Markov chain Monte Carlo approach.

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## Monte Carlo studies of protein aggregation

A. Irbäck<sup>†</sup>

<sup>†</sup>Lund University

The amyloid beta-protein (Abeta) is the main component of the plaques associated with Alzheimer's disease. Abeta is a natively unfolded protein that forms ordered and stable fibrillar aggregates. A growing body of research indicates that the primary pathogenic form of Abeta is small soluble oligomers formed early in the aggregation process, rather than the mature fibrils. Isolating and characterizing Abeta oligomers is a challenge. The main topic of this talk is a Monte Carlo study of the elementary step of Abeta aggregation, the formation of dimers. The simulations are based on an effective all-atom implicit solvent model, and focus on Abeta42, with 42 residues, which is the form of Abeta most closely linked to Alzheimer's disease. For comparison, we consider both wild type Abeta42 and the mutants F20E, E22G and E22G/I31E. These four variants are known from experiments to have very different propensities to aggregate. The E22G mutation is associated with a familial ("Arctic") form of Alzheimer's disease and enhances aggregation, whereas the F20E mutation has the reverse effect. The double mutant shows more complex aggregation properties. Its propensity to form fibrils is almost as high as that of E22G, whereas its propensity to form prefibrillar species is only slightly higher than that of F20E. Our approach was validated by comparing simulation results for the wild type Abeta42 monomer with NMR data (J-couplings and chemical shifts). The observed dimers of the four variants share many overall conformational characteristics. In particular, in all four cases, the most common type of secondary structure is intramolecular antiparallel beta-sheets. Parallel in-register beta-sheet structure, as in Abeta fibrils, is rare. At a more detailed level, conformational differences can be seen among the four variants, especially in the region where there is a loop in fibrils. Our findings hint at reorganization of this part of the molecule as a potentially critical step in Abeta aggregation. In addition, I briefly discuss preliminary results from two other studies based on the same model. The first one focuses on the alpha-synuclein protein, associated with Parkinson's disease. The other study deals with oligomer growth. Here we study a small fibril-forming peptide with 6 residues (a protein tau fragment)

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## Oil shock transmission to stock market returns: wavelet-multivariate Markov switching garch approach

R. Jammazi†

†International finance group-Tunisia, Faculty of Management and Economic Sciences of Tunis, Tunis Cedex, Tunisia

In recent years our understanding of the nature of crude oil price shocks and their effects on the stock market returns has evolved noticeably. Evidence on spillover effects between several kinds of markets has widely been discussed around the globe whereas the transmission of shocks between crude oil market and stock market returns has received little attention. Extending earlier work in the literature, we use data on monthly crude oil returns and stock market returns of five developed countries (USA, UK, Japan, Germany and Canada) to investigate two issues that have been at the center of recent debates on the effect of crude oil shocks on the stock market returns. First, we analyze whether shocks and or volatility emanating from two major crude oil markets are transmitted to the equity markets. We do this by decomposing monthly real crude oil prices and analyzing the effect of the smooth part on the degree of the stock market instability. The motivation behind the use of the former method is that noises can affect the quality of the shock and thus increase erroneous results of the shock transmission to the stock market. Second, under the hypothesis of common increased volatility, we investigate whether these states happen around the identified international crises. In doing so, flexible model involving the dynamic properties of the Trivariate Markov switching garch model and the recent Harr A trous wavelet decomposition, is implemented to achieve prominent prediction of the mentioned issues. The proposed model is able to circumvent the path dependency problem that can affect the prediction's robustness and can provide useful information for investors and government agencies that have largely based their views on the notion that crude oil markets affect negatively stock market returns. Indeed, the results show that the A Haar Trous Wavelet decomposition method appears to be an important step toward improving accuracy of the smooth signal in detecting key real crude oil volatility features. Additionally, apart from UK and Japanese cases, the responses of the stock market to an oil shock depend on the geographic area for the main source of supply whether from the North Sea or from the North America (as we take two oil benchmarks WTI and Brent respectively).  
Keywords: Trivariate BEKK-Markov Switching, wavelet decomposition, oil shocks, stock markets

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## Automatic and unsupervised discovery and recognition of patterns by a dynamical system: a neural network alternative

N. Janson†, C. Marsden

†Department of Mathematics, Loughborough University, United Kingdom

The work is devoted to the creation of a system that can learn from a sequence of input patterns. Learning is understood as a two-stage process: (1) pattern recognition, i.e. attributing the newly arriving pattern to one of the existing classes, and (2) class discovery, i.e. automatic and unsupervised formation of classes from the input patterns. The problem is being solved within the framework of a dynamical systems theory. Famous dynamical systems capable of learning are neural networks: collections of typically identical non-linear units, coupled with each other with different strengths ("weights") [1,2]. The most natural way of learning for them is with supervision, when the teacher provides them with the full information about the number of the existing classes of patterns and their location in the phase space of the network. Before the network can do the job, it needs to be trained: the weights need to be adjusted in response to the training set of patterns. As a result, the phase space of the network is split into several basins of attractors, each containing an attracting set (typically a stable fixed point or a limit cycle) - a typical representative of the given class. An input pattern arrives in the form of the vector of initial conditions, which falls within the basin of a particular attractor. Pattern recognition manifests itself as approaching the respective attractor as time passes by. We propose an alternative solution to the problem of learning within the framework of dynamical equations, that "evolve" the answer by approaching an attractor. A dynamical system is introduced, in the form of a system of partial differential equations with delay, subjected to a continuously varying multi-dimensional vector input, imitating a sequence of patterns arriving at sensors of living systems. This system is constructed in such a way, that it automatically shapes its phase space in response to external, generally stochastic, input. As a result, the phase space appears to have the structure similar to the one we observe in neural networks. A significant advantage of the new system is that it naturally operates without supervision, although permits interference from the teacher when necessary. The performance of a prototype mathematical model is demonstrated using musical input: the system automatically detects musical phrases, and recognizes them from a corrupted input.

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## Long division unites-long union divides, a model for cultural evolution

J. Jiang<sup>†</sup>, R. Wang, M. Pezeril, Q.A. Wang

<sup>†</sup>Institut Supérieur des Matériaux et Mécaniques Avancés (ISMANS), Le Mans, France

One of the historical phenomena in the time evolution of cultural, national and economic system is the transition between union and division of one or several entities. In this work, we propose a union-division model based on the maxim "long union divides and long division unites" in order to investigate the long time behaviours of the networks composed of nodes representing the above mentioned entities. Each node is characterized by several quantities such as cultural identity, ingredient, richness, and age. The time evolution of the network is probabilistic depending on the above quantities and on the interaction between the neighbouring nodes. This work offers a long term view on the apparently periodic dynamics in the number of nodes (network size). The other main features of the model can be summarized as follows: 1) The degree distribution is Poisson at the beginning of evolution. After that, there is a transition between a continuous distribution and a discontinuous one with two peaks. 2) The ingredient distribution also undergoes a periodic transition between power law distribution and uniform one. 3) The age distribution has an exponential form, which agrees with the empirical data from the kingdoms and dynasties in China and Europe. 4) There is no correlation between the cultural richness and the degree. 5) There is a positive correlation between the cultural richness and the ingredient in the early time evolution. After certain time steps, no correlation is found. 6) There is periodic fluctuation in the time evolution of average distance of different cultures. The present model is a phenomenological one without specification about the nature of the nodes and the links. In this probabilistic model, the mechanism of union and division is not imposed deterministically to the evolution. At each step of evolution, union or division may or may not happen, depending on the probability of union or division calculated from the temporal values of richness, age, ingredient and identity. The notions of richness and ingredient are abstract. Hence, it is possible to use it for various networks other than the cultural one with some adaptations if necessary. The networks showing distinct union-division dynamics include language, economic entities, nations, and political opinions. This work offers a long term view on the apparently periodic dynamics of an ensemble of cultural entities.

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## Thermodynamic potentials and non-isolated adiabatic pistons

B. Jiménez de Cisneros<sup>†</sup>

<sup>†</sup>Dept. Aerotecnia, Universidad Complutense de Madrid

The problem of the so-called "adiabatic" piston refers to a system formed by an isolated cylinder which contains two fluids in different initial equilibrium states separated by an adiabatic fixed internal piston. The problem [1] is to predict the final equilibrium state once the constraint fixing the piston is removed. It was soon realized that classical equilibrium thermodynamics and the analysis of the entropy for this isolated system cannot give a simple answer to the question [2]. The problem had to be subsequently studied by means of molecular dynamics simulations and analytical models for the time evolution of the system formed by the fluids and the piston [3-5]. These works show that, although the piston is made of a thermally insulating material, the system evolves towards a state of full thermal equilibrium where both fluids share the same pressure and temperature. The time evolution towards equilibrium has two different time scales for a system with a piston mass much larger than those of the fluid particles. In the first stage the mechanical processes dominate, leading to a fast equilibration of the pressures of both fluids. Then, in a much larger time scale, the piston slowly moves in average to the hotter side, while the pressure of both fluids is constant, and heat is surprisingly transferred from the hotter to the colder one through the "adiabatic" piston until the temperatures of both fluids attain the same value. The reason is that the stochastic motion of the piston induces an effective thermal contact between the two fluids. Most of the models considered up to now focus on isolated systems where the analysis of the entropy fails to predict the final equilibrium state. In the present work are considered simple modifications of the usual adiabatic piston model which allow for interactions with the surroundings and lead to difficulties in the prediction of the final equilibrium state by means of thermodynamic potentials other than the entropy. Analytical results of a simplified microscopic model of this system and molecular dynamics simulations are presented. Finally, a proposal aimed to conciliate the observed facts in different models and the usual methods of classical equilibrium thermodynamics to predict equilibrium states will be discussed.

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## Superstatistics approach to special and doubly special relativity

P. Jizba†, H. Kleinert

†Freie Universitaet in Berlin

Probability distributions which can be obtained from superpositions of Gaussian distributions of different variances  $v$  play a favored role in quantum theory, financial markets or biological systems. This smearing procedure is often referred to as “superstatistics” [1]. Superpositions thus obtained need not necessarily obey the Chapman-Kolmogorov semigroup relation for Markovian processes because smearing distributions may (and invariably do) introduce memory effects. We derive the general form of the smearing distributions for  $v$  which do not destroy the semigroup property (detailed account can be found in Ref. [2]). Aforementioned smearing technique has two immediate applications. It permits simplifying the system of Kramers-Moyal equations for smeared and unsmeared conditional probabilities, and can be conveniently implemented in the path integral calculus. This is relevant, e.g., in financial models with stochastic volatility [2]. Second application is in Feynman path integrals. In many cases, the superposition of path integrals can be evaluated much easier than the initial path integral (detailed account can be found in Ref. [2]). For instance, superstatistics permits the calculation of the Feynman propagator of a relativistic particle in a novel way from a superstatistical average over non-relativistic single-particle paths. We illustrate this for the Klein-Gordon particle in the Feshbach-Villars representation, and for the Dirac particle in the Schroedinger-Dirac representation. Generalization to massive spin-1 particles, such as pions is also straightforward. As a byproduct we recover the world-line representation of Klein-Gordon and Dirac propagators, and discuss the role of the smearing distributions in fixing the reparametrization freedom. In this connection it is interesting to notice that the superstatistics approach implicitly sets the Polyakov gauge fixing condition. The emergent relativity picture that follows from our approach together with a novel representation of the Lorentz group for the Feshbach-Villars particle will be also discussed (for details see [3]). We also present the modifications necessary to accommodate in our scheme the doubly special relativistic dynamics. In this way, an unsuspected, common statistical origin of the two frameworks is brought to light.

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## Bayesian perspective on maximum work characteristics of quantum heat engines

R.S. Johal†

†Indian Institute of Science Education and Research Mohali, India

In Bayesian probability theory [1], any uncertainty associated with the system is treated probabilistically. Bayesian probabilities represent state of knowledge about the system and are in this sense subjective in nature. In this talk, Bayesian perspective is assumed to analyze the performance of an infinitely slow Otto cycle for a quantum engine operating between a hot ( $T_1$ ) and a cold ( $T_2$ ) reservoir. The total work in a cycle can be expressed as  $\mathcal{W}(a_1, \eta)$ , where  $\eta$  is the cycle efficiency and  $a_1$  is an external control parameter appearing in the hamiltonian. For simplicity, consider a two-level system with energy levels 0 and  $a_1$  and functioning at an efficiency  $\eta$ . Imagine that  $\eta$  is specified to us, but  $a_1$  is unknown. Within the Bayesian approach, the expected value of the work per cycle is defined as  $\bar{W}(\eta) = \int_{a_{\min}}^{a_{\max}} \mathcal{W}(a_1, \eta) \Pi(a_1) da_1$ , where  $\Pi(a_1)$  is the prior probability distribution. If the only prior information about parameter  $a_1$  is that it takes positive real values and otherwise we have complete ignorance about its value, then Jeffreys has suggested the prior distribution  $\Pi(a_1) \propto 1/a_1$ . We find that as the range of  $a_1$  tends to  $[0, \infty]$ , the expected work becomes maximal at Curzon-Ahlborn (CA) value, given by  $1 - \sqrt{T_2/T_1}$ . Laplace and Bayes have advocated a uniform probability distribution to quantify the state of complete ignorance. We find that the efficiency at optimal work is then given by

$$\eta^* = 1 + \frac{\theta^{4/3}}{3 \left(1 + \sqrt{1 + \frac{\theta^2}{27}}\right)^{1/3}} - \theta^{2/3} \left(1 + \sqrt{1 + \frac{\theta^2}{27}}\right)^{1/3},$$

where  $\theta = T_2/T_1$ . We also investigate a general class of priors  $\Pi(a_1) \propto (a_1)^\gamma$ . For close to equilibrium ( $T_1 \approx T_2$ ), the efficiency at optimal work for such priors is given by

$$\eta^* \approx \frac{\eta_c}{2} + \frac{(3-\gamma)}{16} \eta_c^2 + O(\eta_c^3),$$

where  $\eta_c = 1 - T_2/T_1$ . The prior probabilities can be updated in the light of experimental data using Bayes theorem and we find expected value of work using the posterior probabilities. Thus the choice of power-law prior  $\Pi(a_1) \propto 1/(a_1)^\gamma$  leads to efficiencies close to CA value and which resemble the results obtained with finite-time cycles [3] and finite heat source/sink models [4,5]. These findings lead us to propose a connection between Bayesian probabilities and thermodynamic behavior.

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## Writhe and twist in DNA circles and their melting behavior

A. Kabakcioglu<sup>†</sup>, A. Bar, D. Mukamel, E. Orlandini, B. Avsaroglu, M. Sayar

<sup>†</sup>Koc University, Istanbul

The physics of DNA melting has been of interest since 60's with some recent theoretical developments. In particular, Kafri et.al. [PRL, 85, 4988 (2000)] showed how self-avoidance between denaturation loops and the rest of the DNA molecule leads to a first-order melting transition. We have recently developed a model for circular DNA rings which takes into account the helical arrangement of the two strands. Also guided by single-molecule forced extension/torsion experiments on DNA strands, we propose that the torsional stress induced by locally denatured regions enforces formation of supercoils whose "writhe" absorbs the "linking number" expelled by the denaturation loops. The problem is investigated in the Grand Canonical framework where the topological constraint (fixed linking number) is imposed through an associated fugacity-like term. The resulting melting scenario we find is in contrast with the earlier models which ignore the writhe of the DNA chain, predicting a softer transition for  $c_2=2$  and merely a crossover for  $c_2>2$ . We also find a topology-induced drop in the melting temperature in qualitative agreement with recent experiments. The interplay between writhe and twist is nontrivial also in DNA minicircles. In a recent work, we consider supercoil formation in DNA minicircles under varying linking number by using molecular-dynamics simulations of a two-bead coarse-grained model. Our model is designed with the purpose of simulating long chains without sacrificing the characteristic structural properties of the DNA molecule, such as its helicity, backbone directionality, and the presence of major and minor grooves. The model parameters are extracted directly from full-atomistic simulations of DNA oligomers via Boltzmann inversion; therefore, our results can be interpreted as an extrapolation of those simulations to presently inaccessible chain lengths and simulation times. Using this model, we measure the twist/writhe partitioning in DNA minicircles, in particular its dependence on the chain length and excess linking number. We observe an asymmetric supercoiling transition consistent with experiments. Our results suggest that the fraction of the linking number absorbed as twist and writhe is nontrivially dependent on chain length and excess linking number. Beyond the supercoiling transition, chains of the order of one persistence length carry equal amounts of twist and writhe. For longer chains, an increasing fraction of the linking number is found to be absorbed by the writhe.

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## Current-voltage characteristic of narrow superconducting wires: bifurcation phenomena

V. Kabanov<sup>†</sup>, V.V. Baranov, A.G. Balanov

<sup>†</sup>Josef Stefan Institute

The current-voltage characteristics of a long and narrow superconducting channels are investigated using the time-dependent Ginzburg-Landau equations for complex order parameter [1]. We found out that the steps in the current voltage characteristic can be associated with bifurcations of either steady state or oscillatory solution. We revealed typical instabilities which induced the singularities in current-voltage characteristics. We demonstrated that the steady state superconducting solution loses its stability as a result of the saddle-node homoclinic bifurcation with the formation of the limit cycle with diverging period when current is close to its critical value. We derived normal form for this bifurcation and obtained the analytical expression for the dependence of the period of the limit cycle on current. The current voltage characteristic is singular and the voltage is proportional to the square root of  $J-J_c$  where  $J_c$  is the critical current. This singularity of current voltage characteristic is similar to the case of very short channel [2]. The phase slip center in that case is localized in the center of the channel [3]. The second singularity in current voltage characteristic corresponds to the period doubling bifurcation. This bifurcation leads to second singularity in current voltage characteristic. This singularity is well pronounced in long channels while it manifests itself as a small kink in the short channels. As a result of period-doubling bifurcation, a new frequency appears in the spectrum of the electromagnetic radiation generated by the current. This new frequency is equal exactly to the half of the frequency before bifurcation point. In that case two neighboring phase slip centers are shifted in opposite directions with respect to the center of the wire [3]. Further increase of the current leads to the next step on current voltage characteristic. The bifurcation associated with this anomaly is not universal. In the short channel the bifurcation occurs when oscillations are chaotic. In the case of the long channel this anomaly is associated with the appearance of a new and more complicated limit cycle. The new extra phase slip center in that case appears in the center of the channel as experimentally observed in Ref.[3]. Our results in the range of higher currents shows that these bifurcations can substantially complicate dynamics of the order parameter and eventually lead to appearance of such phenomena as multistability and chaos.

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## Target location on DNA

Y. Kafri†, M. Sheinman, O. Benichou, R. Voituriez

†Technion

Genomic expression depends critically both on the ability of (typical) regulatory proteins to locate specific target sites on a DNA within seconds and on the formation of long lived (many minutes) complexes between these proteins and the DNA. In the talk I will focus on the challenges posed on this process by the interactions between transcription factors and the DNA. A possible classification of transcription factors according to their interaction energies (weight matrices) with the DNA will be presented and will be illustrated using experimental data. I will then argue that each class calls for a different search process and will discuss the possible application of previously suggested mechanisms to each class. The talk will end with a new proposed mechanism which is based on barrier discrimination. It will be shown that this mechanism applies to all classes of transcription factors and can lead to a fast and specific search. Moreover, it is shown that the mechanism has interesting transient features which allow for stability at the target despite a rapid binding and unbinding of the transcription factor from the target.

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## Physical origin of the power-law tails in statistical physics

G. Kaniadakis†

†Physics department, Politecnico di Torino, Italy

In the last few decades, the power-tailed statistical distributions have been observed in a variety of physical, natural or artificial systems [1]. For instance, in plasma physics, the cosmic ray spectrum,  $f_i \propto \chi(\beta E_i - \beta\mu)$ , obeys the Boltzmann law of classical statistical mechanics i.e.  $\chi(x) \underset{x \rightarrow 0}{\sim} \exp(-x)$  at low energies, while at high energies this spectrum presents power law fat tails i.e.  $\chi(x) \underset{x \rightarrow +\infty}{\sim} x^{-a}$ . Currently, there is an intense debate regarding the physical origin of the experimentally observed non-Boltzmannian distributions. Recently, after noting that the power-law tails are placed in the high energy region, where the particles are relativistic, the question has been posed whether the solution of the problem, i.e. the theoretic prediction of the function  $\chi(x)$  and consequently of the related distribution and entropy, can be explained by invoking the basic principles of special relativity. The present contribution, deals with the statistical theory [2,3] predicting that  $\chi(x) = \exp_\kappa(x)$  where  $\exp_\kappa(x) = (\sqrt{1 + \kappa^2 x^2} + \kappa x)^{1/\kappa}$  and  $\kappa^2 < 1$ . The related, experimentally observed distribution  $f_i \propto \exp_\kappa(\beta E_i - \beta\mu)$ , at low energies behaves as the Maxwell-Boltzmann exponential distribution, while at high energies presents power law tails. Here we show that the function  $\exp_\kappa(x)$  and its inverse  $\ln_\kappa(x) = (x^\kappa - x^{-\kappa})/2\kappa$ , can be obtained within the one-particle relativistic dynamics, in a very simple and transparent way, without invoking any extra principle or assumption, starting directly from the Lorentz transformations. The achievements support the idea that the power law tailed distributions are enforced by the Lorentz relativistic microscopic dynamics, like in the case of the exponential distribution which follows from the Newton classical microscopic dynamics. Subsequently we reconsider critically the kinetic foundations of the statistical theory based on the function  $\exp_\kappa(x)$  and show that the standard principles of ordinary relativistic statistical physics, i.e. kinetic equation, H-theorem, Molecular Chaos Hypothesis (MCH), conduct unambiguously to the relativistic generalization of the classical Boltzmann entropy and Maxwell-Boltzmann distribution. Then we obtain the evolution equation, conducting asymptotically to the distribution  $f_i$ , starting from the relativistic BBGKY hierarchy.

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## A consistent approach for the treatment of Fermi acceleration in time-dependent billiards

A.K. Karlis<sup>†</sup>, F.K. Diakonou, V. Constantoudis

<sup>†</sup>Department of Physics, University of Athens, Greece; Physikalisches Institut, Universität Heidelberg, Germany

More than 60 years ago, Fermi [1] proposed an intuitive mechanism for the explanation of the origin of the highly energetic cosmic ray particles and ever since it has been a subject of intense study. Soon after his seminal paper, his co-worker Ulam introduced a simple mechanical model for testing Fermi's idea [2], known as the Fermi-Ulam model (FUM), linking for the first time Fermi acceleration with the study of time-dependent billiards. From the time of the FUM, the standard description of Fermi acceleration developing in a class of time-dependent billiards is given in terms of a diffusion process taking place in momentum space [3]. Within this framework the evolution of the probability density function (PDF) of the magnitude of particle velocities as a function of the number of collisions  $n$  is determined by the Fokker-Planck equation (FPE). In the literature the FPE is constructed by identifying the transport coefficients with the ensemble averages of the change of the magnitude of particle velocity and its square in the course of one collision [3, 4]. Although this treatment leads to the correct solution after a sufficiently large number of collisions has been reached, the transient part of the evolution of the PDF is not described. Moreover, in the case of the FUM, if the standard simplification is employed, the solution of the FPE is even inconsistent with the values of the transport coefficients used for its derivation. The aim of the work presented [5], is to provide a self-consistent methodology for the derivation of the PDF of particle velocities for all times. The proposed approach obviates any assumptions for the continuity of the random process and the existence of the limits formally defining the transport coefficients of the FPE. Specifically, we suggest, instead of the calculation of ensemble averages, the derivation of the one-step transition probability function and the use of the Chapman-Kolmogorov forward equation. This approach is generic and can be applied to any time-dependent billiard for the treatment of Fermi-acceleration. As a first step, we apply this methodology to the FUM, being the archetype of time-dependent billiards exhibiting Fermi acceleration, considering the simplifying, as well as the exact dynamics.

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## Optimal origin placement in DNA replication

J. Karschau<sup>†</sup>, J.J. Blow, A.P.S. de Moura

<sup>†</sup>Institute for complex Systems and Mathematical Biology, SUPA, University of Aberdeen, Aberdeen, UK; Wellcome Trust Centre for Gene Regulation and Expression, University of Dundee, Dundee, UK

DNA replication is an essential process in biology and its timing must be robust so that cells can divide properly. Random fluctuations in the formation of replication starting points, called origins, and the subsequent activation of proteins lead to the risk of variation in timing. I will discuss an investigation of the stochastic properties of DNA replication using a mathematical model, which elucidates the spatial organisation of origins on the DNA such that brief and faithful duplication of the genome is ensured. If failing origins delay the completion of the replication process more than is allowed by the cell cycle, cells can lose vital pieces of their genomic information. This could lead to cell death or the development of cancer in animal cells. There exists a number of recent theoretical and modelling works on the dynamics of DNA replication (reviewed by Hyrien and Goldar [1]). Previous theoretical works on yeast have used the experimentally determined origin locations as given parameters, without attempting to understand why the origins are located where they are [2-4]. Inspection of the origin locations on a yeast genome map makes it clear that the origin positions are not random. There is also experimental evidence for origin clustering in frog embryos, where origins seem to be distributed in clusters of 10 to 20. Existing mathematical models of replication in frog embryos, such as by Jun and Bechhoefer [5], assume the origin locations to be random and independent of each other, and so they cannot explain the origin clustering. A full explanation of origin clustering and the mechanism for on-time replication is still unknown. Contrary to what one might naively expect - that placing origins as far apart as possible will achieve minimum replication time - I will show that it can actually be more efficient to place origins together in clusters for certain parameter regimes. Whether it is optimal to place origins in clusters or in isolation strongly depends on both the probability that certain sites will bind with origin forming proteins and the length of the activation time window for an origin. I will show that the same phase transition is observed in simulations which include stochasticity in origin activation and in simulations which do not, i.e. the result is independent of the precise details of the model. The model explains the basis of the spatial distribution of origins for two common model organisms, yeast and frog embryos. Overall, the results show that the organisation of origins is an optimisation problem in living organisms which has been solved through evolution rather than being completely random.

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## Diverging equilibration times in long-range quantum spin models

M. Kastner<sup>†</sup>

<sup>†</sup>National Institute for Theoretical Physics, South Africa

Equilibration is one of the central concepts of thermodynamics, but our understanding of the underlying microscopic processes is still far from complete. Modern facets of this topic, attracting much attention in the recent research literature, are relaxation after a quantum quench or (the failure of) thermalization close to integrability, to name but a few. Although experience shows that equilibration takes place in the vast majority of situations, exceptions have always attracted particular interest. One particularly interesting case, going under the name of quasistationary states, has triggered intense activity, reviewed in [1], in the field of long-range interacting systems. *Long-range* refers here to interactions decaying at large distances  $r$  as a power law  $r^{-\alpha}$  with a positive exponent  $\alpha$  not exceeding the spatial dimension  $d$  of the system. The term *quasistationary* is used to describe metastable states whose lifetime diverges with increasing system size  $N$ . Here we study the approach to equilibrium of long-range quantum spin models. For a large class of observables and initial states, the time evolution of expectation values can be calculated. For finite system sizes  $N$ , almost periodic behavior is observed with recurrence times that increase exponentially with  $N$ . In the limit of large system sizes, we analytically prove quasistationary behaviour, i.e. that, at a given instant of time  $t$  and for sufficiently large system size  $N$ , the expectation value of some observable  $\langle A \rangle(t)$  will practically be unchanged from its initial value  $\langle A \rangle(0)$ . This finding implies that, for large enough  $N$ , equilibration effectively occurs on a time scale beyond the experimentally accessible one and will not be observed in practice. Future experiments with ultracold polar gases in optical lattices are expected to permit the experimental observation of such diverging equilibration times in long-range quantum spin systems. This analytic result, reported in [2], is, to the best of the author's knowledge, the first observation of this kind of behavior in quantum systems, and it prepares the ground for further studies, including the above-mentioned quantum quenches or inquiries into the foundations of statistical physics of long-range systems. The results complement the peculiarities of long-range quantum spin systems in equilibrium reported in [3].

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## Nonextensive statistical mechanics distributions and the dynamics of financial observables from the nonlinear stochastic differential equations

B. Kaulakys<sup>†</sup>, J. Ruseckas, V. Gontis

<sup>†</sup>Institute of Theoretical Physics and Astronomy, Vilnius University, Lithuania

The studies of the financial markets using statistical physics approach revealed some stylized facts, including the power-law tails of probability densities of the trading activity and log-returns, as well as the long-term power-law correlations of the traded volume, the absolute value returns and returns standard deviation (the volatility). The exponents that characterize these power laws are similar for different types and sizes of markets, for different market trends and even for different countries - suggesting that a generic theoretical basis may inspire these phenomena. The financial observables may be related to the Tsallis statistical approach (see, e.g., [1] and references herein), resulting in the power-law tails, as well. On the other hand, trades in the financial markets occur at discrete times and may be considered as point processes [2-3]. Here, starting from the multiplicative point process model [4] and nonlinear stochastic models [5] of  $1/f$  noise and power-law distributions, modifications of the nonlinear stochastic differential equations, giving probability distribution functions from the nonextensive statistical mechanics are proposed. We derive nonlinear stochastic differential equations, generating the processes with the  $q$ -exponential and  $q$ -Gaussian distributions of the observables, i.e., with the long-range power-law autocorrelations and  $1/f$  power spectral density. Equations for the  $q$ -Gaussian distributions allow negative values of the variable. On the other hand, the  $q$ -exponential and  $q$ -Gaussian distributions may be obtained in the superstatistical framework as the superposition of different local dynamics at different time intervals. In such approach, the average of the stochastic variable is generated by the simple nonlinear stochastic variable, while the local distribution of the signal is exponential or Gaussian one, conditioned by the slow average. Further we analyze analytically and numerically the properties of solutions of these equations in relation with the nonextensive statistical mechanics framework and relevance of the generalized and adapted equations for modeling of the financial processes. We model the inter-trade durations, the trading activity and the normalized return using the superstatistical approaches with the exponential and normal distributions of the local signals driven by the nonlinear stochastic process.

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## Amplification of uniformity and multiformity in the stressed global market

D. Kenett<sup>†</sup>, M. Raddant, T. Lux, E. Ben-Jacob

<sup>†</sup>Tel-Aviv University

One feature of the current age of "global village" is the interaction between different international stock markets. This has resulted in a global financial village, in which markets across the world are strongly coupled. In dealing with financial crises, such as the 2008 crisis, the issue of market coupling becomes critical. However, the extent and magnitude of such coupling is not fully understood, and who it varies between different types of markets (i.e. developed versus emerging). Here we present a methodology to investigate the coupling between different markets, by investigating the dynamics of their correlations and cross-market similarities of these dynamics. We investigate the dynamics of correlations in six markets - U.S., U.K., Germany, Japan, China and India. For each market, we investigate the daily return of stocks traded between January 2000 and December 2010, focusing on the largest stocks traded in each market. The first step is to investigate correlations between stocks in each market separately - the market Micro-correlations. Next, a cross-market analysis of the micro-correlations is applied, to quantify the similarity of correlation patterns in different markets, denoted as Macro-correlations. Studying the micro versus macro correlations, we investigate whether local events in one market (represented by strong changes in the micro-correlations) will lead to changes in the macro-correlations with other markets, which in turn lead to strong changes in the micro-correlations in other markets. This allows to study the bottom-top-bottom feedback interactions in the global market. We observe significant uniformity in the Macro-correlations of the developed markets (U.S., U.K., Germany, Japan), and significant multiformity in the Macro-correlation of the developing markets (China, India). We have found that the U.S. and Japan, and that the U.K. and Germany are strongly coupled and uniform in their dynamics; in contrast, China and India exhibit a strong multiformity, and are weakly coupled to the other markets. This methodological framework provides a way to quantify uniformity and multiformity in the global market, and changes in these measures. Such changes can precursors to the agitation and stress in the global financial village, and this quantification will help to prevent, or ameliorate, future worldwide financial crises.

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## The sociophysics of collaborative research: Critical mass and the dependence of quality on quantity

R. Kenna<sup>†</sup>, B. Berche

<sup>†</sup>Coventry University

Although the notion of critical mass in research has been around for a long time, it has lacked proper definition. This is remarkable, since the notion is frequently used by policy makers and managers in academia. Critical mass in this sense has been loosely described as some kind of threshold group size above which collaborative-research quality significantly improves. An extended notion is that quality continues to improve through increasing concentration of researchers into a small number of research institutions. However no evidence for such a threshold has been found and critical masses have never been measured - until now. We present a sociophysical model which explains how research quality depends on research-group structure and in particular on group size. In the model, research groups of interacting scientists are considered as many-body, complex sociophysical systems and their cooperative behaviour is analysed from a mean-field point of view. The model shows that interactions between individuals rather than accumulated individual node strengths dominate the strength and quality of group activity, and give rise to phenomena similar to phase transitions, where the extensive relationship between research quality and group quantity reduces. A consequence of the model is that there are, in fact, two critical masses in research, each of which is discipline dependent. Research quality tends to be linearly dependent on group size, but only up to a limit termed the "upper critical mass". The upper critical mass is approximately the average maximum number of colleagues with whom a given individual in a research group can meaningfully interact. In a phenomenon akin to the Ringelmann effect, once the group exceeds this size, communication problems set in and there is a tendency to fragment into sub-groups. Consequently, research quality no longer tends to improve significantly with increasing size. There is also a lower critical mass, which small research groups should strive to achieve stability. Our theory is tested using empirical data on the quantity and quality of scientific research groups, for which critical masses are determined. For example, the upper critical masses in experimental and theoretical physics are about twenty-five and thirteen, respectively, while for pure mathematics it is about four or less.

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### Surface tension calculations of nanodrops

G. Kharlamov†, G.V. Kharlamov, A.A. Onischuk, S.V. Vosel, P.A. Purtov

†Novosibirsk State Technical University

An effective method of nanoparticles production is the condensation of nanodrops from supersaturated vapor. Nucleation, formation of critical nuclei, is the first stage of such condensation. The surface tension of small drops is the most important parameter in the nucleation theory, as it determines the work of the critical nuclei formation. It is common practice to distinguish mechanical and thermodynamic definitions of a droplet surface tension. Mechanical definition assumes equal forces and force moments in real and model systems. Equations of mechanical equilibrium of a drop are employed. The thermodynamic surface tension is included in thermodynamic potentials of the system and their variations in thermodynamic processes. It is supposed that these two definitions of surface tension are to give different values of surface tension and radii of tension, though the cause is not quite clear. J Gibbs defined surface tension of a drop in conditions of equal chemical potentials of comparison phase and real system. Thus the problem is to elucidate the difference between thermodynamic and mechanical definitions of surface tension, and to calculate their dependences on the drop radius and the system temperature. In our report the results of molecular dynamics calculations of nanodrops are presented. The equilibrium states of Lennard-Jones molecular system containing one nanodrop of 50 - 4000 molecules and the saturated vapor have been prepared. The density profiles, the Irving-Kirkwood pressure tensors, the chemical potentials of the systems, the equimolar radii of the drops and the radii of tension, the mechanical and thermodynamic surface tensions have been calculated. The dependence of the surface tension of Lennard-Jones liquid nanodrops on the equimolar radius and temperature has been calculated by the molecular dynamics method. It was found that the mechanical surface tension is different from the thermodynamic surface tension for drops of 50 - 2000 molecules and is equal for drops of more than 2000 molecules. It is shown that both the mechanical and thermodynamic surface tensions decrease with the decrease of the equimolar radius of the drop, and reach zero at the same  $R_0$  depending on temperature. The radii of tension also reach zero. The dependence of the ratio of the surface tension of the drop to the surface tension of the flat surface liquid-vapor on the ratio of the equimolar radius of the drop to  $R_0$  is a universal function [1-4].

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### Entropy and the statistical method of the technological processes describing

V. Khodusov†, O.M. Pignasty

†Karazin Kharkov National University, Ukraine

Modern problems of describing of technological processes gone far beyond the traditional studies of production systems. A qualitative leap in the complexity of the tasks of technological processes control is one of the main reasons for the new directions appearance. A statistical method is an area of the building modern models of technological processes. It allows us to establish the basic laws of technological phenomena through the use of model representations about the stochastic mechanism of technological equipment inspiration for subject of labour and collective interaction of subjects of labor among themselves. We consider the main regularities of technological processes macroparameters behavior. An equation of macroparameters state of technological process is obtained through the aggregation of objects of labor microstates. For a large number of objects of labor a macroscopic streams of characteristics of technological process uniquely related to the microscopic parameters. The objects of labor movement on the technological route default by the dynamic balance transport equations, which describe the evolution in space and time technological process macroparameters. Values fluctuations of the object of labor microparameters determine the behavior of macroscopic quantities of the technological process and, most importantly, participate in the formation of an irreversible process. As a state function of technological process an entropy is introduced, which characterizes the measure of its uncertainty. Entropy of the technological process is determined by the distribution function by the microstates of the objects of labor. The distribution function of the objects of labor is defined by the kinetic equation of the technological process, similar Boltzmann equation. Production function of the technological process is determined the mode of production. The instability of technological trajectories of the objects of labor is the main factor leading to irreversibility. It is shown that the irreversibility of technological phenomena at the motion of objects of labor on the technological route consist in the interaction of objects of labor with production equipment. Using the model representation of the character of the interaction between the object of labor with technological equipment, the increase entropy law is proven for a closed industrial-technological system.

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## Complexity from strategic choices during ship maneuvering

G. Kleppe<sup>†</sup>, K. Christensen, V. Frette

<sup>†</sup>Stord/Haugesund College, Norway

We analyze complex maneuvering histories of ships obtained from training sessions on bridge simulators. Advanced ships are used in fields like offshore oil exploration: dive support vessels, supply vessels, anchor handling vessels, tugs, cable layers, and multi-purpose vessels. Due to high demands from the operations carried out, these ships need to have very high maneuverability. This is achieved through a propulsion system with several thrusters, water jets, and rudders in addition to standard propellers. For some operations, like subsea maintenance, it is crucial that the ship accurately keeps a fixed position. Therefore, bridge systems usually incorporate equipment for Dynamic Positioning (DP). DP is a method to keep ships and semi submersible rigs in a fixed position using the propulsion systems instead of anchors. It may also be used for sailing a vessel from one position to another along a predefined route. Like an autopilot on an airplane, DP may operate without human involvement. The method relies on accurate determination of position from external reference systems like GPS, as well as a continuously adjusted mathematical model of the ship and external forces from wind, waves and currents. Ship simulator exercises forms a necessary part of the training of operators that are to run ships under this advanced automation system. In a specific simulator exercise for offshore crews, a ship is to be taken up to an installation consisting of three nearby oil platforms connected by bridges (Frigg field, North Sea), where a subsea inspection is to be carried out. Due to the many degrees of freedom during maneuvering, including partly or full use of DP, the chosen routes vary significantly. We report results on representations of the complex maneuvering histories: trajectories, with additional dynamical parameters, form varied, complex patterns, as a result of strategic choices of the crews. We suggest and develop several measures that characterize variations between the patterns. The results are compared to expert evaluations from experienced instructors. Expert evaluations are intuitive, integrating assessments of complex situations, based on extensive experience. These evaluations cannot necessarily (or easily) be decomposed into a series of parameter values. We test to which degree expert evaluations correspond to quantitative measures of trajectories.

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## Tunable oscillators based on Josephson junctions.

A.M. Klushin<sup>†</sup>, F. Song, F. Müller, T. Scheller, A. Semenov

<sup>†</sup>Institute for Physics of Microstructures RAS, Nizhny Novgorod, Russia.

Essential applications of terahertz (THz) technology in fields such as astronomy, medicine, security and communications starve for compact, tunable solid-state continuous wave radiation sources. Through intensive investigation in the last decade, quantum cascade lasers have offered satisfactory solutions at frequencies above 1.2 THz. However, it is still under discussion for the frequency range approximately up to 1.0 THz. Fortunately, Josephson junctions have connatural advantages to fulfill this challenge. Here we present coherent tunable non-Josephson radiation of large series arrays of 7000 Josephson junctions embedded in the quasi optical resonator at liquid helium temperature. First we have observed coherent emission at the frequency of 76 GHz from an array of superconductor-isolator-normal metal-isolator-superconductor (SINIS) junctions. This junctions have a small characteristic frequency about 30 GHz. The maximal detected power of  $2 \mu W$  was measured by a room temperature superheterodyne receiver. In the second experiment arrays of superconductor - normal metal - superconductor (SNS) were used. Characteristic frequency was increased to 115 GHz. In this case the detected radiation covered the frequency range from about 0.10 THz up to more than 0.25 THz, with maximum radiation power up to  $7 \mu W$  around 0.143 THz obtained at room temperature. Such power is sufficient for implementation of this technology as local oscillators in heterodyne receivers with superconducting hot-electron bolometers. We have overcome the well-known obstacle to impedance matching by utilizing the resonances on the junction substrates served as dielectric resonator antennae. Considering the substrate as a dielectric resonator antenna, the coherent emission was excited if the locations of each sub-array coincided with the positions of the maximum electric field intensity in the substrate resonance mode. The observation of the self-induced steps without an applied static magnetic field or external irradiation indicates strong coupling of the junctions to their substrate cavity around resonant frequencies  $f_r$ . Through intensive investigation, it was documented that radiation frequency is shifted from the Josephson oscillation frequency  $f_J$  to the resonant frequency  $f_r$ , which originated the so-called non-Josephson oscillation. Moreover, it is very promising to extend this coherent radiation to the sub-THz range with higher power available by coupling the Josephson junction arrays to modified substrates. Furthermore, this approach could be also useful for synchronizing various integrated oscillators including semiconductor devices on one chip to cover the sub-THz frequency range. The work was supported by Russian Agency of Education under the Federal Program "Scientific and educational personnel of innovative Russia in 2009-2013".

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## Derivation of Navier-Stokes equation using Stochastic variational method

T. Koide†, T. Kodama

†Universidade Federal do Rio de Janeiro

The variational principle is one of important guiding principles in physics. Classical equations of motion of particle can be formulated in the variational principle for both of non-relativistic and relativistic cases. This idea is easily extended to field equations. In particular, the variational principle is indispensable from the view point of the theory of relativity. Once the action is chosen to be Lorentz scalar and invariant for coordinate transforms, the requirement from relativity is automatically satisfied for the derived equations. As a matter of fact, the variational principle is unaltered by the modification from Newton's classical mechanics to the theory of relativity. However, the variational principle is difficult to apply to irreversible dynamics. This is because dissipation is caused by the conversion between macroscopic and microscopic motions, leading to the entropy production. In this sense, problems with dissipation will be beyond the scope of the classical variational method. Thus, to include dissipative effects, one usually introduces by hand, for example, the so-called Rayleigh dissipative function, time dependent external factor in the Lagrangian and so on. The form of the function (factor) is changed to reproduce the result which we wish to derive. The stochastic variational method was proposed thirty years ago [1]. In this approach, variational principle is extended to stochastic variables. It was shown that the incompressible Navier-Stokes (NS) equation can be obtained naturally from an action without introducing any additional functions. In this work, we will show that the compressible NS equation can also be derived in the stochastic variational method. From this analysis, we found that 1) Starting from the action which leads to the Euler equation when the traditional variational method is applied, we arrive naturally at the compressible NS equation simply by applying the stochastic variational method considering variation variables as stochastic quantities. 2) The form of the NS equation thus obtained is independent of the detailed structure of interactions among constituent particles of fluid. They enter only through the equation of state. This explains why the NS equation is universal for various fluids. 3) In general, hydrodynamics is expressed only with unique velocity. However, two different velocities naturally appear in the stochastic variational method. That is, the more general hydrodynamics than the NS equation is expressed with the two velocities. This feature is similar to the idea of the generalization of hydrodynamics by Brenner [2].

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## On Cantor sets associated with unstable periodic orbits generated by the logistic map

M. Komatsuzaki†, T. Motoike, T. Arimitsu

†University of Tsukuba

Multifractal Probability Density Function Theory (MPDFT) [1-4] is a statistical mechanical ensemble theory for analyzing intermittent phenomena providing fat-tail PDFs under the assumption that the singularities due to the intermittency distribute themselves multifractal way in real physical space. In order to extract the intermittent character of the fully developed turbulence, it is necessary to have information of hierarchical structure of the system by observing a series of PDFs for those quantities responsible for intermittency with different length  $\ell_n = \ell_0 \delta^{-n}$  ( $n = 0, 1, 2, \dots$ ) that characterize the sizes of regions in which the physical quantities are coarse-grained. Since the value  $\delta (> 1)$  of magnification is determined freely by observers, the choice of  $\delta$  should not affect observables. This demand provide us with a new scaling relation  $(\ln 2)/(1 - q) \ln \delta = 1/\alpha_- - 1/\alpha_+$  where  $\alpha_{\pm}$  are the zeros of the multifractal spectrum  $f(\alpha)$ . MPDFT tells us that each turbulent system has its own multifractal spectrum. On the other hand, it has been revealed [5] that the new scaling relation is intimately related to a  $\delta$ -scale Cantor set associated with  $\delta^\infty$  super-stable periodic orbits (SSPOs) in the system of logistic map. This leads us to a conjecture that the system of fully developed turbulence is constituted of the accumulation of the Cantor sets characterized by  $\delta^\infty$  SSPOs with different values of  $\delta$ . Observation of the system with the magnification  $\delta$  extracts the information of the  $\delta$ -scale Cantor sets constituting the turbulence. Note that the scaling relation for the  $\delta^\infty$  SSPO has a slight difference of the order of  $1/\delta$  compared with the above new scaling relation within MPDFT found for fully developed turbulence. Note also that each  $\delta^\infty$  SSPO has its own multifractal spectrum, which is different from the case for fully developed turbulence. In this paper, we propose a refined conjecture that the system of the fully developed turbulence is constituted of the accumulation of the Cantor sets associated with  $\delta^\infty$  unstable periodic orbits (USPOs) especially at the fully developed chaotic state with different values of  $\delta$ , and check its validity by investing precisely the characteristics of the orbits of  $\delta^\infty$  USPOs generated by the logistic map and their topological structures.

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## Paradoxes of subdiffusive infiltration in disordered systems

N. Korabel<sup>†</sup>, E. Barkai

<sup>†</sup>Physics Department, Institute of Nanotechnology and Advanced Materials, Bar-Ilan University

Infiltration of diffusing particles from material A to material B through some interface is a widely investigated process. In recent years much focus was diverted to the problems when the diffusion in one material or in both is anomalous, namely  $\sim t^\alpha$  and  $\alpha \neq 1$  [1]. We consider two semi-infinite materials located in  $x < 0$  and  $x > 0$ , where particles exhibit anomalous sub or super-diffusion. We find several peculiar behaviors unique to anomalous infiltration: in the case of a composition of two sub-diffusive systems the infiltration of particles from material  $x < 0$  to  $x > 0$  leads to a net drift. We also show that in some cases the flow of particles is opposite to the drift. In fact we find a situation when asymptotically all the particles are say in sample  $x > 0$  but the average drift is oppositely directed  $< 0$ . This seemingly paradoxical behavior is explained [2]. Secondly, if materials in  $x < 0$  and  $x > 0$  have different diffusive properties, in the long time limit all particles will be accumulated in the material with slower diffusion, which will act as a trap producing a flow of particles from one material to another. Interestingly, in the long time limit the drift depends only on the properties of the slower medium. This is a surprising result since  $\langle x(t) \rangle$  can be very far from the interface, deep in the faster sample, but still be independent of the properties of that region. A second model we study is a sub-diffusive material (for example in  $x < 0$ ) coupled to super-diffusive sample (in  $x > 0$ ). For super-diffusion motion we consider a Lévy walk model. To analyze this infiltration problem we need the distribution of the first passage times for a Lévy walk on a semi-axes. Using the first passage time density in  $x > 0$  and  $x < 0$ , we calculate the average of occupation fractions and find the scaling of the particles distribution [3]. For a sub-diffusive system coupled to a super-diffusive material a net drift is found even for unbiased motion on the boundary. This drift is always directed to the super-diffusive material, while the particles flow to the material with longer sticking times. Although phenomena such as drift against the flow and flow without the drift are known for systems with normal diffusion where they are generated by geometrical constraints, or by thermal or external field inhomogeneities, in our case these effects are only due to the anomalous nature of diffusion and are not present for normal diffusion. These phenomena are explained by the competition of the diffusion processes which are slower or faster than normal spreading.

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## Quantum Refrigerators: The quest for the absolute zero

R. Kosloff<sup>†</sup>, Y. Rezek, A. Levi, T. Feldmann

<sup>†</sup>Hebrew University Jerusalem Israel

Quantum thermodynamics was initiated by Einstein in 1905 demanding consistency of black body radiation with thermodynamics. This was the seed from which quantum mechanics was born. In modern context we reverse the argument checking the consistency of thermodynamics with quantum mechanics. We follow thermodynamical tradition of studying model engines based on quantum principles. I will show the emergence of Carnot efficiency in a model 3-level amplifier. Reciprocating and continuous quantum refrigerator are employed with the purpose of determining the limitations of cooling to absolute zero. We seek a dynamical description of the the third law of thermodynamics: The scaling of the optimal cooling power as a the cold bath temperature vanishes  $T_c \rightarrow 0$ . We study a cycle is based on demagnetization/magnetization of a working medium. We find that if the energy spectrum of the working medium possesses an uncontrollable gap, and in addition there is noise on the controls, then there is a minimum achievable temperature above zero. The reason is that even a negligible amount of noise, prevents adiabatic following during the demagnetization stage. This results with a minimum temperature,  $T_c(min) > 0$  which scales with the energy gap. The refrigerator is based on an Otto cycle where the working medium is an interacting spin system with an energy gap. For this system the external control Hamiltonian does not commute with the internal interaction. As a result during the demagnetization and magnetization segments of the operating cycle the system cannot follow adiabatically the temporal change in the energy levels. We connect the nonadiabatic dynamics to quantum friction. An adiabatic measure is defined characterizing the rate of change of the Hamiltonian. Closed form solutions are found for a constant adiabatic measure for all the cycle segments. We have identified a family of quantized frictionless cycles with increasing cycle times. These cycles minimize the entropy production. Such frictionless cycles are able to cool to  $T_c = 0$ . External noise on the controls eliminates these frictionless cycles. The influence of phase and amplitude noise on the demagnetization and magnetization segments is explicitly derived. An extensive numerical study of optimal cooling cycles was carried out which showed that at sufficiently low temperature the noise always dominated restricting the minimum temperature.

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## Nonextensive entropy approach versus fractional model to describe subdiffusion

T. Kosztolowicz<sup>†</sup>, K.D. Lewandowska

<sup>†</sup>Institute of Physics, Jan Kochanowski University, Kielce, Poland

We study the similarities and differences between different models concerning subdiffusion. We look at two sets of subdiffusive models having different physical origins. The first contains the models derived from nonextensive entropy and the second contains the fractional model (this model utilizes a differential equation with fractional time derivative). In general, both of these sets can describe the same physical processes. However, there also occur processes which can be described by models from one set alone, the other set of models cannot be applicable to describe this processes. We are going to find the accordance conditions between models from both sets using the first passage time (FPT) distributions. We calculate FPT distributions for subdiffusion derived from Green's functions of nonlinear equations obtained from Sharma-Mittal's, Tsallis's and Gauss's nonextensive entropies. Then we compare these with FPT distributions calculated from a fractional model using a subdiffusion equation with a fractional time derivative. All of Green's functions give us exactly the same standard relation  $\langle(\Delta x)^2\rangle = D_\alpha t^\alpha$  which defines subdiffusion ( $0 < \alpha < 1$ ), but generally FPT's are not the equivalent of to one another. Our considerations are based on the following assumptions: the first, since all models describe the same subdiffusion process, all of Green's functions should provide the same relation which defines subdiffusion; the second, all Green's functions depend on two parameters  $\alpha$  and  $D_\alpha$ ; the third, the parameters  $\alpha$  and  $D_\alpha$  are measured experimentally within the fractional model. Our last assumption is supported by the fact that the subdiffusion parameters  $D_\alpha$  and  $\alpha$  are measured experimentally by means of, for example, the time evolution of the near membrane layer. We will show here that the FPT distribution for the fractional model is equal to the Sharma-Mittal model only if in the latter the parameters characterizing this model depends on  $\alpha$ , and satisfy the specific equations, whereas the other two models mentioned above give FPT distributions different from the fractional model. Green's functions obtained from the Sharma-Mittal and fractional models - for the parameters obtained from the particular equations - are very similar to each other. We will also discuss the interpretation of subdiffusion models based on nonextensive entropies and the possibilities of experimental measurement of the subdiffusion models parameters.

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## Multifractal height cross-correlation analysis

L. Kristoufek<sup>†</sup>

<sup>†</sup>Charles University in Prague, Czech Republic

The research of long-range dependence and multifractality in various time series has grown significantly during the last years. An efficient detection of long-range dependence and estimation of Hurst exponent is crucial for financial analysts as its presence has important implications for a portfolio selection, an option pricing and a risk management. There are several methods for the long-range dependence detection, among the most popular are the rescaled range analysis, the modified rescaled range analysis, the rescaled variance analysis, the detrended fluctuation analysis and the detrending moving average. For the detection of multifractality, there are three popular methods - the multifractal detrended fluctuation analysis (MF-DFA), the generalized Hurst exponent approach (GHE), which is based on the height-height correlation analysis, and the wavelet transform modulus maxima (WTMM). The precision of various methods has been discussed as well. Recently, the examination of long-range cross-correlations has become of interest as it provides more information about the examined process. Podobnik et al. generalized the detrended fluctuation analysis for two time series and introduced the detrended cross-correlation analysis (DCCA). Zhou further generalized the method and introduced the multifractal detrended cross-correlation analysis (MF-DXA). In this paper, we introduce a new method for detection of long-range cross-correlations and cross-multifractality - multifractal height cross-correlation analysis (MF-HXA). MF-HXA is a multivariate generalization of the height-height correlation analysis. We show that long-range cross-correlations can be caused by a mixture of the following - long-range dependence of separate processes and additional scaling of covariances between the processes. Similar separation applies for cross-multifractality - standard separation between distributional properties and correlations is enriched by division of correlations between auto-correlations and cross-correlations. Efficiency of the method is showed on various simulated series - Mandelbrot's Binomial Multifractal measures, correlated ARFIMA processes and two-component ARFIMA processes. We further apply the method on traded volume and volatility of NASDAQ and S&P500 indices as well as on Crude Oil spot and futures returns and volatility and uncover some interesting results.

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## Nanoscale phase separation in materials with strongly correlated electrons

K.I. Kugel<sup>†</sup>, A.L. Rakhmanov, A.O. Sboychakov

<sup>†</sup>Institute for Theoretical and Applied Electrodynamics, Russian Academy of Sciences, Moscow, Russia

Nanoscale phase separation, that is, a spontaneous formation of random or ordered inhomogeneities in a chemically homogeneous medium, is a common property of the materials with strongly correlated electrons. The nature of such inhomogeneities is usually related to electron correlations, but their specific manifestations can include the effects of different degrees of freedom existing in the solids: spin, charge, orbital, and lattice. Based on the two-band Hubbard model, we analyze a possibility of the phase separation in a system with two types of charge carriers and demonstrate that in the limit of strong on-site Coulomb repulsion, such a system has a tendency to phase separation even in the absence of magnetic or any other ordering [1]. This tendency is especially pronounced, if the ratio of the bandwidths is large enough. The characteristic size of inhomogeneities is estimated accounting for the surface energy and the electrostatic energy related to the charge imbalance. The model is used to study the phase separation in doped manganites [2]. A similar approach is applied to describe characteristic features of band structure and phase separation in multiband superconductors, especially in cuprates [3]. In the systems with the orbital ordering, charge carriers introduced due to doping favor the formation of nanosize inhomogeneities with the orbital structure different from that in the undoped material. Depending on the ratio of the electron hopping integral  $t$  and the interorbital coupling energy  $J$ , their shape can vary drastically. In particular, there exists a critical value of  $t/J$ , corresponding to an abrupt transition from the nearly circular to needle-like inhomogeneities [4]. We also study the situation characteristic of compounds with spin-state transitions, e.g. those containing  $\text{Co}^{3+}$  ions. In this case, the competition of single-site effects and kinetic energy of doped carriers can favor a change in the spin state. Based on a model similar to that used for manganites [2], we predict the possibility of a jump-like change in the density of itinerant charge carriers accompanied by the change in the spin state of cobalt ions at a certain doping level. A tendency to the electronic phase separation within a wide doping range is demonstrated [5]. We also show that the electron-lattice interaction produces a pronounced effect on the conditions of the electronic phase separation since it influences the value of the bandwidth ratio and the relative positions of the bands. If this interaction is strong enough, there appears a competition between states with different values of strains and the transition between them can occur in a jump-like manner.

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## A stable form of two-dimensional crystals: is graphene a glass?

K.I. Kugel<sup>†</sup>, A. O'Hare, F.V. Kusmartsev

<sup>†</sup>Loughborough University

Original motivation of this work was to resolve an issue with the existence and stability of two-dimensional crystals such as graphene and silicene. However, in the course of the research we have not only found a solution of the issue but also discovered a new principle for nanotechnology, which is essential to create and control a bandgap in graphene. The latter finding can open new avenues for graphene applications. The issue is that, at present, there is a contradiction between the fundamental theorem, which was introduced originally by Landau, and Peierls and then proved and formulated into a theorem by Mermin and Wagner (the Mermin-Wagner theorem) and the existence of the flat graphene. Working on this controversy, we found a very general form or shape, in which any two-dimensional crystal may exist, and this shape is not flat but significantly deviates from the flat, plane form. Although the results presented in the paper are very general and applicable to arbitrary two-dimensional crystals we have mainly focused on applications to graphene and compared obtained results with recent experimental work. Analyzing the several existing experiments we found evidence confirming our predictions of the distorted buckled shape of the graphene. We have not only described the form of new structures created by lattice distortions, which may arise on the graphene plane, but also made an estimation of their parameters and expect that these new described structures will be observed and investigated in a great detail in the near future. But what is the most important from our findings is that due to an interaction of graphene with a substrate there may arise a bandgap, whose value depends on the strength of the interaction of the substrate with the graphene in a buckled form. We estimate the range of the bandgap values, which can be created in graphene and indicate that this phenomenon may be very important for various technological applications of graphene. The results are general and applicable to any other two-dimensional crystals and in particular for silicene. Due to the bandgap, there may arise new phenomena, and in particular a rectification of AC current induced by microwave or infrared radiation. We show that the amplitude of direct current arising at wave mixing of two harmonics of microwave electromagnetic radiation is huge. Moreover, we predict the existence of mini-excitons and a new type of fermionic mini-polaritons whose behavior can be controlled by the microwave and terahertz radiation.

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## Statistical mechanics of economics

F.V. Kusmartsev†

†Loughborough University

We demonstrate that methods of statistical mechanics may be very useful in description of many different phenomena existing in economics, and, in particular, in a description and characterisation of free market processes and a formation of financial crisis. A market has been treated as open (in general chaotic), complex system consisting of an arbitrary number of trading economic agents. Taking a large amount of instant snapshots of the market during its operation and then considering ensembles of such snapshots we are able to find money, debt, income and wealth distribution. Depending on the market constraints they may take different forms. The most general form of these distributions, so-called, Bose-Einstein one, arises when the average amount of money and number of economic agents on a market remain constant at least during a some time period, which is arbitrary large in a comparison with single trading processes. Typically, such a period can be taken as a quarter or one year. Using the described methods we study the evolution of market in USA during last one and half decade. We shown t hat during these years the income of the major population is well described by Bose-Einstein(BE) distribution, that complements by a high income tail (the Pareto Law). The parameters of the BE distribution are changing with years and very close to its classical limit- the Boltzmann distribution found before, which also describe this part of income distribution well. Using the data from USA tax return we also show that a formation of financial crisis corresponds to an increase in the value of an absolute activity coefficient (a fugacity) arising from the BE distribution. The analysis indicates correctly the crisis arisen in the past and predicts the future one. We use statistical mechanics to describe financial networks. Taking a large amount of instant snapshots of a financial network over an interval of time we construct their ensembles and study their statistical interference. This results in a probability description of the financial market considering this as a network o f trading agents. Treating the network as statistical ensemble we find the degree distribution of the network and other properties that include the main relation of a market in a connection with a number of transactions, bank capital, demand of the goods and their price. We derive equations connecting the bank capital, money, income, wealth, debt distributions and other econometric parameters.

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## Huge linear magnetoresistance in graphene on graphite

F.V. Kusmartsev†, T. Hewitt, M.B. Gaifullin, F. Al Mari and O.E. Kusmartseva

†Loughborough University

In recent years the magnetoresistance of inhomogeneous systems has been the subject of much research. Solin et al. discovered a new magnetoresistance effect (namely extraordinary magnetoresistance, EMR) in 2000 where magnetoresistance values of up to 1,000,000% at room temperature in an applied magnetic field of 5T were reported. The systems responsible for these large values were inhomogeneous but very well defined, containing exclusively non-magnetic constituents. The dependence of electrical resistivity on magnetic field, known as magneto-resistance was discovered by Lord Kelvin in 1857 in metals. The effect was very weak and fast saturating at large field but because of strong potential for enormous number of useful applications for magnetic sensors and in information storage technologies for century it was an intensive search for materials and structures having a strong magnetoresistance Here we found that graphene on graphite has huge linear magnetoresistance in very broad temperature range. The graphene on graphite and metal-oxide hybrid samples have been fabricated and characterized in varies magnetic fields at both room temperature and low temperature conditions. There is a two dimensional inhomogenous distribution of graphene islands on the graphite layer that has been confirmed via 2D band mapping by Raman scattering. The found linear positive magnetoresistance reaches 200% already in the magnetic field of 0.5 Tesla and at temperature of 35K. this is much larger than other inhomogeneous magneotresistive materials such as the silver chalcogenides. The samples structure has been verified by Raman spectroscopy. A theoretical model has been shown to fit well with the experimental data, with this model allowing for estimates of the samples materials parameters to be estimated. These have suggested mobility values of graphene of up to  $180,000 \text{ cm}^2/\text{Vs}$  at  $T = 35\text{K}$ . A theoretical model has been fitted to the results with very good agreement. The model was previously used to describe arbitrary two phase systems in a magnetic field. This model allows for an estimate of the material parameters in the sample. Such strong linear magnetoresistance arising even in small field, that have been never observed before, may be used in a wide variety of technologies that involves high sensitivity magnetic field sensors such as ultrahigh-density information storage, automotive control systems, industrial applications, medical devices and consumer electronics.

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## Feigenbaum graphs: a complex network perspective of chaos

L. Lacasa†, B. Luque

†Universidad Politécnica de Madrid

We expose a remarkable relation ship between nonlinear dynamical systems and complex networks by means of the horizontal visibility (HV) algorithm [1-3] that transforms time series into graphs. In low-dimensional dissipative systems chaotic motion develops out of regular motion in a small number of ways or routes, and amongst which the period-doubling bifurcation cascade or Feigenbaum scenario is perhaps the better known and most famous mechanism. This route to chaos appears an infinite number of times amongst the family of attractors spawned by unimodal maps within the so-called periodic windows that interrupt stretches of chaotic attractors. In the opposite direction, a route out of chaos accompanies each period-doubling cascade by a chaotic band-splitting cascade, and their shared bifurcation accumulation points form transitions between order and chaos that are known to possess universal properties. Low-dimensional maps have been extensively studied from a purely theoretical perspective, but systems with many degrees of freedom used to study diverse problems in physics, biology, chemistry, engineering, and social science, are known to display low-dimensional dynamics. The horizontal visibility (HV) algorithm converts the information stored in a time series into a network, setting the nature of the dynamical system into a different context that requires complex network tools to extract its properties. Relevant information can be obtained through this methodology, including the characterization of fractal behavior [3,4] or the discrimination between random and chaotic series [1, 2], and it finds increasing applications in separate fields, from geophysics, to finance or physiology. Here we offer a distinct view of the Feigenbaum scenario through the HV formalism, and provide a complete set of graphs, which we call Feigenbaum graphs, that encode the dynamics of all stationary trajectories of unimodal maps. We first characterize their topology via the order-of-visit and self-affinity properties of the maps. Additionally, a matching renormalization group (RG) procedure leads, via its flows, to or from network fixed-points to a comprehensive view of the entire family of attractors. Furthermore, the optimization of the entropy obtained from the degree distribution coincides with the RG fixed points and reproduces the essential features of the map's Lyapunov exponent independently of its sign.

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## Mechanical self-organization of growing actin networks

A. Laio†, L. Cardamone, A. de Simone, V. Torre

†SISSA, Trieste

Actin networks are remarkable biological materials essential for the structural stability of eukariotic cells and for many of their vital functions such as mitosis, cytokinesis, and motility [1]. They are formed by oriented and branched actin fibers and are able to produce compact and relatively stiff two-dimensional structures such as lamellipodia. Remarkably, actin networks are capable of growing at an approximately constant velocity, independently of the strength of the external force that opposes their growth [2]. This constant force-velocity response confers to motile cells the ability to adjust easily to sudden changes of the environment in which they are moving. Materials with such "smart" properties are a material scientist's dream, while they are manufactured routinely in the lamellipodium of a crawling cell. One wonders how evolution has been able to achieve such an optimal "design". We put forward the hypothesis that the growing actin network is a critically self-organized system, in which long range mechanical stresses arising from the interaction with the cellular membrane provide the selective pressure leading to organization. The actin network forms by a stochastic process, in which at every "time step" an actin "monomer" is added at the tip (barbed end) of a random filament, forming time-evolving truss structure. Filaments repel each other at short distance and, with a given probability, branch at a random point. This network is kept under compressive stress by a flexible membrane, directly interacting with the leading edge of the network. If the stress in a monomer becomes higher than a critical threshold, the monomer breaks and the filament depolymerizes from its pointed end. We show that this simple model reproduces several of the features observed in experiments: a growth velocity initially insensitive to the external force [2]; the capability of the network to organize its orientation [3]; a load-history-dependent growth velocity [4]. In addition, our model predicts that the power spectrum of the time series of the height of a growing lamellipodium decays with the inverse of the frequency ( $1/f$  spectrum). This behavior is a well known signature of self organized criticality [5]. This prediction is confirmed by novel accurate in vivo optical tweezer measurements on neuronal growth cones.

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## Dynamics and rheology of vesicle suspension in shear flow

A. Lamura<sup>†</sup>, G. Gompper

<sup>†</sup>IAC CNR, Bari, Italy

We consider a two-dimensional suspension of deflated vesicles under shear flow at finite temperature for different concentrations. The system is made of several vesicles, whose evolution is described via molecular dynamics. Vesicles are embedded in a hydrodynamic solvent modeled by using the multi-particle collision technique [1] and the system as a whole is placed in a rectangular channel with shearing walls. Both thermal and hydrodynamic effects are properly taken into account since their role is relevant when considering the dynamic behavior of vesicles in shear flow [2]. First we analyze the interaction of two vesicles in the tank-treading regime. When approaching, they slide over and around each other increasing their vertical separation after scattering with strong fluctuations of vesicle shape and inclination angle. This result is in a very good quantitative agreement with recent experimental results [3]. The dependence of the suspension intrinsic viscosity on the viscosity ratio between inner and outer fluids,  $\lambda$ , is found to be monotonic with  $\lambda$  at the fixed outer fluid viscosity for  $\lambda \geq 1$  in dilute and semi-dilute regimes. Viscosity measurements in the dilute regime are extremely important since not accessible by experiments [3]. We do not find any indication of non-monotonic behavior of viscosity when approaching the tank-treading to tumbling transition as theoretically predicted in the case of a single quasi-circular vesicle. Our model allows to study at the same time several vesicles with arbitrary shape. The behavior of concentrated suspensions is even more interesting. We simulate a system made of 12 vesicles and find a shear thinning behavior with the intrinsic viscosity decreasing with the shear rate over more than one decade. This happens for different values of  $\lambda \geq 1$  both in the tank-treading and in the tumbling regime. Moreover, there is a clear indication of the formation of vesicle-free boundary layers of finite thickness,  $\Delta$ , next to the system walls. Vesicles arrange themselves through the formation of lanes along which they stream. This will be shown by using animations.  $\Delta$  increases with the shear rate reaching saturation value. When increasing the system width, the values of  $\Delta$  grow as also observed in two-dimensional simulations of red blood cells in small vessels. Finally, it is found a non-linear dependence of  $\Delta$  on  $\lambda$  with the boundary layer decreasing at high values of the viscosity contrast attaining a maximum at the tank-treading to tumbling transition.

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## Limit theorem leading to Bose-Einstein, Boltzmann, Zipf Law and application to complexity science

T. Lapinski<sup>†</sup>, V. Kolokoltsov

<sup>†</sup>PhD student at University of Warwick

A new limit theorem which originated from work of Prof. Maslov will be presented. The theorem yields three limiting distribution: Zipf Law, Bose-Einstein statistics and Boltzmann distribution, depending on the asymptotic (i.e. in the thermodynamic limit) behavior of system.

The presented result emerge from four assumptions, formulated independently of application context (i.e. physics, economics, linguistics, etc.). These assumptions are (in bracket for the ideal gas in physics): 1. Sufficiently large number of elements in system (ex. particles); 2. Fixed parameter of the system (ex. total energy); 3. If two elements has the same value of parameter they are considered as indistinguishable. (ex. the particles having the same energy level are indistinguishable); 4. Number of elements having the same parameter value asymptotically depend on the system size (ex. number of energy level degenerations depend on the number of particles in system).

The three distributions appear in the limit when we consider different asymptotic dependencies: a) Zipf Law, if the number of energy level degenerations increases much faster than number of particles in the system; b) Bose-Einstein statistics is when the rate of increase is similar for both, degenerations and particles; c) Boltzmann distribution is when the number of levels degenerations increase much slower.

Out of these three, particularly interesting is the Zipf Law, first noticed in linguistics. What more, various power laws appear across many fields of science. For instance, in geology, Horton's law for river system or Gutenberg-Richter law for earthquakes as well as Pareto law in economics. Many empirically observed networks, such as WWW, citation or some social networks seem to be scale-free networks, characterized by power laws, too. In the Complexity Science the Self Organized Critical Systems (SOC) phenomena manifests as various power laws. The presented result might have 'identified' some common property of power law systems.

In addition, there are two mathematical results associated with main limit theorem. They are might be important from the practical point of view. The Central Limit theorem describes the possible deviation from our calculations. The theorem for large deviation theorem provides estimates for probability of rare events.

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## Survival of the Fattest, the Flattest, or the Fastest? The role of fluctuations in biological evolution

M. Lässig<sup>†</sup>

<sup>†</sup>University of Cologne

Natural selection is an important factor in biological evolution. This is expressed in the famous Darwinian principle of survival of the fittest. According to this principle, populations should evolve towards peaks of a fitness landscape. However, selection competes with stochastic evolutionary forces, which include mutations and reproductive fluctuations (genetic drift). Moreover, selection itself is often time-dependent and sometimes stochastic: fitness becomes a dynamic seascape rather than a static landscape. Stochastic forces drive populations away from fitness peaks - but where do they end up? In this talk, I discuss fluctuation principles of molecular evolution, which establish links to the statistical physics of entropy and entropy production [1,2]. These principles will be applied to the evolution of gene regulation [3], of RNA structures, and of the influenza virus.

These applications illustrate the diverse sources and biological consequences of fluctuations in molecular evolution: (1) We show that regulatory sequence elements can be described by evolutionary equilibrium in a fitness landscape. The equilibrium distribution is shaped by the fatness of the fitness peak, that is, the number of sequence states with near-optimal fitness. (2) Micro-RNA processing in plants is a complex function under significant mutational load. The processing mechanism and its evolution are influenced by the flatness of the fitness peak, that is, the susceptibility of the system to deleterious mutations. This is shown to generate genomic modularity, that is, a mechanism for independent evolution of different functions. (3) In the evolution of the human influenza virus, adaptation of the viral proteins to the host immune challenge is a process far from equilibrium. This dynamics is determined by the fastest strains, which produce beneficial mutations away from host immunity. We show that adaptation leads to interference with protein stability and other viral functions. Hence, adaptation and conservation are tightly coupled in this system, which affects the predictability of influenza evolution.

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## Persistence of incomplete mixing: A key to anomalous transport

T. Le Borgne<sup>†</sup>, M. Dentz, P. Davy, D. Bolster, J. Carrera, J.-R. de Dreuzy, O. Bour

<sup>†</sup>CNRS, Université de Rennes, France

Anomalous dispersion has been ubiquitously observed for transport in complex environments (Bouchaud and Georges, 1990). It can be traced back to the fact that projection of microscale transport dynamics in terms of stochastic or spatial averaging leads to non-Markovian behavior (Le Borgne et al., 2008). For Markovian systems, memory of previous states is destroyed by complete mixing on the support scale. Mean field approaches typically rely on support scales that are not well mixed, or in other words, that are not in local equilibrium. These approaches are characterized by non-Markovian dynamics which (i) can lead to anomalous dispersion and (ii) may not preserve the full concentration variability. The latter is of critical importance for the quantification of phenomena that depend non-linearly on local concentration such as reactive transport or population biology. Thus, the understanding of anomalous dispersion and the quantification of concentration variability in heterogeneous environments depend critically on the notion of incomplete mixing and the evolution of the local mixing scale compared to the support scale of the projected transport dynamics. For example, as the mixing scale reaches the support scale, projected transport is expected to become Markovian and represent the full spectrum of concentration values. For homogeneous environments, mixing is due to diffusion only and the local mixing scale grows like the square root of time. For heterogeneous environments, mixing dynamics are more complex and governed by the creation of concentration gradients due to heterogeneity and its destruction by microscale mixing. The dynamics of mixing have been studied for turbulent flows, chaotic flows and heterogeneous porous media flows in terms of global mixing states, as quantified by scalar dissipation rates, entropy measures, Lyapunov exponents and pair-dispersion (Ottino, 1990). While these approaches give invaluable insight into the mechanisms of mixing, they fail to provide a quantification of a local mixing scale, i.e. , the support scale for scalar gradients (Villermaux and Duplat, 2006). In this study, we introduce the concept of the mixing scale based on the distribution of pair separations. We analyze the evolution of the mixing scale due to the competition of shear action and diffusion and relate it to the anomalous evolution of the global dispersion scale.

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### Dynamics of neural network with depressing synapses: synchronization, global oscillations and chaotic behavior

K. Lee†, A.V. Goltsev, A.F. Ioffe, J.F.F. Mendes

†Department of Physics and I3N, University of Aveiro, Campus Santiago, Portugal

Nervous systems are complex networks and complex network structure strongly influences on dynamics of neural networks. We study dynamics of stochastic neural networks with complex network architecture consisting of two types of neurons and depressing excitatory synapses. Neurons are connected at random by synaptic links and form a directed random network. We consider a neuron as a generator of random spikes with a Poisson inter-spike intervals distribution (cells in visual cortex) [1]. Dependencies of firing rates of excitatory and inhibitory neurons versus inputs are described by sigmoid functions. Synaptic inputs from extrinsic afferents are also modeled as an independent Gaussian process with a mean rate in the range from 0 to 20 kHz. This is the only source of the noise in our model. All excitatory-to-excitatory connections (there are  $5 \times 10^6$  dynamical synapses in our network with 10000 neurons) are dynamical and display depressing transmission, i.e., every spike decreases by a certain value of the synaptic efficacy (short-term plasticity from Abbot et al (1997) [2], Tsodyks and Markram (1997) [3]). We carried out analytical calculations of neuronal dynamics and compared with numerical simulations of networks with 10000 neurons. We found that neural networks with complex network structure show various patterns of self-organization, phase transitions in neural activity driven by noise or stimulus, hysteresis phenomena, neuronal avalanches and a rich repertoire of dynamical phenomena. If synaptic noise is larger than a threshold value, global oscillations may appear. Global neuronal oscillations with suppressed excitatory synapses reveal amazing robustness against noise and depression of synapses in a broad range of parameters. Furthermore, if synaptic bombardment is strong enough, the oscillations show period-doubling, then triplication and so on. We also observed spindle-like oscillations which are similar to ones observed in brain. We found a transition to chaotic behaviour in a certain range of parameters characterizing depressing synapses. In order to prove emergence of chaos we used the 0-1 test proposed by Gottwald and Melbourne (2004) [4]. Emergence of chaos is very intriguing phenomenon which was observed in the brain activity (for example, in EEG). The chaotic behaviour takes place on a strange attractor while global oscillations evolve to a limit cycle. Thus, depressing excitatory synapses not only suppress neural activity but also on the contrary demonstrate a rich pattern of global oscillations, spindles and chaotic neural activities.

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### Return-to-origin probability in scale-free networks: crossover behaviors and the impact of shortcuts

D.-S. Lee†, S. Hwang, B. Kahng, D. Kim

†Inha University, Korea

Random walk has attracted much attention as a model for various dynamical processes on complex networks such as epidemic spreading in social networks, signal transmission in biological networks, and data packet transport in the Internet [1]. The time-evolution of the occupation probability of a random walker reflects the spatiotemporal pattern of information spreading in a given network. The return-to-origin (RTO) probability, the occupation probability at the starting node at a given time, is determined by the Laplacian matrix of the underlying network and characterizes the speed and range of information diffusion. Here we address how the network topology, local as well as global, affects the time-evolution of the RTO probability. Applying the renormalization-group approach to a Gaussian model, we obtain exactly the spectral dimension characterizing the asymptotic behavior of the RTO probability averaged over all possible starting nodes in the hierarchical scale-free networks [2] that may be either fractal or non-fractal depending on the weight of shortcuts. It is found that the spectral dimension is larger for the non-fractal case than for the fractal one. More importantly, the spectral dimension is shown to be constant under possible variations of network structure, such as the degree distribution, for the non-fractal case, whereas it varies for the fractal case [3]. We also introduce the link accessibility, the relative probability of a random walker to cross a link at a given time, and study its time-evolution. We find that the accessibility of a link depends not only on its distance to the starting node but also on the degree of the nodes at its ends. In scale-free networks, the links incident on the hub nodes find their accessibilities increase with time much slowly. Such impact of the local connectivity on the link accessibility gives rise to a crossover behavior in the RTO probability for a specific starting node, between a slow decay in the early-time regime and a fast decay in the late-time regime. We derive the crossover time and the power-law exponents characterizing the time decays in both time regimes, which are related to the spectral dimension and the degree exponent. We check the robustness of our results against the numerical simulations performed for several model and real-world networks. Finally, we discuss the implications of our findings.

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### Effects of intraday patterns on analysis of stock market index

J.W. Lee<sup>†</sup>, D.H. Kim, S.E. Maeng, Y.S. Bang and H.W. Choi

<sup>†</sup>Department of Physics, Inha University, South Korea

The statistical properties of the stock market have been extensively studied recently. The time series of the stock index in the world's stock market were accumulated every second. Therefore, the stock index is a good candidate for the time series analysis. Many statistical properties have been found for many stock markets [1-3]. The stock markets have many common features such as the long-range autocorrelation of absolute returns or volatilities, the long fat tail of the probability distribution functions of return and volatility. Stock markets are very complex system with many interacting agents. Intraday patterns of activity exist in stock markets. For example, the heaviest trading occurs around opening and closing of the market. In many statistical analyses, the intraday patterns are eliminated from the time series of the stock index. Some studies have removed intraday patterns or weekly patterns from stock index and stock volume analysis. We consider the effects of intraday patterns on a time series analysis of the Korean stock market index. We analyzed one-minute tick data from January 2, 2004, to October 30, 2009, on the Korea composite stock index (KOSPI). The total number of data was  $N = 508,984$ . We applied two eliminating method of the intraday patterns: elimination from the index itself and from the absolute returns. The autocorrelation function (ACF) of the absolute return showed a power law with an exponent that did not depend on the method used to eliminate the intraday patterns. The periodicity of the ACF was removed when we eliminated the intraday patterns from the absolute return. The period corresponds to the length of the trading day. Removing intraday patterns from the absolute return led to clear power-law decays of the autocorrelation functions. The probability distribution function of the returns showed a power law, but its exponents of the power law depended on the method of removing the intraday patterns. The power law exponents of the probability distribution function for the volatility intraday patterns have higher values than those of the return for the original time series and the return with intraday patterns eliminated from the index. Though the autocorrelations of the absolute return are insensitive to the type of the intraday pattern, the probability distribution function of the return depend on how the intraday patterns are eliminated. A similar behavior was observed for the probability distribution of the volatility. The probability distribution function of the return of the original time series is considered to be more suitable because the patterns of the daily activity themselves are part of the intrinsic market dynamics.

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### Wealth redistribution models on complex network and evolution of nation's wealth in world trade network

J.W. Lee<sup>†</sup>, S.E. Maeng, H.W. Choi, Y.S. Bang

<sup>†</sup>Department of Physics, Inha University, Korea

The wealth growth and redistribution of the wealth are very interesting topics in econophysics. The wealth of a nation is changed by the internal economic growth of a nation and by the international trade among countries. Trade between countries is one of their most important interactions and thus expected to affect crucially the wealth distribution over countries. Wealth distribution in real economy follows a power law in the wide range of the wealth and exponential function in the small wealth values. Bouchaud and Mezard proposed a stochastic growth model which has multiplicative stochastic growth and transactions between agents. Their model showed the power law of the wealth distribution. We suggested some asymmetric wealth exchange models and found that the asymmetric transaction is one of main origin of power law wealth distribution. We applied our model to the network structures among the agents. We also analyze data sets of world trade and the gross domestic products (GDP) of a total of 190 countries from 1950 to 2000. With these data sets, we construct the world-trade web (WTW) [1-3], where each node is a country and directed links connecting them represent money flow from a source node to a target one. The topology of this WTW tells us the patterns of the international trades - exports and imports. For example, the GDP of each country increases with its node degree - number of connected countries, and the network topology is found to be crucial in determining the wealth distribution through a modeled wealth dynamics. Link weights - trade volumes - and the node strength - sum of the link weights - seem to take log-normal distributions when averaged over countries and time. On the other hand, the link weight distribution appears to be slowly moving from a log-normal one towards a power-law one with time. In this work, we focus on detailed relationships between the GDP of each country and the international trade on the WTW. We explore the trade patterns of the 190 countries for a period of 50 years from various aspects and, in particular, measure their correlations with the wealth of each country. Our observation is that the effective money flow is asymmetric: more money is flowing into richer countries. Such asymmetric wealth exchange is expected to affect the wealth distribution, so we check this possibility in an agent-based model system.

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## Induced magnetic field transition in a strong anchoring Cholesteric-Nematic model

I. Lelidis<sup>†</sup>, G. Barbero, A. M. Scarfone

<sup>†</sup>Université de Picardie Jules Verne, Laboratoire de Physique des Systemes Complexes

The spontaneous twisted arrangement of the molecules in Cholesteric Liquid Crystals (CLC) is at the origin of their unique electro-optical and photonic properties [1,2,3] such as selective reflection flexoelectricity, photonic band edge lasing, etc, that are widely used in many device applications (displays, thermometers, optical storage, laser, etc). These properties may be tuned by modifying reversibly the pitch of the helix with an electric or magnetic field, the temperature, and light irradiation. The effect of an external bulk (magnetic or electric) field on helix unwinding of cholesteric liquid crystals has been described theoretically by de Gennes and independently by Meyer in the case of an infinite (unbounded) sample. The untwist of the cholesteric under magnetic field perpendicular to the helical axis has been observed by using the NMR technique and by a de electric field.

Other aspects of the model as the critical field dependence on the undisturbed cholesteric pitch and the dielectric anisotropy have been also experimentally verified. In the case of a bounded sample with rigid anchoring conditions Dreher predicted that the pitch of the CLC can change only stepwise by a magnetic field. The step unwinding of a CLC by a non uniform electric field has been optically observed using the Grandjean-Cano texture. Note that the same problem appears also in smectic ferroelectric liquid crystals.

In this work, we investigate the Cholesteric-Nematic transition induced by an external bulk field in a sample of finite thickness  $\ell$  [4]. The analysis is performed by considering a tilted magnetic field with respect to the easy direction imposed by rigid boundary conditions inducing planar orientation. In the case of parallel orientation between the magnetic field and of the easy direction, in the limit of  $\ell \rightarrow \infty$  we reobtain the results of de Gennes where the effective pitch of the Cholesteric is a continuous function of the magnetic field diverging at the critical field related to the cholesteric-nematic transition. For finite  $\ell$  we obtain a cascade of transitions, where the bulk expels a half-pitch at a time to avoid divergences in the elastic energy, in a similar manner as solids expel defects in the presence of strong deformation. In the ease of oblique orientation between the magnetic field and the easy direction, only the completely untwisted state depends on the tilt angle. Therefore, only the cholesteric-nematic transition depends on the tilt angle whilst all the other magnetic transition values are unchanged.

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## Nonextensive network statistics and hierarchically nested structure scales

M.P. Leubner<sup>†</sup>

<sup>†</sup>Institute for Astro- and Particle Physics, University of Innsbruck, Austria

Long-range interactions, as manifestation of nonextensivity in nature, appear ubiquitously as generic property of any physical structure on all scales where strong, weak, gravitational or electromagnetic interactions provide the source of non-local couplings and correlations. On the other hand, networks are discrete structures made up of a set of nodes joint by a set of links and can be characterized by their entropy. One approach to capture the global properties of systems composed by a large number of highly interconnected ensembles is to model them as graphs whose nodes represent the dynamical units, and whose links stand for the interactions between them. Nonextensive statistics along with network science, an emerging branch of graph theory, are increasingly recognized as potential interdisciplinary frameworks whenever systems are subject to non-local interactions and memory. Such settings are characterized by a non-local communication, evolving in a non-Euclidean fractal/multi-fractal space-time which makes their behavior nonextensive. Statistical mechanics is fundamentally based on the adoption of a specific entropy functional serving as shortcut for the vast, detailed information stored in a system and providing the connection to the macroscopic behavior. Consequently, after summarizing the theoretical foundations linking nonextensive statistics with complex networks from first principles, a novel subject under development and debate, we focus on the generalized nonextensive node and link entropy in a growing complex network, which can be formulated independently of any specific physical force allocation. Upon link entropy extremization, a particular hierarchical network tree structure of nested ensembles is obtained. Identifying nodes by physical mass scales and introducing corresponding length scales on statistical grounds, the growth of physical structures is governed by simple recurrence relations. As application we show that the system reflects precisely the global sequence of discrete cosmic inhomogeneity scales, as observed between Planck scale and large scale astrophysical systems. In summary, starting from a homogeneous distribution, the analysis demonstrates how the elements of systems subject to long-range interactions and memory tend to merge into discrete complex clusters, which defines also an arrow of time pointing in the direction of growing inhomogeneity. The universe can be viewed as a nonextensive self-organizing network, evolving into increasingly complex higher order structures scales.

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**The Levy lattice: a new kind of small world lattice and its implementation to the study of complex and disordered systems. The instance of spin-glasses**

L. Leuzzi<sup>†</sup>, G. Parisi, F. Ricci-Tersenghi, J.J. Ruiz-Lorenzo

<sup>†</sup>CNR-IPCF

Small word lattices interpolate among usual finite dimensional lattices and infinite-range lattices. In a nutshell, they are regular lattices of given linear dimension, in which there is a (small) finite probability that two points are connected independently on their distance. We introduce a network in which two nodes  $i$  and  $j$  are connected with a Levy walk-like probability depending from their distance as  $|i - j|^{-\rho}$ . Many models can be naturally defined and are interesting to investigate on Levy lattices: ferromagnet in random magnetic field, percolation, localizations, spin glasses, Kardar-Parisi-Zhang equations for smoothed interfaces, domain limited aggregation equations for branched interfaces. They can be studied at high effective dimensions, near the effective upper critical dimension, if it exists. In this talk, we will implement Levy lattice in the study of the spin-glass transition. In the one-dimensional spin-glass system on Levy lattice, varying the power  $\rho$  corresponds to change the dimension in short-range models. The spin-glass transition and the nature of the frozen phase at low temperature is studied in and out of the range of validity of the mean-field approximation in order to discriminate between different theories. Since each variable (Ising spin) interacts only with a finite number of others, the cost for simulating the model is drastically reduced with respect to the fully connected version, and larger sizes can be studied. In particular, the one dimensional nature of the Levy lattice allows to probe lengths extremely longer than in numerical simulations of Bravais lattices and reduce finite size effects strongly influencing the inference of the thermodynamic behavior from simulations of finite size systems. The numerical improvements are such that we are able to find much clearer static and dynamic indication that a spin-glass transition occurs both within and outside of the limits of validity of mean-field theory. We present the finite size scaling analysis of the critical behavior at equilibrium, the study of the overlap parameter probability distributions at low temperature and the investigation of off-equilibrium four-spin time correlation functions. We studied the system both with zero and non-zero external magnetic field. In the latter case finite size effects are very strong and existing finite size scaling techniques have been critically reviewed leading to a new method for the determination of the critical point.

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**Hyperbolic subdiffusive impedance**

K. Lewandowska<sup>†</sup>, T. Kosztolowicz

<sup>†</sup>Department of Radiological Informatics and Statistics, Medical University of Gdansk, Poland

We use two different hyperbolic subdiffusion equations with fractional time derivatives (the generalized Cattaneo equations) to study the transport process of electrolytes in media where subdiffusion occurs. These equations read

$$\tau \frac{\partial^2 C(x, t)}{\partial t^2} + \frac{\partial C(x, t)}{\partial t} = D_\alpha \frac{\partial^{1-\alpha}}{\partial t^{1-\alpha}} \frac{\partial^2 C(x, t)}{\partial x^2},$$

$$\frac{\partial C(x, t)}{\partial t} + \tau^\alpha \frac{\partial^{1+\alpha} C(x, t)}{\partial t^{1+\alpha}} = D_\alpha \frac{\partial^{1-\alpha}}{\partial t^{1-\alpha}} \frac{\partial^2 C(x, t)}{\partial x^2},$$

where  $\alpha$  is the subdiffusion parameter and  $D_\alpha$  is the subdiffusion coefficient; here we assume that the subdiffusive flux  $J$  is delayed in time by  $\tau$  with respect to the concentration gradient

$$J(x, t + \tau) = -D_\alpha \frac{\partial^{1-\alpha}}{\partial t^{1-\alpha}} \frac{\partial C(x, t)}{\partial x},$$

what provides the finite propagation velocity of the particles. We present a theoretical foundations for studies of subdiffusion impedance using a hyperbolic equations. In particular, we obtain the formulae of electrochemical subdiffusive impedance of a spatially limited sample in the limit of large and small pulsation of the electric field. The boundary conditions at the external wall of the sample are taken in the general form as a linear combination of the subdiffusive flux and the concentration of transported particles. We discuss the influence of the equation parameters (the subdiffusion parameter and the delay time) on the Nyquist impedance plots. For the limit of large pulsation the Nyquist plot is the linear function passing through the origin of coordinates with the slope angle depending on the subdiffusion parameter  $\alpha$ . For low pulsation the plots are dependent on the boundary conditions. There are a few methods for extracting the value of subdiffusion parameters from experimental data. The considerations presented in this contribution show that it is possible to determine the value of parameter of the system from the Nyquist plots obtained experimentally. We also briefly discuss the properties of the hyperbolic Cattaneo equations which we use and we show that the presence of the fractional time derivatives in the equations presented above is physically well motivated.

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## Reinforcement learning in complementarity game and population dynamics

W. Li†, J. Jost

†Max-Planck-Institute for Math. in the Sci.

We study a complementarity game as a systematic tool for the investigation of the interplay between individual optimization and population effects and for the comparison of different strategy and learning schemes. The game randomly pairs players from opposite populations. It is symmetric at the individual level, but has many equilibria that are more or less favorable to the members of the two populations. Which of these equilibria is then attained is decided by the dynamics at the population level. Players play repeatedly, but in each round with a new opponent. They can learn from their previous encounters and translate this into their actions in the present round on the basis of strategic schemes. The schemes can be quite simple, or very elaborate. We can then break the symmetry in the game and give the members of the two populations access to different strategy spaces. Typically, simpler strategy types have an advantage because they tend to go more quickly toward a favorable equilibrium which, once reached, the other population is forced to accept. Also, populations with bolder individuals that may not fare so well at the level of individual performance may obtain an advantage toward ones with more timid players. By checking the effects of parameters such as the generation length or the mutation rate, we are able to compare the relative contributions of individual learning and evolutionary adaptations. We find a new version of Roth-Erev (NRE) reinforcement learning within the framework of our complementarity game. In this NRE scheme, the probability of choosing a certain action  $k$  for any player  $n$  at time  $t$  is proportional to the accumulated rescaled reward by playing  $k$  during the time steps prior to  $t$ . The formula of the rescaled reward is a power law of payoff, with the optimal value of the power exponent being 1.5. NRE reinforcement learning outperforms the original Roth-Erev-, Bush-Mosteller-, and SoftMax reinforcement learning when all of them choose optimal parameters. NRE reinforcement learning also gains advantage over most evolutionary strategies, no matter whether information is processed directly from players' opponents or indirectly from their friends. However, most simpler and more flexible versions, without look-up tables, of the evolutionary strategies can have very significant advantage over NRE reinforcement learning. The reason is that the former converge more quickly than the time-costly reinforcement learning, and can settle more quickly at some values which are more favorable to themselves.

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## Finite time thermodynamics of a single-level quantum dot

K. Lindenberg†, M. Esposito, R. Kawai, C. Van den Broeck

†University of California, San Diego

Single quantum dot devices provide a theoretical testbed for complete characterization of efficiency of operation at maximum power. We do so for a number of single quantum dot systems using the methods of stochastic thermodynamics. These methods are particularly suited for the study of small entities subject to thermal fluctuations. First we identify the operational conditions for maximum power of a nanothermoelectric engine consisting of a single resonant quantum level embedded between two leads at different temperatures and chemical potentials. A single electron with sharply defined energy can move from one lead to the other through the quantum level. We assess the efficiency of the thermal motor function, in which electrons are pumped upward in chemical potential under the impetus of a downward temperature gradient. We next consider the optimal protocol to extract the maximum work from a single-level fermion system interacting with a single particle reservoir as the energy level is moved between an initial higher value and a final lower value in a finite time. It turns out that the optimal protocol displays discontinuous jumps at the initial and final times. The time evolution of the quantum state is described using a master equation formulation. While discontinuities in the protocol that minimizes the work on a device operating under given constraints appears surprising at first glance, it is easy to understand these discontinuities on a physical basis. On the other hand, these are discontinuities on the coarse-grained time scale of the master equation. At a more microscopic level they are merely rapid but continuous changes of the energy level. This calculation also yields the optimal protocol to raise the energy level with the expenditure of the least amount of work. Finally, we provide a full analysis of a thermal engine undergoing a Carnot cycle, the auxiliary system consisting of a single-level quantum dot that is switched between a hot and a cold reservoir. We run this engine through the four standard stages of a Carnot cycle: isothermal raising of the energy level at a cold temperature, adiabatic shifting of the level, isothermal lowering of the energy level at a hot temperature, adiabatic restoration of the energy to the starting value. We seek the protocol that maximizes the power output of this engine and calculate the associated efficiency. We point to extremely interesting universal features of the problem not only at the level of linear response but also in the nonlinear response regime.

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## Anomalous diffusion and morphogen gradients

K. Lindenberg<sup>†</sup>, S.B. Yuste, E. Abad

<sup>†</sup>University of California, San Diego

One view of morphogenesis relies on the formation of morphogen gradients that evolve as a result of morphogens created at a source point at a constant rate and that then move through the embryonic environment coupled to a degradation mechanism that removes them from the system. The morphogen concentration at the location of a cell then determines the evolution of that cell. The shape of the morphogen gradient as a function of location is thus an essential ingredient of embryonic evolution according to this point of view. Morphogen gradient nonmonotonicities can give rise to interesting patterning, where spatially separated cells develop in the same way. A monotonic morphogen gradient, on the other hand, leads to spatially distinct cell development. Morphogen gradient shapes depend on the balance between the morphogen motion and the space (and time) dependence of the degradation mechanism. Almost all current models of morphogen gradient formation assume that the motion of the morphogens is diffusive. However, in cellular crowded environments morphogen motion is more likely to be subdiffusive, requiring a complete reassessment of the balance between motion and degradation. We investigate this balance for morphogens moving subdiffusively as described by a continuous time random walk model, and with a variety of (time-independent) degradation mechanisms as a function of distance from the morphogen source. We derive a fractional reaction-subdiffusion equation that incorporates the subdiffusive motion and the degradation mechanism. In particular, we consider constant reactivity (i.e., space independent degradation rates). We also consider degradation mechanisms that decay as a function of distance from the morphogen source including piecewise constant reactivity, exponentially decaying reactivity, and reactivity decaying as an inverse power law. We find morphogen profiles that exhibit behavior not seen in the case of normal diffusion. They include the absence of steady states, nonmonotonic profiles, and transitions between monotonic and nonmonotonic profiles. The methodology that we have developed for the morphogen problem may be useful in other situations that involve creation of a species at a source, movement of this species through a complex medium, and simultaneous degradation. One example of such a scenario might arise in the context of point-source pollutants in ground water. Also, while a continuous time random walk approach is but one of the possible models giving rise to subdiffusion (e.g. another might be a fractional Brownian motion approach), we expect some of our results to extend to these models as well.

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## The entire N-particle kappa distribution

G. Livadiotis<sup>†</sup>, D.J. McComas

<sup>†</sup>Southwest Research Institute, USA

Space plasmas from the solar wind to planetary magnetospheres and the outer heliosphere are complex systems residing in nonequilibrium stationary states. These are described by the empirical kappa distribution, recently connected with the theoretical framework of nonextensive Statistical Mechanics, where the kappa index  $k$  that governs these distributions is related to the entropic index  $q = 1 + 1/k$  [1]. So far, however, the kappa distributions were constructed as one-particle distributions, while the attempt to derive the  $N$ -particle kappa distribution was facing major problems, such as the fact that the possible values of the kappa index are restricted by the degrees of freedom  $f$ , i.e.,  $k > f/2$  [2]. For example, the kinetic degrees for all the  $N$  particles,  $f = 3N$ , is a huge number and the same holds for the kappa index which is restricted to be  $k > 3N/2$ . Hence the kappa index is practically infinite corresponding to thermal equilibrium, and the entire,  $N$ -particle distribution function is always a Maxwellian. Obviously, this cannot be correct. Moreover, the kappa index depends on the number of the degrees of freedom [3], and this is opposed to the physical meaning of the kappa index as to identify each stationary state [4]. Indeed, a nonequilibrium system of particles can be residing in any stationary state, independently of the number of the degrees of freedom. For example, when we study the one-particle, two-particle, or,  $N$ -particle distributions, the kappa index is always different, but obviously this must not affect the specific stationary state that the system is residing in. These major problems can be solved at once by modifying accordingly the kappa index to remain invariant under varying the degrees of freedom, and thus, to give a suitable measure for characterizing the identity of stationary states [3]. Then we develop the entire,  $N$ -particle kappa distribution function that describes space and laboratory plasmas in terms of the nonequilibrium temperature and the modified kappa index [3]. Now that the connection is complete between non-extensive Statistical Mechanics and nonequilibrium plasmas, the full strength and capability of non-extensive statistical tools are available for the space and plasma physics community to analyze and understand the nonequilibrium properties of the various particle and energy observed distributions.

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## Thermostatistics of plasmas in nonequilibrium stationary states

G. Livadiotis<sup>†</sup>, D.J. McComas

<sup>†</sup>Southwest Research Institute, USA

Recent advances in plasma physics theory have established the connection between non-extensive Statistical Mechanics and nonequilibrium systems that reside in stationary states [1]. We have developed (i) a theoretical basis for the empirically derived kappa distribution which coincides with the escort q-Gaussian distribution and is commonly used to describe space and laboratory plasmas [1-3]; (ii) a set of proven tools for understanding and analyzing these distributions. This includes, among others, a consistent definition of the nonequilibrium temperature that coincides with the so-called physical temperature [1,3], and the physical meaning of the kappa index as a measure of how far the stationary state that the system resides in is from thermal equilibrium [3,4]. The classically only stationary state at equilibrium now degenerates to a spectrum-like arrangement of stationary states, in which the identity of each state is indicated by the Tsallis' entropic index q, or its equivalent, the kappa index  $k$  [3,4]. While thermal equilibrium corresponds to infinite kappa index, the opposite boundary state that corresponds to the smallest kappa index, specifies the furthest possible stationary state from equilibrium. We call this the "q-frozen state", because as a system approaches this state, it behaves analogously to when its temperature approaches absolute zero [3]. Therefore, the nonequilibrium temperature and kappa index are independent parameters characterizing nonequilibrium stationary systems and they have a key role in understanding the governing thermodynamical processes and transitions. We introduce a novel isothermal procedure that describes a system's transition into different stationary states by varying the kappa index, and show how the variation of temperature can be realized using an "iso-stationary" procedure, in which the system remains in a fixed stationary state. Finally, by expressing the entropy in terms of the kappa index, we show the detailed paths by which the transition of the system through stationary states evolves toward, or away from thermal equilibrium, following discrete dynamics along the kappa indices. This naturally exhibits certain stationary states out of equilibrium that are frequently observed in space and laboratory plasmas [3,5].

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## The fine structure of spectral properties for random correlation matrices: an application to financial markets

G. Livan<sup>†</sup>, S. Alfarano, E. Scalas

<sup>†</sup>Dipartimento di Fisica Nucleare e Teorica, Università degli Studi di Pavia, Italy

In this work some properties of empirically observed eigenvalue spectra of financial correlation matrices are presented and analyzed (see ref. [1]). Typically, such spectra display a complex structure made of a main bulk around 1 (for standardized data) plus a small number of larger eigenvalues "leaking out" of the bulk. Such a picture, in recent years, has been interpreted from the perspective of Random Matrix Theory (RMT). In particular, the main eigenvalue bulks of empirical spectra have been identified with the Marcenko-Pastur distribution, i.e. the mean spectral density (in the infinite matrix size limit) for random correlation matrices of the kind  $C = RR^T$ , all entries in  $R$  being Gaussian random numbers (see refs [2-3]). So, according to this framework, the main part of empirical spectra can be regarded as a consequence of the supposedly large amount of noise contained in financial data. On the other hand, the large eigenvalues out of the bulk are interpreted as the only ones yielding some information on the correlation structure of the financial data under study. In the work to be presented, the common knowledge described above is challenged and critically revised. In particular, we act on the empirical correlation matrices of two data sets with a filtering procedure which highlights some of the cluster structure they contain, and we analyze the consequences of such filtering on eigenvalue spectra. We show that empirically observed eigenvalue bulks emerge as superpositions of smaller structures, which in turn emerge as a consequence of cross-correlations between stocks. We interpret and corroborate these findings in terms of factor models. Such models aim at describing the time evolution of each item in a collection of assets in terms of a few "driving forces", or factors, which typically describe the impact that a market sector or the whole market itself have on a given asset. As one can easily verify, each factor yields, for the correlation matrix of such a model, one large eigenvalue plus  $K-1$  ( $K$  being the number of stocks influenced by the factor under consideration) smaller eigenvalues. These latter, when considering several different clusters of stocks driven by different factors, contribute to form the aforementioned superpositions of structures eventually forming large eigenvalue bulks. Moreover, by adopting a "mean field" approach on the correlation matrices of the datasets we consider, we are able to perform significant comparisons between empirically observed facts and predictions of RMT for correlation matrices with degenerate eigenvalues (see refs [4-5]). We find a good qualitative agreement between data and theory, showing the discrepancies between the two to be due to further unresolved correlation structures.

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## Electrostatic self-energy of a partially formed spherical shell in salt solution: the weak and strong coupling limits

A. Losdorfer Bozic<sup>†</sup>, A. Siber, R. Podgornik

<sup>†</sup>Department of Theoretical Physics, Jozef Stefan Institute, Ljubljana, Slovenia

We study the electrostatics of a partially formed, charged spherical shell in a salt solution. We investigate the problem in two different limits [1], the weak and strong coupling regimes [2]. The former is described by the Poisson-Boltzmann (PB) theory (and its linearised Debye-Hückel (DH) version) [2], based on the mean-field approximation, and is applicable for low ion valencies, small charge densities, high medium dielectric constant, or high temperatures. The limitations of the PB approximation become practically important only in highly charged systems (especially ones with high ion valencies), where ion-ion correlation effects begin to affect the electrostatic properties of the system. There, one has to give up the idea of a mean-field description altogether, taking recourse in a fundamentally different reformulation of the electrostatic theory, based on the concept of strong coupling [3]. Since our system in this regime includes both monovalent salt and polyvalent counterions, we describe it by using the strong coupling dressed counterions theory. Thus, the monovalent salt is treated in the DH limit, giving rise to a strong coupling description of dressed polyvalent counterions interacting via a screened DH pair potential. For each of the two cases described above we numerically calculate the dependence of the free energy on the parameters of the system, and derive analytical approximations to clarify its behaviour. From the results on the energetics of partially formed shells we then examine the stability of tethered (crystalline) and fluid shells towards rupture. We delineate different regimes of stability, where, for fluid shells, we also include the effects of the bending elasticity of the shells. By combining all of this we compare the results with previous studies (e.g. [4, 5]), which have almost exclusively treated a similar class of problems at the Debye-Hückel level. We then show how these results apply to the stability of both viruses and vesicles, drawing out phase diagrams for the rupture of small spherical capsids and the poration of charged liquid vesicles depending on the properties of the system. Additionally, our analysis gives some insight regarding the growth of charged shells, providing a characteristic radius of the charged shell without invoking the spontaneous curvature of the shell material.

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## Price spikes in electricity markets and stochastic resonating spiking

C. Lucheroni<sup>†</sup>

<sup>†</sup>University of Camerino

Hourly electricity price time series show peculiar patterns very different from those found in the more studied stock and bond price series, like spikes and antispikes that are fired occasionally but only in specific parts of the day. Standard top-down models try to capture electricity prices behaviour in path and distributional properties only, whereas hybrid models try to include in a few degrees of freedom physical and organizational features coming from the intrinsic network structure of power markets. In such markets agents coordinate strategically through partially competitive market institutions about electricity delivery through the physical power grid. Consequently, network congestions and other tight market conditions can show up in price series as strong nonlinearities and threshold effects, and often appear as price spikes. Effective models should be able to relate microeconomics with complex financial behaviour. In this talk, a class of nonlinear hybrid models for electricity prices will be presented, that use a Hopf critical point stochastic dynamics to generate price spikes, and that are able to include in their degrees of freedom typical and more general tight markets behaviour. A first model will be presented in the frame of SETARX (Self-Excited Threshold Autoregressive externally driven) and switching regime modelling of electricity time series, a second model will be presented in the frame of FitzHugh-Nagumo models. The major microeconomic feature included in the models is the presence of capacity constraints and power grid congestions, that can act on prices at varying levels of demand. These factors introduce a demand threshold in the price formation mechanism. Below the threshold prices react smoothly to demand variations, above the threshold prices can react in a non-smooth way, with spike-like patterns. In the SETARX frame, a three-regimes SETARX is chosen, where one AR sector is set in the usual stable regime, two other sectors are set respectively in unstable and metastable regimes in a specific sequence. These two not-stable regimes together allow of nonlinear deviations from the stable regime, sustaining spikes. TARX self-excitation avoids linking the regime thresholds to data different from the prices themselves, and demand data are unnecessary to calibration as far as a sinusoidal driver is embedded in the model. If desired, real world demand data can be used as an external driver process in substitution of the embedded driver. In the presentation, it will be discussed how to estimate the models on hourly electricity data.

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## Distance between states of an open quantum system

J. Luczka<sup>†</sup>, J. Dajka

<sup>†</sup>Institute of Physics, University of Silesia, Poland

One of the central issues of quantum engineering is to prepare a quantum system in a state having desired features. Doing this one is faced with many difficulties: unavoidable decoherence, imperfect implementation of quantum operations and many other sources of unpredictable mess. For a wide class of systems any two states of an open quantum system become more and more similar as they approach a unique equilibrium. Finally, after a long time one cannot distinguish them. There are various ways of introducing a notion of distance between two quantum states. Examples of such distance measures include the relative entropy, trace distance, Hilbert-Schmidt distance, Hellinger distance, Bures distance and closely related fidelity. They satisfy some properties and not all of them have a clear operational interpretation. However, they are related to the problem of distinguishing two systems or two states and therefore the distance measures in quantum information are referred to as distinguishability measures. One of the most popular quantifiers for the distinguishability of states is the trace distance [1]. The trace distance (also called the variational or Kolmogorov distance) is a genuine metric on quantum states. Any positive and trace-preserving map defined on the whole space of trace class operators on the Hilbert space is contractive, i.e. the distance cannot increase in time. It means that the distinguishability of any states can not increase above an initial value. Motivated by findings recently reported in Ref. [2], we study an open quantum system which is initially correlated with its environment. We consider a dephasing model of decoherence of a qubit interacting with a bosonic environment. If the qubit and its environment is initially prepared in an uncorrelated state, the reduced dynamics is completely positive and hence contractive. In consequence, the distance between two states tends to zero when the system approaches a steady-state. In the presence of initial correlations one is left with positive (but non completely positive) dynamics and the contractivity may fail. We show that under tailored regimes the contractivity fails for some initially entangled states. We consider two cases: (a) sub-ohmic and ohmic environments and (b) super-ohmic environments. For the case (a), the distance of states tends to zero in the long-time limit. Much more interesting is the super-ohmic case: the trace distance of different states can grow above its initial value even in the long time limit. In consequence, the distinguishability of the long-time limit states can increase above its initial value [3]. This feature seems to be favorable for the potential experimental verification.

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## Power law and scaling analysis for the Bucharest Stock Exchange

E.-O. Lungu<sup>†</sup>

<sup>†</sup>National Scientific Research Institute for Labour and Social Protection

During the last years, researchers spend a lot of time and effort in showing that different physical and social phenomena follow a power law distribution. Even though it is hard to prove the existence of a power law, or if you are sure you have one, to explain its significance, the studies published cover subjects as: World Wide Web, metabolic networks, internet router connections, intensity of wars, severity of terrorist attackers, number of academic papers author or co-authored, financial variables. In this article I investigate the distributions of two well-studied financial variables: return and volume. Even though researches of this type have been done on most of the financial markets; none has been conducted on the Bucharest Stock Market. I intend to fill this gap and test if we can empirically prove that the price and volume fluctuations of the shares listed at the Bucharest Stock Exchange (BSE), present power law distributions. Studies done on several markets, reported that returns and volumes follow power law distributions, with exponents 3 for returns and 3/2 for volumes. I collected the daily closing prices and volumes for all the shares listed at the Bucharest Stock Exchange (BSE), both domestic and foreign, from their first quotation day until 14.03.2010. The data were extracted from the Bucharest Stock Exchange internet site. In total there are 22 I tier shares 48 II tier shares, from different industrial sectors (financial, services, pharmaceuticals etc.), with a wide range of market capitalization and average daily volume. The length of the 69 time series varies from 455 to 3879 trading days. In order to evaluate the shares price and volume dynamics, I employ three basic measures: (1) the price variation, return  $R$ , (2) the magnitude of the price change, volatility  $V$ , (3) the volume variation  $Q$ . My findings are that power law isn't a good approximation for the return CDF, but might be a good one for the volume CDF. Furthermore, I investigate the dynamic behavior of BSE performing a sliding windows analysis with R/S analysis and DFA. First I estimated the global Hurst exponent for BET and RASDAC- $Q$  return series and observed that they are greater than 0.5, which means that the returns series presents positive correlations. To see if this behavior is constant in time we perform an R/S moving-window analysis. I computed the local scaling exponent for different window sizes  $l$  and sliding steps  $p$  and represented these variations, with respect to time. In the end, I discuss the statistical significance of the results.

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## Patterns in the occupational mobility networks of the higher education graduates

E.-O. Lungu<sup>†</sup>, A.M. Zamfir, C. Mocanu

<sup>†</sup>National Scientific Research Institute for Labour and Social Protection

Occupational mobility represents an important feature of the labour market. Although there is a rich literature on the rate of occupational mobility, there are important gaps in understanding patterns of movement among occupations (career trajectories). Such a challenge should rely on understanding individual histories of transitions among occupations as coherent sequences, if such patterns do exist. Y. Isoda (2009) tried to visualize the flow of occupational change using a job map created with Geographical Information System (GIS) on the base of occupational change data from Quarterly Labour Force Surveys (QLFS, 2001 - 2005, Britain). His idea was to treat occupational mobility the same as geographical mobility. In this paper we further explore this topic using the Flexible Professional in the Knowledge Society (REFLEX) data set and networks. We employ a network based approach to analyse the occupational mobility of the university graduates from 14 EU countries included in REFLEX (Italy, Spain, France, Austria, Germany, Netherlands, United Kingdom, Finland, Norway, Czech Republic, Switzerland, Portugal, Belgium and Estonia) in the first years after graduation (1999 - 2005). We use the survey data about their career mobility to build an empirical occupational mobility network (OMN) for each country, which covers all their job movements in the considered period. We construct the network as directed and weighted. The nodes are represented by the occupations (post coded at 3 digits according to ISCO-88) and the links are weighted with the number of persons switching from one occupation to another. We also take into account people that are changing their job within the same occupation (self-loops). On average our networks have a low density around 0.02 and a fixed number of nodes (190, because we considered all the occupations post coded at 3 digits according to ISCO-88). This representation of data permits us to visualize paths of mobility and to use the statistical techniques developed in the framework of weighted directed networks in order to extract a set of stylized facts that cast light on the patterns of occupational mobility: node degree/strength distribution, node's centrality, community structures and network motifs. We identify groups of occupations characterized by the transferability of skills among which there are higher flows of individuals. Further, we analyze patterns and direction of mobility and discuss our results in the view of job matching and human capital theories.

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## Network properties of bipartite ecological networks

S.E. Maeng<sup>†</sup>, J.W. Lee, H.W. Choi, Y.S. Bang

<sup>†</sup>Department of Physics, Inha University, Korea

The study of the network properties in ecological systems have been focused on the aspects of unweighted and unipartite networks. However, the bipartite network is a general type of the ecological network, for example, plant-pollinator networks, plant-seed disperser network, host-parasite networks, plant-herbivore networks etc. In ecological networks the nodes are species, and the links are their interactions. In plant-pollinator networks, pollinators visiting flowers comprise a link between them. In contrast to the complex food webs, plant-pollinator mutualistic networks embed not only the trophic relations among the interacting partners but also the complexities of coevolutionary effects. Such interactions give the two species beneficial relations. The probability distribution functions of the degrees for ecological networks show no unique functional form  $s$ . Observed probability distribution functions are power laws, truncated power laws, exponential functions, and uniform distributions[1-3]. The network properties of weighted complex networks have been studied for many social and economic networks. In our previous work, we reported the cumulative probability distribution function (CDF) of plant-pollinator networks. We observed the functional dependence of the CDF of the plant was different from that of the pollinators. However, we did not consider the network properties of the weighted complex networks in the previous works[4]. In this work we consider seven bipartite ecological networks consisting of plants and pollinators. We investigated the network properties such as the distribution functions of degree and strength, disparity, and the correlations for degrees and strengths. The frequencies of pollinator's visits to plant species in a certain season were recorded in data sets which were collected from published papers and web sites. We consider the visiting frequencies as the interaction strength between the plants and pollinators. The cumulative probability distribution function of the plants followed a stretched exponential function for both degree and strength. However, the CDF of the pollinators followed a power law. The average disparity followed a power law dependence on the degree of the networks. The disparity indicated that the strengths were distributed heterogeneously on the links of the ecological networks. We proposed a simple growing network model to understand the emerging properties of the bipartite ecological networks. We assigned the different connecting probability between plants and pollinators when they are coming into the network. We reproduced the observed power law and stretched exponential cumulative distribution functions for degree.

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## Ergodic properties of anomalous diffusion

M. Magdziarz<sup>†</sup>, A. Weron

<sup>†</sup>Wroclaw University of Technology, Poland

The foundation of statistical mechanics and the explanation of the success of its methods rest on the fact that the theoretical values of physical quantities (phase averages) may be compared with the results of experimental measurements (infinite time averages). L. Boltzmann in his papers on the kinetic theory of gases introduced a special hypothesis according to which leaving a system in free evolution and waiting for a sufficient long time, the system will pass through all the states consistent with its general conditions, namely with given value of the total energy. This hypothesis was later called the Boltzmann ergodic hypothesis.

Recently, ergodic properties of systems exhibiting anomalous behavior have attracted growing attention of researchers in various fields of physics and related sciences. In this talk we concentrate on the ergodic properties of some classes of anomalous diffusion processes [1,2]. We start with discussing the ergodic properties of stationary Gaussian processes. We show that ergodicity of the fractional Brownian motion and the fractional Langevin equation follows immediately from the very old results of Maruyama and Grenander. Next, using the so-called Lévy autocorrelation function, which is an analogue of autocorrelation for Lévy-driven processes, we derive a generalized Khinchin theorem. This result allows us to verify ergodic properties of Lévy-driven stochastic processes, in particular Lévy flights. Finally, we investigate the ergodic properties of two fractional Ornstein-Uhlenbeck processes, both exhibiting subdiffusive behavior. The first one is defined as the stationary solution of the fractional Fokker-Planck equation, the second one is derived via Lamperti transformation of the force-free subdiffusion process. We prove that the second process is mixing (and therefore ergodic), whereas the first process is known to display ergodicity breaking.

The presented results can be applied to verify ergodicity for a number of cases, including: transport of light in special optical materials (Levy glass); motion of mRNA molecules inside bacterial cytoplasm; Levy searching strategies. The obtained results allow to verify Boltzmann hypothesis for system, which can be successfully described in terms of Levy flights. Verification of the Boltzmann hypothesis for a given physical system is one of the fundamental problems of statistical physics.

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## Simple cubic lattice random-site percolation thresholds for complex neighborhoods

K. Malarz<sup>†</sup>, L. Kurzawski

<sup>†</sup>AGH University of Science and Technology, Krakow, Poland

Percolation is mathematical description of the geometrical phase transition. This allows for purely theoretical studies of all phenomena occurring near the critical point with computer only experiments or sometimes even analytically. The mixture of occupied and empty sites on given lattice may exhibit some features of real physical systems. Among typical applications of the percolation theory one may find material science, immunology or forest fires problems and studies of liquids moving in a porous media, etc. General speaking, the percolation theory deals with statistical properties of the clusters of occupied nodes (site percolation) or occupied edges (bond percolation) for a given graph, network or regular lattice. In a site percolation problem, the cluster is defined as a group of lattice vertices which belong to the same site neighborhoods. When each site is occupied with some probability  $p$  there is a critical probability of sites occupation  $\theta$  above which the cluster spanning through the whole system appears for the first time. This special probability is called percolation threshold  $\theta$  and it separates two phases (in a language of material science a conductor and an isolator). The value of percolation threshold  $\theta$  depends on kind of percolation (site/bond), lattice/graph/network topology and an assumed sites neighborhoods. In the simplest case only the nearest-neighbors constitute the neighborhoods (von Neumann's neighborhood) or the nearest-neighbors and second-nearest neighbors are considered (Moore's neighborhood). In this paper with computer simulation we evaluate the random-site simple cubic lattice percolation thresholds [1] for neighborhoods including the nearest-neighbors (NN), the second-nearest neighbors (2NN) and the third-nearest neighbors (3NN). Our estimations base on finite size scaling analysis of the percolation probability vs. site occupation probability plots. The Hoshen-Kopelman algorithm [2] has been applied for cluster labeling. The calculated thresholds  $\theta$  are 0.137(1), 0.142(1), 0.097(1), 0.199(1), 0.103(1), 0.245(1) for (NN + 2NN), (NN + 3NN), (NN + 2NN + 3NN), 2NN, (2NN + 3NN), 3NN neighborhoods, respectively. In contrast to the results obtained for a square lattice [3, 4, 5] the calculated percolation thresholds decrease monotonically with the site coordination number  $z$ , at least for inspected neighborhoods.

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## Applications of non-equilibrium relations to biophysics

K. Mallick†

†CEA Saclay

Biophysics provides numerous examples of systems far from equilibrium. For example, a significant part of the eukaryotic cellular traffic relies on 'motor' proteins that move along filaments similar in function to railway tracks or freeways (kinesins and dyneins move along tubulin filaments; myosins move along actin filaments). The filaments are periodic (of period 10nm approximatively) and have a fairly rigid structure; they are also polar: a given motor always moves in the same direction. These molecular motors appear in a variety of biological contexts: muscular contraction, cell division, cellular traffic, material transport along the axons of nerve cells... Molecular motors typically operate far from equilibrium, in a regime where the usual thermodynamical laws do not apply. Generically, molecular motors have been described theoretically either by continuous ratchet models or by models based on master equations on a discrete space. In recent years, a renewed interest has arisen in ratchet models in the context of non-equilibrium statistical physics. It has been realized that if the ratchet is kept far from equilibrium by coupling it to some external 'agent' (e.g., a chemical reaction) that continuously drives the system out of equilibrium, then, under certain conditions, work can be extracted. Again, there is no contradiction with thermodynamics here: the system is far from equilibrium and the ratchet plays simply the role of a transducer between the energy put in by the agent (e.g. chemical energy) and the mechanical work extracted. The analysis of the energetics of such devices far from equilibrium requires concepts that go beyond the classical laws of thermodynamics and this remains a very challenging and important open issue. During the last decade, a general organizing principle for non-equilibrium systems has emerged which is known under the name of fluctuation relations. These relations, hold for non-equilibrium steady states but arbitrarily far from equilibrium and can be seen as macroscopic consequences of the invariance under time reversal of the dynamics at the microscopic scale. An interesting ground to apply these concepts is the field of molecular motors. Indeed molecular motors are systems of molecular size which operate with a small number of molecules, and for these reasons undergo large thermal fluctuations. The fluctuation relations impose general constraints on their operation of these nanomachines and provide us a way to understand the non-equilibrium energetics of molecular motors.

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## Relaxation of quasi-stationary states in long range interacting systems and a classification of the range of pair interactions

B. Marcos†, A. Gabrielli, M. Joyce

†Laboratoire J.A. Dieudonne, University of Nice - Sophia Antipolis, France

Systems of particles interacting with long range interactions have the particularity to present generically "quasi-stationary states" (QSS), which are approximately time-independent out of equilibrium states [1]. Examples in nature of such objects are galaxies, globular clusters, vortices, etc. In this contribution, we explore the generalization of the formation of such "quasi-stationary states" and their relaxation from the much studied case of gravity to a generic pair interaction in three dimensions of the asymptotic form of the potential  $V(r) = 1/r^g$  with  $g > 0$ . We compute analytic estimations of the relaxation time calculating the rate of two body collisionality in a virialized system approximated as homogeneous, which are simply generalizations of the corresponding calculations for the case of gravity [2]. We show [3] that gravity is the marginal case between two different regimes of the relaxation: for  $g < 1$ , the collision integral is dominated by large impact parameters, and for  $g > 1$ , it is dominated by small impact parameters. In addition, the lifetime of QSS increases with the number of particles if  $g < 2$  (i.e. the force is not integrable) and decreases if  $g > 2$ . QSS can therefore only exist in the infinite system limit for a system of particles interacting with a non-integrable force. Using numerical simulations performed with an appropriate modification of the GADGET2 code, we confirm the formation of QSS for a range of the index  $g$  and show that the parametric dependence of their relaxation times are very well explained by our analytic results. We illustrate, using different cores in the potential, the important qualitative difference between the cases  $g < 2$  and  $g > 2$ : in the latter case the dynamics is very sensitive to such a core, and its size cannot be taken to zero. A corollary of our work gives a "dynamical" classification of interactions, which is in agreement with a mathematical study given in [4]. The dynamical properties of the system (i.e. the existence of QSS and their relaxation dynamics towards thermodynamical equilibrium) depend whether the force is integrable or not, which differs from the classification of interactions in function of the properties of the thermodynamical equilibrium, which is known to be given by the integrability of the potential.

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### Metabolic networks: a thermodynamical analysis based on the Von Neumann approach

E. Marinari<sup>†</sup>, A. De Martino, D. De Martino, M. Figliuzzi  
<sup>†</sup>Sapienza Università di Roma

We analyze metabolic networks. Thanks to a modelization suggested from Von Neumann we are able to use ideas from statistical mechanics, looking at "typical" solutions of the problem. We infer thermodynamical properties of the networks. We discuss examples where we have a good control over all details of the metabolic network. Let us give a few more details about the basic features of our analysis. Understanding the organization of reaction fluxes in cellular metabolism from the stoichiometry and the topology of the underlying biochemical network is a central issue in systems biology. In order to do that it is important to devise reasonable approximation schemes that rely on the stoichiometric data only, because full-scale kinetic approaches are computationally affordable only for small networks (e.g. red blood cells, about 50 reactions). Methods commonly employed are based on finding the stationary flux configurations that satisfy mass-balance conditions for metabolites, often coupling them to local optimization rules (e.g. maximization of biomass production) to reduce the size of the solution space to a single point. Such methods have been widely applied and have proven able to reproduce experimental findings for relatively simple organisms in specific conditions. We have defined and studied a constraint-based model of cellular metabolism where neither mass balance nor flux stationarity are postulated, and where the relevant flux configurations optimize the global growth of the system. In the case of *E. coli*, steady flux states are recovered as solutions, though mass-balance conditions are violated for some metabolites, implying a non-zero net production of the latter. Such solutions furthermore turn out to provide the correct statistics of fluxes for the bacterium *E. coli* in different environments and compare well with the available experimental evidence on individual fluxes. Conserved metabolic pools play a key role in determining growth rate and flux variability. We have also been able to connect phenomenological gene essentiality with 'frozen' fluxes (i.e. fluxes with smaller allowed variability) in *E. coli* metabolism. We also introduce modifications of this technique that allow to compute the free energy landscape of a given metabolic network flux state. We provide an algorithm to sample together fluxes and chemical potentials.

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### Dynamic approach to flowing liquids in confined systems

U. Marini Bettolo Marconi<sup>†</sup>S. Melchionna  
<sup>†</sup>University of Camerino and INFN-Perugia, Italy

We present the detailed analysis of the transport of spatially inhomogeneous fluids and the interplay between structural and dynamical properties varying on the atomic scale. The present treatment is based on different areas of liquid state theory, namely kinetic and density functional theory (DDFT) and their implementation as an effective numerical method via the Lattice Boltzmann approach. By combining the first two methods we are able to describe on equal footing their structural and hydrodynamic properties beyond the usual Smoluchowski overdamped description. The method focuses on the evolution of the singlet phase space distribution function involving an Enskog interaction kernel, of the type introduced many years ago by Ernst and van Beijeren, further simplified by separating the long lived hydrodynamic modes from the fast kinetic modes and by invoking a coarse graining ansatz. The DDFT dynamics is recovered under the application of a stochastic thermostat, physically representing an inert solvent. For particles evolving under Newtonian dynamics, the system displays propagating modes and the damping occurs only when gradients of the velocity and temperature fields are present. At first we show that the resulting system has the correct equilibrium behavior and non-trivial transport properties. Next, we discuss how the transport equation is a self-consistent dynamical equation amenable to numerical simulations in the framework of the Lattice Boltzmann method, as routinely employed to study fluid flows. The use of the LB method is particularly appealing regarding the applications to confined fluids, where standard hydrodynamics becomes questionable. In addition, the method is flexible and offers a powerful alternative to Molecular Dynamics simulations. The devised numerical method has been applied to simple fluids as well as to mixtures of particles having unequal sizes, repulsive short-range hard-sphere potentials and weak attractive long-range interactions. Oscillatory density profiles and modulations of the velocity field induced by confining walls result from the simulations of selected testcases. Within this framework, we also present numerical data concerning the mutual diffusion properties both in the case of a quiescent bulk fluid and shear flow inducing Taylor dispersion.

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## Nonlinear dynamics of two semiconductor lasers with time-delayed orthogonal mutual coupling

C. Masoller<sup>†</sup>, D. Sukow, A. Gavrielides, M. Sciamanna

<sup>†</sup>Universitat Politècnica de Catalunya

In many different configurations, the dynamics of a semiconductor laser is significantly perturbed by a time-delayed optical coupling or optical feedback. The naturally damped oscillatory transient dynamics (so-called relaxation oscillations) may become undamped and, depending on the coupling conditions, the laser diode can exhibit a rich variety of bifurcation scenarios possibly leading to optical chaos. An in-depth understanding of these instabilities leading to laser nonlinear dynamics is crucial in several applications that require a stable laser output, or by contrast, that target the development of a controllable periodic or chaotic light source. Moreover, the conclusions drawn from the laser system analysis can typically be extended to other systems in biology, chemistry, mechanics, economy where the combination of nonlinearity, mutual coupling and time-delay is ubiquitous. Even more richness in the laser dynamics can be observed in the presence of mode competition and noise. The mode competition dynamics is very sensitive to noise (spontaneous emission noise or external noise source), which not only may drive transitions between stable modes but also may lead to constructive dynamical effects such as coherence resonance and stochastic resonance. In this presentation we focus on a particular example of a two-mode laser in the presence of time-delayed coupling and noise. It comes from a recent work by Sukow et al. [Sukow PRE 2010] where two semiconductor lasers with natural TE lasing polarizations are mutually coupled through the delayed injection of their orthogonal TM polarizations. More specifically, the dominant TE mode of each laser is rotated by 90 degrees before being delayed coupled to the other laser. Such a configuration - hereby called orthogonal mutual coupling - allows for the observation of square-wave switching between TE and TM modes simultaneously in the two lasers, at a period related to the coupling time-delay and with duty cycles that are tunable as a function of the coupling strength and pump currents. We will present a theoretical study of the square-wave dynamics. As the coupling strength increases, the laser dynamics exhibits either a steady two-mode dynamics (mixed mode solution) or a steady one-mode dynamics (pure mode solution) but both fixed points bifurcate to pulsating dynamics that evolve into either transient or long term stable square wave dynamics. We will discuss the role played by various model parameters, and we will also present experimental observations that are in very good agreement with the numerical results.

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## Canonical and grand canonical ensembles : discrepancy in thermodynamical quantities

S.K. Mehdi<sup>†</sup>, S. Kessal, N. Daoudi

<sup>†</sup>LSNIRM Faculty of physics, USTHB. Algiers Algeria

For many particles quantum systems, calculating thermodynamics quantities in the canonical ensemble is a very hard task, while it's tractable in the grand canonical ensemble. The second ensemble is then used. The results are supposed to be the same in the thermodynamic limit [1]. We study the difference between the canonical ensemble and grand canonical ensemble for systems with a small number of energy levels. Because of the indistinguishability of identical particles (bosons, fermions), the canonical partition function of quantum systems is difficult to calculate when the number of particles is high. The grand canonical partition function is more convenient to calculate. Therefore the thermodynamic quantities of these systems are calculated in the grand canonical ensemble. However, some systems have a constant number of particles and their study is more appropriate for the canonical ensemble. We want to know if the canonical and the grand canonical ensembles are equivalent for the calculation of the average energy. We begin by applying this method to a system with two levels of energy  $\epsilon_1$  and  $\epsilon_2$  and contain  $N$  particles. We define the partition function  $Q_n$  for the canonical ensemble of the system :  $Q_n$  is the partition function for  $n$  energy levels,  $E_i$  is the energy of the system,  $\beta = 1/kT$  and  $N$  is the number of particles. We consider a system with a macroscopic number of bosonic atoms in equilibrium at the temperature  $T$  in two energy levels. For reasons of simplicity but without loss of generality, we set in this case the first level of energy equal to 0 and the second level equal to  $\epsilon$ . The partition function in the canonical ensemble for a system of two energy levels is then: The mean energy is given by the equation: For the grand canonical ensemble, we obtain : Where  $z$  is the fugacity of the system, satisfying the equation: We calculate then the relative discrepancy in the mean energy We calculate as a function of for different values of the number of particles . It takes a maximum value of 0.21. This relative discrepancy does not vanish, even when  $N$  goes to infinity. We have done the same calculations for bosonic systems with three, four and until ten equidistant energy levels.  $\Delta$  is not vanishing but the maximum value is diminishing with the energy level number: it is only 0.07 for ten level . We can conclude that the two ensembles (canonical and grand canonical) are not equivalent for system of a small number of energy levels. A nonequivalence of micro-canonical and canonical ensembles has been stated recently [2].

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## Multi-stage cascades

S. Melnik<sup>†</sup>, J.A. Ward, J.P. Gleeson, M.A. Porter

<sup>†</sup>University of Oxford

The global propagation of cascades starting from initially small localized events is a phenomenon that manifests itself in many areas of human life and society, and that has attracted a considerable amount of recent interest [1-4]. The examples of global cascades include the transmission of epidemics and the dissemination of fads and innovations in social networks, or the spread of computer viruses and cascading failures in infrastructure networks. Studying models of cascades allows us to gain insights into these processes, to determine conditions for the appearance of global cascades, and to open new prospects for applications ranging from the estimation of systemic risk in financial systems to finding the best strategies for viral marketing. To date researchers have mostly focused on single-stage cascade models wherein the propagation of a cascade is characterized by a single subpopulation of active agents [1-4]: the agents in these models are either active (i.e., form part of the cascade), or inactive. Active agents influence their inactive friends in order to persuade them to adopt a certain behaviour, idea or product. When the total amount of influence exceeds a certain specific to each inactive agent threshold, the inactive agent becomes active and starts influencing its own inactive friends, thereby promoting the spread of the cascade. Note that it is usually assumed that all active agents exhibit the same amount of influence on their peers. In reality however, regular users of, say, a certain product are often more enthusiastic in recommending it to their friends than just casual users. We present a model of multi-stage cascading dynamics in which agents can exert different amount of influence on their friends depending on the stage of their adoption (i.e., the level of their commitment to a certain product or idea) [5]. This model allows us to track the evolution of several subpopulations which can describe, for example, agents who are completely unaware of a certain idea, those who have somewhat accepted the idea, and those who have become keen adopters of that idea. As the agents transition through different stages of adoption, they can exert a different amount of influence on their peers. Therefore, the multi-stage model should provide a more adequate description of cascades running in real systems, and enable the observation of new dynamical behaviour. We investigate the dynamics of our multi-stage cascade model on networks and provide an analytical method for solving this model. This method, for example, gives excellent prediction for the expected sizes of cascades on configuration model networks.

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## Neuronal (bi)polarity driven by membrane growth

S. Menchon<sup>†</sup>, A. Gartner, P. Roman, C.G. Dotti

<sup>†</sup>Department of Molecular and Developmental Genetics, VIB and Center for Human Genetics, Leuven, Belgium

Polarization refers to the asymmetric changes in cellular organization in response to external or internal signals. The generation and maintenance of polarity are very important for many complex biological activities. Neurons are among the cell types with the most prominent asymmetry, by establishing dendritic vs. axonal domains which are different in function and morphology. The correct establishment of polarized domains in neurons enables their directional migration and polarized axon-dendrite formation and is thus one of the most critical steps in brain development. Neuronal polarization starts with the selection of the site from which the first neurite will grow before morphological changes are evident. It happens shortly after cell division and it is followed by the growth of a second neurite at the opposite pole [1]. The site from which the first neurite emerges is defined by the localized accumulation or activation of molecules with the capacity to directly or indirectly induce a local deformation in the plasma membrane. However, it is not known if a direct relationship exists between the formation of the second, opposite, neurite and the mechanisms involved in the formation of the first. In other words, it is not clear whether the formation of the second neurite also requires the same triggering mechanisms or if it is the consequence of a "passive" mechanism, derived from the first one. We tackled this issue through mathematical modeling, based on membrane traffic (exocytosis-endocytosis), and lateral diffusion. We based our model on those proposed by Altschuler et al. [2] and Turing [3]. Our model assumes an activator-inhibitor dynamics and diffusion-driven instabilities. In our model, the activator is identified with membrane proteins and the inhibitor is characterized by modulators of endocytosis. We study spontaneous symmetry breaking and how polarity domains are affected by membrane growth. With this approach, we demonstrated that a single pole of molecular asymmetry is sufficient to induce a second one at the opposite side, upon induction of growth from the first pole. To experimentally validate predictions of our model we compare results of our simulations with intensity distributions of sec8 on cell surfaces. Our work suggests that the occurrence of a single asymmetry in a round cell is sufficient to warrant morphological bipolarism.

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## Bootstrap percolation on complex networks

J.F. Mendes<sup>†</sup>, G. Baxter, S.N. Dorogovtsev, A.V. Goltsev

<sup>†</sup>University of Aveiro

Bootstrap percolation serves as a useful model to describe in detail or in analogy a growing list of complex phenomena, including neuronal activity, jamming and rigidity transitions and glassy dynamics, and magnetic systems [1]. Chalupa et al. introduced bootstrap percolation in a particular cellular automaton used to study some magnetic systems. The standard bootstrap percolation process on a lattice operates as follows: sites are either active or inactive. Each site is initially active with a given probability  $f$ . Sites become active if  $k$  nearest neighbors are active (with  $k = 2, 3, \dots$ ). In the final state of the process, the fraction  $S_a(f)$  of all sites are active. Remarkably, the function  $S_a(f)$  may be discontinuous. It may have a jump at a bootstrap percolation threshold  $f_c$ . I will show that when this process takes place on a network, this is not the only threshold in this system. Bootstrap percolation is closely related to another well known problem in graph theory, that of the  $k$ -core of random graphs [1,2]. The  $k$ -core of a graph is the maximal subgraph for which all vertices have at least  $k$  neighbors within the  $k$ -core. It is important to note the difference between the stationary state of bootstrap percolation and the  $k$ -core. Bootstrap percolation is an activation process which starts from a subset of seed vertices and spreads over a network according to the activation rules [1]. I will show the phase diagram for this process [3] with respect to two parameters:  $f$ , the fraction of vertices initially activated, and  $p$ , the fraction of undamaged vertices in the graph. Show the existence of two transitions: the giant active component appears continuously at a first threshold. There may also be a second, discontinuous, hybrid transition [1,2] at a higher threshold. Avalanches of activations increase in size as this second critical point is approached, finally diverging at this threshold. Describe the existence of a special critical point at which this second transition first appears. In networks with degree distributions whose second moment diverges (but whose first moment does not), we find a qualitatively different behavior. In this case the giant active component appears for any  $f > 0$  and  $p > 0$ , and the discontinuous transition is absent. This means that the giant active component is robust to damage, and also is very easily activated. I will also show how to formulate a generalized bootstrap process in which each vertex can have an arbitrary threshold.

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## Random motion in complex systems, ergodicity breaking, and single-particle trajectories

R. Metzler<sup>†</sup>, J.-H. Jeon, E. Barkai

<sup>†</sup>Technical University of Munich

In a wide variety of systems deviations from the standard laws of diffusion are observed. Such systems range from geophysical scales (tracer transport in subsurface aquifers) down to microscopic scales, for instance, the passive transport of biopolymers or artificial tracers in living cells. Related anomalous statistics are observed in the semi-classical domain, for instance, for the blinking statistics of quantum dots, or in the dynamics of subrecoil laser cooling. By the term anomalous diffusion we specifically understand the deviation from the linear growth in time of the mean squared displacement, in the form of a power-law with dynamic exponent  $\alpha$ . For  $0 < \alpha < 1$  we encounter subdiffusion. Anomalous diffusion is typically modelled mathematically in terms of continuous time random walk processes, fractional Brownian motion, generalised Langevin equations, or as diffusion on a fractal support. In particular on small scales single particle tracking has become a standard tool to probe the passive motion of test particles. To this end, either directly by video microscopy or indirectly by optical tweezers, the position time series of the tracer particle is recorded. Physical observables are then determined by calculation of the long-time average, instead of ensemble quantities. While in an ergodic system the long time average from individual trajectories produces information equivalent to the ensemble average of quantities such as the mean squared displacement, this is no longer true in systems exhibiting anomalous diffusion and ageing effects. After an introduction to stochastic models for anomalous diffusion a framework for single trajectory averages in complex systems will be presented. In particular it will be shown that in non-ergodic subdiffusive systems the time averaged mean squared displacement in the presence of a confining potential shows a turnover from a linear dependence on the lag time to a power-law increase, an a priori quite unexpected behaviour. The scatter between individual trajectories due to both small sample size and dynamic effects will be discussed. The theoretical results will be compared to data from living biological cells, and further tools to analyse single particle tracking data will be presented.

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### Universality classes of first-passage time distribution in confined media

B. Meyer<sup>†</sup>, C. Chevalier, O. Benichou, R. Voituriez

<sup>†</sup>Université Pierre et Marie Curie, France

The first-passage time (FPT) of a random walker to a given target site of a domain is a quantity which is involved in the quantification of the kinetics of various processes. A striking example is given by transport limited reactions, which often occur when a small number of reactants is involved. This is the case of many chemical reactions taking place in living cells: for example, the search for their specific DNA sequences by transcription factors. Determining the complete FPT distribution is essential to quantify the kinetics of reactions on all time scales. I present a method to calculate analytically this FPT distribution and discuss extensively its range of applicability. The derivation of the FPT probability distribution is presented for discrete random walks and continuous-time random walks, and it is shown that this method applies from small volumes to infinite systems, and for a wide range of time scales. More precisely, I investigate the following questions: i) How do FPT properties depend on the volume of the confining domain? ii) How do FPT properties depend on the initial positions of the reactants? I answer and show that the FPT distribution falls into universality classes indexed by the walk dimension  $d_w$  and the fractal dimension  $d_f$  of the medium, that show qualitatively different behaviours with respect to the transient versus recurrent nature of the transport. This leads to the concept of so-called "geometry-controlled diffusion" (this expression was chosen to reflect that considering a transport process from the point of view of its transient or recurrent nature is, in that context, more relevant than to keep a normal versus anomalous analysis frame). The results are applied to various models of disordered media, such as fractal networks, percolation clusters, random trap models, and continuous time random walks (CTRW). Such models have been frequently used to describe transport processes in real complex media, for example in the case of exciton trapping on percolation systems or in the case of anomalous motions induced by obstruction and binding in crowded environments such as biological cells (in particular recent experiments recently revealed a fractal structure of the chromatin). Thus, the results I introduce could, for example, help understand the crucial role of spatial organization of genes in transcription kinetics, such as the fact that successively activated genes are often localized in the very same nuclear regions.

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### Statistical equilibrium and the process of competition

M. Milakovic<sup>†</sup>, S. Alfarano

<sup>†</sup>University of Bamberg

We consider the empirical distribution of returns on assets (or profit rates) across firms, and find that it obeys an exponential power distribution that is remarkably stable over time. To explain the dynamics of firm profitability, we propose a diffusion process that has the exponential power distribution as its stationary density. Essentially, the diffusion decomposes the process of competition into the contemporaneous presence of idiosyncratic noise and a systematic tendency towards profit rate equalization. Finally, we estimate the diffusion process and find that firms do in fact appear to be in statistical equilibrium when competing with each other. The profit rate is perhaps the most crucial variable in the metabolism of capitalist economics, revolving around the reallocation of capital among competing productive uses. The basic economic idea of competition is that capital will seek out sectors with an above-average profitability, thereby leading to a negative feedback in sectoral profitability: additional activity in these sectors lowers prices and increases wages, thereby depressing the sectoral profit rate. In turn, capital will leave the sector, thus initiating another cycle of higher profitability for the surviving firms. Essentially, we argue here that this perpetual reallocation of capital can be modeled by a diffusion process that results in an exponential power distribution. According to the model, the shape parameter of the distribution measures the 'competitive pressure' in and across industries, and the estimation of the diffusion process suggests that it is a very reasonable description of actual firm destinies?

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## Two-band superconductors in Ginzburg-Landau theory

M.V. Milosevic†

†University of Antwerp, Belgium

Several years ago [1], Babaev and Speight predicted existence of the semi-Meissner state in two-band superconductors, the superconducting state with intercalating regions of high and low vortex density. The naive explanation for such a state was found in the possibility that two coupled Cooper-pair condensates in two-band superconductors may be of type-II and type-I respectively, and could therefore cause short-range repulsive and long-range attractive vortex-vortex interactions. Such a vortex state, with vortices exhibiting clear clustering and stripe formations was imaged experimentally by Moshchalkov et al. in 2009 [2] in single-crystal MgB<sub>2</sub>, and was coined as "type-1.5 superconductivity". To date, the theoretical Ginzburg-Landau (GL) approach of Babaev has been heavily criticized (e.g. by Kogan and Schmalian [3]) for its general noncompliance with the strict conditions of the GL derivation, while the experiments of Moshchalkov experienced reservations of the MgB<sub>2</sub> community (where MgB<sub>2</sub> is widely recognized as a purely type-II superconductor). As it seems, the estimated parameters of single-crystal MgB<sub>2</sub> vary extremely from experiment to experiment (with e.g. penetration depth differing even by an order of magnitude), which overshadows any conclusion on the type of superconductivity in this compound. On the other hand, to observe unusual vortex patterns, one must perform experiments at very low fields, below the first critical field, where the existence of intermediate state is not energetically favorable, and where pinning effects may play a large role. On top of all that, Brandt and Das [4] pointed out that inhomogeneous vortex distributions do not necessarily signify type-1.5 behavior, as such states were observed earlier even in single-band superconductors. In this talk, I present the analytic and numeric results revealing the true nature of the "type-1.5" vortex behavior and conditions for its appearance, based on the microscopic parameters of superconductivity, and the two-band Ginzburg-Landau theory derived to higher order terms [5]. I further discuss the unique vortex states predicted by our formalism, and their behavior as a function of field, temperature, and applied dc current. Finally, I show predictions for first two-band type-I superconducting compounds.

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## Statistical properties of Japanese business firm network and a probabilistic model based on creation, annihilation and coagulation

W. Miura†, H. Takayasu, M. Takayasu

†Tokyo Institute of Technology

The network that shed light on money flows of business transaction among firms is known to have a scale-free property, that is, the cumulative distribution of the link number follows a power law with an exponent about 1.3 [1]. In this study we aim to construct a probabilistic model of business firm network that is directly comparable with the real one. We analyze the huge data offered by the Japanese governmental Research Institute of Economy, Trade & Industry, which contains basic financial reports for almost all firms in Japan together with the information about business partners. From the data we find three basic properties relating to the growth of the business firm network. 1: The distribution of firm's age nearly follows an exponential function implying that firms die randomly. 2: The link number of a firm increases exponentially with the age on average. 3: New firms tend to have a link with an existing firm proportional to the partner's number of links, which is consistent with the basic assumption of the preferential attachment. Taking into account these properties we introduce a time evolution model of business firm network that includes bankruptcy, merger and new entry of firms as a stochastic event. Our numerical simulation shows that there exists a unique statistically steady state, that is, the number of firms follows a random walk, so it is non-stationary, however, other basic statistical properties converges to steady state statistics. For examples, the exponential distribution of the age and the exponential growth of the link numbers are fulfilled in fairly good agreement with the real one. The degree distribution also converges independent of the initial condition and it coincides nicely with that of real network. From the viewpoint of statistical physics the basic properties of this model is similar to the steady state realized in coagulation process with injection like the case of aerosols' power law size distribution. By applying the theoretical analysis for aerosols we can explain the power law behaviors of our model. As another important property of the business firm network, we pay attention to the asymmetry between money flow and goods flow [2]. This property is quantitatively described by the fact that the distributions of authorities and hubs are different, although authorities and hubs are conjugate quantities defined by the calculation of HITS [3] algorithm. We show that the mechanism of producing this asymmetry comes from higher order correlation of business firms with link-distance 2. Taking this mechanism into account, we can reproduce the asymmetric HITS properties by the simulation of the modified probabilistic model.

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## Motion and merging of Dirac points in two-dimensional crystals

G. Montambaux<sup>†</sup>, F. Piéchon, J.-N. Fuchs and M.-O. Goerbig

<sup>†</sup>Laboratoire de Physique des Solides, CNRS, Université Paris-Sud

We study under which general conditions a pair of Dirac points in the electronic spectrum of a two-dimensional crystal may merge into a single one. In the specific case of graphene, this situation is reached by a modification of one among the three hopping parameters between nearest neighbors. More generally, we consider the most general lattice Hamiltonian of a 2D crystal with two atoms or molecules per unit cell, and we show that Dirac points merge always at a symmetric point  $G/2$  of the reciprocal space, where  $G$  is a reciprocal lattice vector. The merging signals a topological transition between a semi-metallic phase and a band insulator. At the transition, the spectrum has the remarkable property to be linear in one direction and quadratic in the other direction (semi-Dirac point). We derive a universal Hamiltonian that describes the vicinity of the transition, characterized by three parameters, a mass, a velocity and a driving parameter  $\Delta$  whose values are related to the band parameters of any 2D crystal with time-reversal and inversion symmetries. This model describes continuously the coupling between valleys associated with the two Dirac points, when approaching the transition  $\Delta = 0$ . We calculate thermodynamic quantities. The topological aspect of the transition is revealed by the cancelation of Berry phases associated to the two Dirac points. The spectrum in a magnetic field  $B$  is related to the resolution of a Schrödinger equation in an asymmetric double well potential. We find a general scaling law for the energy levels which evolve continuously square-root to a linear dependence, with a new dependence at the transition (the semi-Dirac point). The spectrum in the vicinity of the topological transition is very well described by a semi-classical quantization rule. The universal Hamiltonian is applied to the description of graphene-like structures (two atoms per unit cell). It reproduces analytically the low field part of the Rammal-Hofstadter spectrum for the honeycomb lattice, when one hopping integral is varied. We discuss the existence of such a merging of Dirac points in graphene-like structures, in the organic salt (BEDT-TTF)<sub>2</sub>I<sub>3</sub> and in optical lattices of cold atoms, d-wave superconductors with a charge density wave. We also show that there is a similar transition in the square lattice (one atom per unit cell) with half magnetic flux per elementary plaquette.

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## Entanglement of two-level atoms and two photonic modes in Kerr media

A.M. Moosavi<sup>†</sup>, M.M. Golshan

<sup>†</sup>Physics department, Shiraz university, Shiraz, Iran

Entanglement is a quantum effect that is not possible in the classical picture [1]. Entanglement of quantum states has recently attracted much interest because of its applications in quantum information processing. In fact manipulation and control of entanglement play a major role in quantum information processing, quantum teleportation, quantum computing, etc. [2,3]. One approach to the realization of entanglement, for quantum information processing, is its use of atom-photon systems [4]. The especially combined system of atoms and photons is then described by the tensor product of atomic and photonic states. In the present work, the temporal behavior of entanglement between two-level atoms and the union of two-mode photons in a nonlinear cavity is studied. We use a generalization of the linear atom-photon interaction (the linear Jaynes-Cummings model) which includes a Kerr-type term and intensity-dependent couplings in a bimodal nonlinear cavity. Since entropy is a measure of disorder or equivalently, the missing information, it is well established that entanglement is quantified through entropy [1]. Therefore, the time evolution of the von Neumann entropy reflects the time evolution of the degree of entanglement. Assuming that the atom-photon system forms a pure ensemble, we calculate the von Neumann entropy as a function of time. We show that the von Neumann entropy as the degree of entanglement between the two-level atom and the union of two-mode photons undergoes periodic oscillations along with dips. Our results indicate that under specific conditions, the dips turn into a plateau which is important in applications. We also present the effect of the initial photon number, detuning, Kerr constant and intensity dependent coupling on the degree of entanglement. Maxima of the manner in which the period of oscillations, entanglement and the depth of dips vary with the variations of these agents are also discussed. To be more specific, we show that as the detuning and/or the Kerr-parameter and/or the initial photon number are increased, the von Neumann entropy (the degree of entanglement) is decreased. We also address the effect of most widely used intensity-dependent couplings on the evaluation of von Neumann entropy. Our study offers a new approach to control the entanglement.

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## Generalized model of polypeptide chain (GMPC) and the helix-coil transition in biopolymers

V. Morozov<sup>†</sup>, Y.Sh. Mamasakhlisov, A.V. Tsarukyan, S.V. Tonoyan, G.N. Hayrapetyan

<sup>†</sup>Yerevan State University

We summarize the results of our theoretical investigations of helix-coil transition in biological macromolecules such as polypeptides, polynucleotides etc. The Hamiltonian of the Generalized Model of Polypeptide Chain (GMPC) is introduced to describe the system in which the conformations are correlated over the scale. The Hamiltonian does not contain any parameter designed especially for helix-coil transition and uses pure molecular microscopic parameters: the energy of hydrogen bond formation, reduced partition function of repeated unit, the number of repeated units fixed by one hydrogen bond, the flexibility of chain, the energies of interaction between the repeated units and the solvent molecules. We evaluate the partition function and other thermodynamic quantities using transfer-matrix approach. The important problem of relation between GMPC and one-dimensional Potts model with many-particle interaction is solved. A number of problems were solved using this unified approach. For example, we describe the influence of the different types of biopolymer-solvent interaction on the thermodynamics of the system. In particular, the issues of interactions competing and non-competing for hydrogen bonds formation have been addressed [1]. We also investigate the comparative effects of the stacking interaction and one-strand flexibility. The influence of the sequence heterogeneity also has been addressed [2]. We have described the system cooperativity in terms of two-particle correlation function and correlation length. In particular, the transition temperature and interval have been estimated in dependence on the solvent-macromolecule interaction parameters. We obtained, that two type interaction of solvent brings to appear low temperature coil-helix transition, which we connect with cold denaturation. We jointly considered the stacking and hydrogen bond interactions, using two-scale GMPC [3]. Stacking increases stability and decreases cooperativity of melting on the background of hydrogen bonding. We also address the issue of the compact packing of macromolecule in the area of helix-coil transition. The compact packing brings to appear new correlation peak, originate from anticooperativity of long helical sequence formation. We also investigated the effect of inter-chain interactions [4], which results to different effects. In case of effective attraction, the cooperativity rises sharply. In case of effective inter-chain repulsion the system exhibits two-step melting with high correlation length and wide interval of transition. The obtained results are relevant for the conformational transitions in biological macromolecules in the context of molecular biophysics and structural biology.

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## Wave propagation and Landau-type damping in liquids and dense gases

D. Mostacci<sup>†</sup>, V. Molinari

<sup>†</sup>University of Bologna

As is well known, the behavior of real gases and of liquids differs from that of ideal gases mainly as a consequence of the effect of cohesive intermolecular forces. The detailed form of the interaction function can be investigated only through quantum mechanics and much work has been done in this direction. However the problem is very complex and many effects are involved; moreover the structure of the molecules is often not very well known. Therefore the existing results contain significant approximations and are applicable only to specific situations. This being the case, it becomes essential to resort to a phenomenological potential. This latter potentials are variously described with several possible models, among which a prominent place is held by the Lennard-Jones potential, together with its most usual approximation, the Sutherland potential. Other models have been proposed, however they introduce much complication while adding information for specific instances. The present work aims at gaining a glimpse of the role played by the cohesive intermolecular forces in the propagation of waves, and particularly how the dispersion relation is affected: this is effected analyzing the system in the framework of kinetic theory. Wave propagation can be investigated either deriving a wave equation from macroscopic equations, or directly from kinetic equations, which do not involve the derivation of a wave equation. This latter approach will show details otherwise hidden, being as they are strictly related to the form of the distribution function, and in particular a form of damping akin to that shown by Landau to exist in plasmas: these kinetic details are of course completely lost in the fluid-dynamic equations approach. To write the relevant kinetic equations, the Sutherland prescription is used to model the intermolecular potential and calculate the Vlasov self-consistent field for liquids and dense gases, and a Vlasov-type kinetic equation is written. This resulting equations is then investigated through integral transforms, leading to the relevant characteristic equation. This equation then yields both a dispersion relation (for the oscillatory part of the wave frequency) and an expression for a damping coefficient. It is worthwhile to observe that the damping coefficient is strongly dependent on the shape of the distribution function.

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## Quantum relativistic distribution function for bosons and fermions

D. Mostacci†, V. Molinari, F. Pizzio

†University of Bologna

Kinetic theory, understood in its widest sense, is a microscopic theory of the processes in many-body systems that are in equilibrium or non-equilibrium state; from kinetic theory the properties of the system considered can be derived from the microscopic interactions of the particles [1,2]. The theory can be used to analyze a great variety of phenomena, both in the framework of classical and of quantum and relativistic mechanics. In this work interest is focused on systems for which neither quantum nor relativistic effects can be neglected: such systems are present in many areas, e.g., of astrophysics and free electron lasing, as well as in certain approaches to thermonuclear fusion and so forth. Aim of this work is to obtain the relativistic equilibrium distribution function for Fermions and Bosons, with particular attention to the appropriate normalization factor, in the same vein as the relativistic Maxwellian distribution function is obtained, e.g., in [1,3]. The particle distribution function is the key to describing the physical status of the system and the equilibrium distribution function must be obtained as a starting point for the analysis of problems in non-equilibrium conditions. The starting point of the analysis are the Bose-Einstein and, respectively, Fermi-Dirac distributions expressed in terms of the relativistic hamiltonian  $H$ . The analysis is conducted separately for three regimes: Bosons above the critical temperature and weakly degenerate fermions (the degree of degeneracy being estimated from the Sommerfeld parameter); highly degenerate fermions at essentially vanishing temperature; highly degenerate fermions away from zero temperature. The number density is obtained integrating the distribution function: in calculating the integrals appropriate expansions are taken, and the solution is given as a series. Reversing the series yields the normalization constant as a function of number density, and hence the chemical potential is expressed as a function of number density as well. The complete distribution function at equilibrium is thus written for the different cases considered. If fugacity is considered, upon expansion in series of the number density it can be recognized how the first term coincides with the fugacity found in [3]: there however the higher order terms are not present, and as a consequence the difference between fermions and bosons could not be seen. The results presented here account correctly for relativistic effects in quantum distribution functions.

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## Hierarchical structures of periodic orbits and multifractal structure of fully developed turbulence

T. Motoike†, T. Arimitsu

†Faculty of Media Information Resources, Surugadai University, Japan

In the system of fully developed turbulence, the physical quantities, such as velocity derivatives, energy transfer rates and so on, reveal strong intermittency for smaller spatial scales and for shorter time scales. Within the formalism of Multifractal Probability Density Function Theory (MPDFT), the intermittent character of the fully developed turbulence is analyzed under the assumptions that 1) the intermittency arises from the invariance of Navier-Stokes equation under the scale transformation with an arbitrary exponent  $\alpha$ , and that 2) the distribution  $P(\alpha)$  of the singularity indices  $\alpha$  can be given by a Tsallis-type distribution function. It is assumed that the hierarchical structure similar to the  $\delta$ -scale cantor set can be extracted from the turbulent flow through the observation conducted by varying the distance of two measurement points as  $\delta^{-k}$ . Since observers can freely choose the value of  $\delta$ , the choice of  $\delta$  should not affect the physical quantities characterizing the turbulent flow. The A&A model within the framework of MPDFT tells us that this requirement is satisfied if the entropic exponent  $q$  of  $P(\alpha)$  is given by the scaling relation  $(\ln 2)/(1-q) \ln \delta = (1/\alpha_- - 1/\alpha_+)$ . Here  $\alpha_{\pm}$  are zeros of the multifractal spectrum. We investigate the relationship between the structure of  $\delta$ -scale cantor set and the scaling relation deduced from the requirement of MPDFT by making use of  $\delta^{\infty}$  periodic orbits of one dimensional discrete dynamical system.  $\delta^{\infty}$  periodic orbits appear in a  $\delta$  periodic window in which there are infinite  $\delta^k$  periodic windows. Investigating the structure of  $\delta^K$  ( $K \gg 1$ ) superstable periodic orbits, we find out that the hierarchical structure of  $\delta^{\infty}$  periodic orbits is equivalent to the  $\delta$ -scale cantor set, and that the expansion rate of the distance between the  $\delta^{\infty}$  periodic orbit and the neighboring orbit at time  $t = \delta^k$  ( $k = 1, 2, \dots$ ) increases in power-law manner with respect to time  $t$ , of which the exponent is given by the similar scaling relation to the one for MPDFT with  $\delta = 2$ . Further investigating the behavior of orbits in the neighborhood of the  $\delta^{\infty}$  periodic orbit, we have determined the new hierarchical structure which reveals power-law divergence with the exponent given by the scaling relation proposed by MPDFT. In this study, we will show the details of this new hierarchical structure and the relationship between this structure and the intermittency of the fully developed turbulence.

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## Exact solution of Boolean dynamics with random connections, thermal noise and strong memory effects.

A. Mozeika<sup>†</sup>, D. Saad

<sup>†</sup>Aston University, Birmingham, UK

In 1969 Stuart Kauffman [1] introduced a very simple dynamical model of biological gene-regulatory networks. The state of each gene was modeled by a binary ON/OFF variable, interacting with other genes via a coupling Boolean function which determines the state of a gene at the next time-step. There are  $N$  such genes (sites) in the network and each gene is influenced by exactly  $k$  other genes from the same network. In Kauffman's approach, the network topology and Boolean functions are random but fixed for all subsequent time-steps. This model, also known as Random Boolean network (RBN), belongs to a larger class of Boolean networks (BN) which found its use in the modeling of genetic networks, neural networks, social networks and in many other branches of science [2]. For over two decades the annealed approximation [3] has proved to be a valuable tool in the analysis of large scale Boolean networks as it allows one to predict the time evolution of network activity (proportion ON/OFF states) and Hamming distance (the difference between the states of two networks of identical topology) order parameters. The main assumption in this method is to ignore the fact that both Boolean functions and random connections in a BN are quenched variables and enables one to reshuffle them at each time-step thereby ignoring the correlations. Remarkably, the annealed approximation provides accurate activity and Hamming distance results for many BN models with quenched disorder, but the broad validity of the annealed approximation in the models of this type has remained an open problem. Furthermore, there are models that have very strong memory effects in specific regimes, where the annealed approximation is no longer valid. We study dynamics of BN model with quenched disorder and thermal noise using the generating functional method of statistical physics. Our analysis is very general and covers a large class of recurrent BNs and related models. We show that results for the Hamming distance and network activity obtained via the quenched and annealed approaches, for the models in this class, are identical. In addition, stationary solutions of Hamming distance and two-time autocorrelation function (inaccessible via the annealed approximation) coincide, giving insight into the uniform mapping of states within the basin of attraction onto the stationary states. In the presence of noise, we show that above some noise level the system is always ergodic and explore the possibility of spin-glass phase below this level. Finally, we show that our theory can be used to study the dynamics of models with strong memory effects.

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## Downhill protein folding peculiarities of living in a world without barriers

V. Muñoz<sup>†</sup>

<sup>†</sup>CIB, CSIC, Madrid, Spain

Protein folding has been commonly characterized as a simple all or none process in which protein molecules interconvert between two states (Unfolded and Native) that are separated by a high free energy barrier. Recent progress in the field has led to the realization that folding barriers are much smaller than previously thought as well as to the identification of examples of folding over marginal or no barriers (downhill folding). The discovery of downhill folding opens a new avenue of experimental research in which it becomes feasible to characterize the whole unfolding process with atomic resolution, to measure conformational dynamics during folding, and to resolve entire folding trajectories on single molecules. In parallel, experimental studies on downhill protein folding challenge many of the conventional assumptions used for the analysis of protein folding experiments. In this talk I will review some of the peculiarities of downhill folding and present new experimental progress aimed towards defeating the three resolution limits in protein folding: structural (ref. 1,2), temporal (ref. 3 and 4), and statistical (ref. 5). Most particularly, for this presentation I will focus on the experimental study of downhill protein folding at atomic resolution, and how these studies permit to make a direct connection between deceptively simple macroscopic observations (bulk equilibrium unfolding curves) and the inherent microscopic complexity of protein folding reactions. By now we have completed the atom-by-atom analysis of downhill folding by nuclear magnetic resonance (NMR) on two proteins, one is the downhill paradigm BBL (ref. 2 and 4). The second protein is GPW, a midsize single gene product protein domain from bacteriophage  $\phi$  with a 3D structure consisting of a two stranded, antiparallel  $\beta$ -sheet packed against two  $\beta$ -helices to form a single hydrophobic core. The size and  $\beta$ - $\beta$  topology of this domain suggest slow two-state-like folding over a large barrier. However, nanosecond temperature-jump experiments show that this protein folds in the microsecond time scale, expanding the realm of ultrafast folding. Both, thermodynamic and kinetic experiments indicate that the maximal folding barrier height for GPW is about 1RT, indicating that this protein is an incipient downhill folder, which makes it an ideal candidate for the high resolution NMR analysis.

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### Statistical analysis for air quality assessment. A case study from Iasi region - Romania

G. Murariu†, C. Daniel, L. Georgescu, F. Mingireanu, M.I. Razvan

†University "Dunarea de Jos", Galati, Romania

An important statistical development of the last 30 years has been the advance in regression analysis provided by generalized linear models (GLMs) and generalized additive models (GAMs). In this way, it should be developed multivariate statistical modelling based on generalized linear models. Generally speaking, the multivariate analysis is concerned with the interrelationships among several variables. The data may be metrical, categorical, or a mixture of the two. Multivariate data may be, first, summarized by looking at the pair-wise associations. Beyond that, the different methods available are designed to explore and elucidate different features of the data. This case study reports a systematic study basing on different multivariate statistical procedure applied for evaluation of air quality using an extensive data set obtained during a series of fly- monitoring missions of Iasi area from Romania. The studied area was chosen because of extremely strong growth in traffic in the city of Iasi as a result of strong economic growth. In this aim were used a series of UAV devices which allowed to cover an significant area. The huge data base contains over 300.000 records and were measured a series of 30 physical and chemical parameters. The complex data matrix was treated with different multivariate techniques such as cluster analysis, factor analysis/principal component analysis (FA/PCA) and discriminant analysis (DA). The huge volume of computations were covered by using StatSoft STATISTICA v.10 tool and, for a series of specific problems were developed a series of MATLAB procedures set. For numerical evaluations was used in the same time FORTRAN programs. In this paper we present some preliminary results are extremely interesting. This is quite promising, especially because they are phenomena that have a unique character, such as heat island phenomenon. The present study proposes a linear model of addiction based on physical environmental parameters. The model degree model was considered successively increased. Thus the second rank model, for this preliminary stage, the results are good enough and are a promising first step for future research

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### Statistical approaching for an unitary evaluation of water quality A case study for three lagoon systems from Italy and Romania

G. Murariu†, A. Caldaru, L. Georgescu, C. Topa

†University "Dunarea de Jos", Galati, Romania

A key step of European Water Framework Directive (WFD) implementation is the ecological status classification and the achievement of 'good water statuses for all waters, by 2015. In this way it should be considered a coherent study on the correlations between the physics and chemistry parameters and the biotic components in order to succeed in reaching a complete picture. The study presents a systematic approach performed on environmental parameters measured in the three areas. For this purpose, were considered and measured a set of 12 chemical and physical parameters were also evaluated at the three biotic zones - Leahova and Sinoe lagoon system from Romania and respectively Cesine from Italy. Because the study areas correspond approximately to the same type, the biotic proved to be common in all three lagoons. The statistical analysis was performed based on regressive approaching methods used in determining the dependencies type between environmental parameters and concentration of micro-organisms. The dimensionality of the well thought-out representations was progressive increased. Only for the bi-dimensional representations, have been analyzed a total of 91 sets of such models. For the three-dimensional representations the number of considered models was about 273. Based on these results, we could identify a set of parameters that best describe the influence of environmental factors on the biological component. The obtained results for all the three studied areas was similar. In the last part is addressed, namely the possibility of coherent use as real indicators of biotic component. In this respect, basing on models above obtained and considering the biotic magnitudes as parameters, we explored the possibility of predicting on environmental parameters. The main aim was to unify in the same class the physics, chemical and biological components for an unitary approaching. The obtained results are attained for second rank order polynomial model approaching found out using the StatSoft STATISTICA v10 software with specialized packages tool. In addition, for such advance, was developed a specific MATLAB procedures set. In this way, the obtained results for the three study areas attended in this manner are encouraging, although they are just getting in a starting point.

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## Economic policies with endogenous innovation and Keynesian demand management

M. Napoletano<sup>†</sup>, G. Dosi, G. Fagiolo

<sup>†</sup>OFCE

The current global crisis has not only strikingly shown the importance of banking and financial markets for the dynamics of real economies, but it has also revealed to be a "natural experiment" for economic analysis, showing the inadequacy of the predominant theoretical frameworks. The basic assumptions of mainstream models (e.g. DSGE models, cf. Woodford, 2003; Gal <sup>?</sup>i and Gertler, 2007), e.g. rational expectations, optimizing, representative agents etc. are to a large extent responsible for the failure to forecast the crisis and seem also unable to propose a therapy to put back economies on a steady growth path (Colander et al., 2008; Kirman, 2010). In fact, the crisis sets a tall challenge for alternative, evolutionary theories linking micro-behavior and aggregate dynamics. In this work, we develop an evolutionary, agent-based model to try to fill the theoretical vacuum present nowadays in macroeconomics. The model addresses three major, interrelated, questions. First, it explores the processes by which technological change affects macro variables such as unemployment, output fluctuations and average growth rates. Besides this "Schumpeterian" question, we also ask how such endogenous, firm-specific changes in the supply side of the economy interact with demand conditions. This is a basic "Keynesian" issue. Finally, we explore the possible existence of long-term effects of demand variations. Is the long-term growth just driven by changes in the technology, or does aggregate demand affect future dynamics? Are there multiple growth paths whose selection depends on demand and institutional conditions? To do so, we refine and expand the model contained in Dosi et al. (2010)<sup>1</sup>, which we use also as a sort of "policy laboratory" where both business-cycle and growth effects of different public policies can be evaluated under different institutional scenarios. In this respect, the model allows to experiment with an ensemble of policies, related to the supply side of the economy (e.g. technology) as well as to the demand side macro-management (e.g. fiscal and monetary policies). Before carrying out policy analysis exercises, we empirically validate the model showing that it is able to replicate a wide spectrum of macroeconomic and microeconomic stylized facts. Our policy-simulation exercises show a strong complementarity between Schumpeterian technology policies and Keynesian fiscal policies. Furthermore, we show that monetary policy can have strong effects on growth as well. In particular, high interest rates not only exacerbate volatility and unemployment rates, they are also able to worsen the long-run growth prospects of the economy.

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## Supersymmetry in models of strongly correlated electrons

J. Naudts<sup>†</sup>, T. Verhulst

<sup>†</sup>Universiteit Antwerpen

We study [1] a supersymmetric model of electrons on a lattice, interacting with phonons. The spectrum of the Hamiltonian can be calculated explicitly. As a consequence, the model can be simulated in the canonical ensemble using the Monte-Carlo technique. Simulations of the model on a cubic lattice with 1,000 sites at half band filling give evidence for a phase transition of the gas-fluid type. Earlier attempts (see [2] and references quoted there) to find supersymmetry in lattice models of strongly correlated electrons made it clear that supersymmetry would be rather exceptional. However, by adding bosonic degrees of freedom to the model the supersymmetry can be realised in a more natural manner. We start from the original Witten model [3] and note that the Jaynes-Cummings model [4] is a special case of it when at resonance. In particular, it is supersymmetric. Next we add to the free electron model harmonic oscillators, as many as there are electronic degrees of freedom, and let them interact in a way similar to that of the Jaynes-Cummings model. The resulting model is still supersymmetric involving an algebra with 8 generators. Only electrons with equal or opposite wavevector do interact. This results in pair forming. However, as a consequence of Pauli's exclusion principle, all electrons are correlated [5] at low enough energies. When heating up a transition occurs from a phase with electrons staying close to the Fermi surface to a gaseous phase containing high velocity electrons. A breaking of the supersymmetry is required to induce coherence of the liquid phase. To this purpose an external field term is added which breaks the spin inversion symmetry of the electron pairs. The induced currents carry a spin rather than a charge.

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## The exponential family in statistical physics and beyond

J. Naudts†

†Universiteit Antwerpen

The exponential family of statistical models is an important notion in statistics. In statistical physics most models are based on the Boltzmann-Gibbs probability distribution and therefore do belong to the exponential family. Recently, generalisations of the notion of an exponential family have been introduced [1-3]. They originate from research in non-extensive statistical physics but have been introduced almost simultaneously in statistics. In a short time, the generalised concept has found many applications, for instance in statistical physics, in statistics, in machine learning. A related concept which also originated from non-extensive statistical physics is the notion of escort probability distributions. A special subclass is that of the  $q$ -exponential families [3]. The  $q$ -Gaussian models belong to this family. So do the Tsallis distributions. Also the configurational probability distribution of any mono-atomic gas as a function of the microcanonical energy belongs to this family [4]. In this case the  $q$ -parameter is given by  $2/(1-q) = n-2$ , where  $n$  is the number of degrees of freedom of the gas. An important feature of the models in the  $q$ -exponential family is the dually affine geometry of Amari. The dual structure is based on Legendre transforms and is related to the well-known coordinate transformations of thermodynamics. The insight obtained from the geometric approach is important for many applications. Orthogonal projection onto parametrised manifolds is a powerful tool for optimal approximations. An example of physics is found in the work of Ohara who has shown that the time evolution of the porous medium equation follows geodesic orbits. In my talk I will give a review of the present state of the research in this domain. In addition, I will discuss some attempts to formulate the notion of an exponential family in the context of abstract information theory without relying on the use of probability theory. This line of research was started in [5]. The main quantity in this approach is an entropy function which is a measure for the negative amount of information. The information-theoretic approach, avoiding notions of probability theory, is of special interest for statistical physics because it allows to unify the notions of the exponential family in classical and in quantum statistical physics. It can also yield a contribution to quantum information theory.

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## Nonlinear Fokker-Planck equations with competing diffusive terms: Solutions and associated entropies

F. Nobre†, M.S. Ribeiro, F.D. Nobre and E.M.F. Curado

†Centro Brasileiro de Pesquisas Físicas, Rua Xavier Sigaud, Rio de Janeiro, Brazil

The applicability of linear differential equations in physics is usually restricted to idealized systems, which are valid for media characterized by specific conditions, like homogeneity, isotropy, and translational invariance, with particles interacting through short-range forces and with a dynamical behavior characterized by short-time memories. However, many real systems - specially the ones within the realm of complex systems - do not fulfill these requirements, e.g., those characterized by spatial disorder and/or long-range interactions. In such cases, the associated equations have to be modified, and very frequently, nonlinear terms are considered in order to take into account such effects. Among these equations one could mention the nonlinear Fokker-Planck equations [1] that are intimately related to anomalous diffusion. One important result associated with nonlinear Fokker-Planck equations is the H-theorem, and its generalizations, which leads to relations involving terms of these equations and entropic forms [1-4]. In the present work we study a Fokker-Planck equation with two nonlinear diffusion terms, characterized respectively, by the pairs of parameters,  $(D_1, q_1)$  and  $(D_2, q_2)$ , where  $D_i$  represent the diffusion constants and  $q_i$  the exponents of the probability of each term. In a recent work [4], the particular case  $(D, 2)$  and  $(k_B T, 1)$  was shown to be relevant for a system of interacting vortices performing an overdamped motion in a random pinning landscape, which is known to be an appropriate model to describe flux front penetration in disordered type-II superconductors. Particularly, at zero temperature, the interactions among vortices leads to a strongly-correlated state, and such a system is well described in terms of the nonextensive statistical mechanics formalism. For finite temperatures one has an interesting competition between the temperature and the strength of interactions among vortices, leading to a stationary solution that is expressed in terms of a  $W$ -Lambert function; only in the high-temperature limit this solution approaches the Gaussian distribution, associated with the Boltzmann-Gibbs entropy. We show that the more general case  $(D_1, q_1)$  and  $(D_2, q_2)$  exhibits other interesting particular cases, where stationary solutions may be calculated exactly, leading to possible physical applications. Particularly, the ratio  $\eta = (D_1/D_2)$  plays an important role, leading to a crossover between the two anomalous regimes.

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## Multiplication and addition laws for large random matrices

M.A. Nowak<sup>†</sup>

<sup>†</sup>Jagiellonian University, Cracow, Poland

Random matrices play an increasingly important role in mathematics, physics, multivariate statistics and interdisciplinary research. Among various formulations of random matrices one particular formalism is flourishing — free random variables calculus (hereafter, FRV). FRV formalism [1] can be viewed as a striking analogue of classical probability calculus, where the role of random variables is played by random operators, represented by large (infinite) matrices drawn from some probability distribution. The large size of the matrices is by no means a restriction: first, contemporary data include samples of several orders of magnitude, with dimensionality ranging from thousands (physics), through millions (telecommunication, internet) up to billions (genetics). Second, the formalism becomes exact precisely in the limit when the size of the random matrix tends to infinity. The cornerstones of this success are the so-called R and S transforms. The R transform allows one to infer the spectral properties of the sum of random operators, provided the individual spectral measures are known for each of them and they are independent in the noncommutative sense a.k.a. free. The S transform plays a similar role for the multiplication of free random operators. These constructions allow for fast decomposition of several problems for complicated random operators into simple ingredients. In this talk we present these transformations as analogues to conventional Fourier and Mellin transforms for sums and products of random variables in classical probability theory. After short summary of the known results corresponding to the cases when eigenvalue support is one-dimensional (real axes in case of hermitian random operators and unit circles in case of the unitary random operators) we will present new results by the author [2-4] on addition and multiplication laws of random operators which are non-hermitian, i.e. their eigenvalues are complex. In this case the support of eigenvalues is in general two-dimensional and standard analytic methods of random matrix theory fail. Such case is by no means academic – important applications cover dissipative phenomena, directed percolations in random media, analysis of various kinds of so-called lagged correlations and several other examples from various branches of physics. Explicit illustrations will be provided, and new potential domains of applications will be proposed.

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## Dynamics of maximal entropy random walk: Solvable cases

J. Ochab<sup>†</sup>, Z. Burda

<sup>†</sup>Marian Smoluchowski Institute of Physics, Jagiellonian University, Kraków, Poland.

We consider the Maximal Entropy Random Walk (MERW) on graphs (Burda, et al. 2009), which maximises entropy globally (which means all paths of a given length and endpoints are equally probable) in contrast to the Generic Random Walk (GRW), which maximises entropy locally (moves to neighbouring nodes are chosen with equal probabilities). The two random walks behave in the same way and reach the same stationary distributions on regular graphs, but any impurities affect the two cases very differently: specifically, the effect on MERW is non-local, in contrast to GRW. The most spectacular difference is the localisation phenomenon, which arises in the case of weakly diluted lattices, where a particle performing MERW eventually gets trapped in the largest nearly spherical region which is free of impurities (Burda, et al. 2010). We obtain the stationary state probability distribution (given by the eigenvector to the largest eigenvalue of the graph's adjacency matrix) for the case of Cayley tree (Bethe lattice) with arbitrary number of generations and arbitrary branching parameter. We also generalise the results (Zimmer and Obermair, 1978) by varying the degree of the root independently of the branching parameter. While the probability distribution for GRW concentrates in the last but one generation of the tree, the stationary state for MERW is well described by the sine, cosine or exponent of the number of generations (depending on the parameters), with high occupation probability of the root. We also show that the second largest eigenvalue of the adjacency matrix does not suffice to describe relaxation process of the probability distribution to the stationary state. In fact, the associated notion of mean passage time between two given nodes strongly depends on the initial and final nodes. We also consider ladder graphs, which are divided in half by gaps (i.e. a number of rungs taken out of the ladder). We check the relaxation dynamics of the probability flowing from one of the halves to the other. In the case of GRW, the stationary probability is proportional to the degree of a node, and we observe the usual scaling relation of the relaxation time with respect to the size of the ladder with the exponent of two, independent of the gap size. In the case of MERW, however, where the stationary distribution is localised in the regions free of defects, we find that for a given size of the ladder the relaxation time grows exponentially with the gap size, and only then the size effects come in.

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## Conformal geometry of escort probabilities and its application to Voronoi partitions

A. Ohara<sup>†</sup>, H. Matsuzoe, S. Amari

<sup>†</sup>Osaka University

Escort probability is naturally induced from researches of multifractals [1] and nonextensive statistical mechanics [2] to play an important but mysterious role. Testing its utility in the other scientific fields would greatly help our understanding about it. This motivates us to approach the escort probability by geometrically studying its role in information science. The first purpose of this presentation is to investigate the escort probability from viewpoints of information geometry [3] and affine differential geometry [4]. The second is to show that escort probability with information geometric structure is useful to construction of Voronoi partitions (or diagrams) [5] on the space of probability distributions. Recently, it is reported [6] that alpha-geometry, which is an information geometric structure of constant curvature, has a close relation with Tsallis statistics [2]. The remarkable feature of the alpha-geometry consists of the Fisher metric together with a one parameter family of dual affine connections, called the alpha-connections. We prove that the manifold of escort probability distributions is dually flat by considering conformal transformations that flatten the alpha-geometry on the manifold of usual probability distributions. On the resultant manifold, escort probabilities consist of an affine coordinate system. The result gives us a clear geometrical interpretation of the escort probability, and simultaneously, produces its new obscure links to conformality and projectivity. Due to these two geometrical concepts, however, the obtained dually flat structure inherits several properties of the alpha-geometry. The dually flatness proves crucial to construction of Voronoi partitions for alpha-divergences, which we shall call alpha-Voronoi partitions. The Voronoi partitions on the space of probability distributions with the Kullback-Leibler, or Bregman divergences have been recognized as important tools for various statistical modeling problems involving pattern classification, clustering, likelihood ratio test and so on. The largest advantage to take account of alpha-divergences is their invariance under transformations by sufficient statistics (See also [3] in a different viewpoint), which is a significant requirement for those statistical applications. In computational aspect, the conformal flattening of the alpha-geometry enables us to invoke the standard algorithm [5] using a potential function and an upper envelop of hyperplanes with the escort probabilities as coordinates.

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## Characteristics of bubble in house price distribution of Japan

T. Ohnishi<sup>†</sup>, T. Mizuno, C. Shimizu, T. Watanabe

<sup>†</sup>Canon Institute for Global Studies and University of Tokyo

We empirically investigate the house price distributions in the Greater Tokyo Area by using a housing information which is published on a weekly basis by Recruit Co., Ltd.. This dataset contains individual listings of 724,416 condominiums from 1986 to 2009 including the period of housing bubble. The attributes of a house are also included such as its size, location, age, and so on. This dataset covers more than 95 percent of the entire transactions in the central part of Tokyo (the 23 special wards of Tokyo). We find that the cross-sectional distribution of house prices has a fat upper tail, and the tail part is close to that of a power law distribution with exponent  $\approx 3$ . On the other hand, the cross-sectional distribution of house sizes measured in terms of floor space has less fat tails than the price distribution and is close to an exponential distribution with mean 25 square meters. We also find a positive linear relationship between the log price of a house and its size. An increase in the house size by a square meter leads to a 1.3 percent increase in the house price. We construct a size-adjusted price by subtracting the house size (multiplied by a positive coefficient) from the log price, which is consistent with these findings. We find that the size-adjusted price follows a lognormal distribution except for the period of the asset bubble and its collapse in Tokyo for which the price distribution remains asymmetric and skewed to the right even after controlling for the size effect. As for the period of the bubble and its collapse, we find some evidence that the sharp price movements were concentrated in particular areas, and this spatial heterogeneity is the source of the fat upper tail. These findings show that the cross-sectional distribution of size-adjusted prices is very close to a lognormal distribution during regular times but deviated substantially from a lognormal during the bubble period. This suggests that the shape of the size-adjusted price distribution, especially the shape of the tail part, may contain information useful for the detection of housing bubbles. That is, the presence of a bubble can be safely ruled out if recent price observations are found to follow a lognormal distribution. On the other hand, if there are many outliers, especially near the upper tail, this may indicate the presence of a bubble, since such price observations are very unlikely to occur if they follow a lognormal distribution. This method of identifying bubbles is quite different from conventional ones based on aggregate measures of housing prices, and therefore should be a useful tool to supplement existing methods.

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## Is Shannon entropy always additive?

T. Oikonomou<sup>†</sup>, G.B. Bagci

<sup>†</sup>Institute of Physical Chemistry, National Center for Scientific Research "Demokritos", Athens, Greece.

A sample space  $\omega$  in probability theory consists of a set of outcomes (objects, elements, events) associated with a probability measure  $P$ . The notion of a statistical ensemble is then founded on the collection and simultaneous consideration of sample spaces. Ensembles can be categorized according to the composition rule/model of the joint probabilities. In the current study we consider a specific class of ensembles characterized by Multiplicative Joint Probability Composition Rule (MJPCR), which is the standard probabilistic model (a mathematical description of an uncertain situation) in almost every textbook on statistics and probability theory. Commonly, the former rule is discussed in the context of independent outcomes. However, this rule is not only satisfied by the aforementioned outcomes, as has been demonstrated in Ref. [1]. Particularly, we explore the ensemble  $\omega_{\max}$  containing the maximum number of configurations and a symmetric reduced ensemble  $\omega_{\text{red}}$ , that is an ensemble with missing configurations. Setting the following three axioms: i) the constituent sample spaces are identical, ii) all possible configurations are probable as well and iii) the configuration (or joint) probabilities are computed according to MJPCR, we are able to determine the former probabilities and the function describing the number of configurations within the composed ensemble. Interestingly, the joint probabilities are inherently normalized in the current ensemble approach. After having determined the primary statistical quantities of the ensemble, i.e., joint probabilities and configuration function, we inquire a probabilistic functional being proportional to the size of the configurations. We denote this property as "extensivity". It is demonstrated that a structure which presents extensivity in  $\omega_{\max}$ , corresponds to the well known Shannon Entropy (SE) [2]. Moreover, in the former ensemble SE is shown to be additive as well. Exploring on the other hand, Shannon measure within  $\omega_{\text{red}}$ , we observe that although it remains extensive, it does not preserve the additivity property. This implies that the appearance of the maximum number of configurations is a necessary condition for the SE additivity. The application of our formalism on a further statistical measure, i.e., the Rényi entropy [3], reveals that the former is neither extensive nor additive for MJPCR.

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## Non-equivalence of ensembles for quantum spin systems

G. Olivier<sup>†</sup>, M. Kastner

<sup>†</sup>University of Stellenbosch, South Africa

In general, equivalence of ensembles hold for all systems with short-range interactions, this can be proved by partitioning a system into smaller subsystems and in the thermodynamic limit neglecting the boundary interactions of the various subsystems. In systems with long-range interactions this is not always the case since the boundary effects cannot be neglected because of the long-range nature of the interactions. This gives rise to various anomalies unique to systems with long-range interactions like negative specific heat and non-equivalence of ensembles. The discovery of non-equivalence of ensembles can be credited to Thirring, who in 1968 used this idea to account for the negative heat capacities shown to exist in bounded self-gravitating gas spheres by Lynden-Bell and Wood [1]. For classical systems there are numerous examples of systems with non-equivalent ensembles, for a review on this see [2]. In quantum systems it is not yet well established how to define a quantum microcanonical entropy and therefore microcanonical calculations for quantum systems are very rare in the literature. The paper by Pflug [3] on gravitating fermions is an example where negative specific heat is found for a certain energy range. The reason for the need to calculate the microcanonical entropy arises from the fact that the canonical free energy can always be recovered from the microcanonical entropy by means of a Legendre-Fenchel transform but not vice-versa. This implies that the microcanonical entropy is richer than the canonical free energy for some cases and that some macrostates in the microcanonical setting are not present in the canonical setting. In this study we consider a quantum spin system with long-range interactions and follow a suggestion made by Truong [4] to interacting systems and calculate the entropy by taking into account that the energy and magnetization correspond to a pair of non commuting operators for the model chosen. The model we consider is the anisotropic quantum Heisenberg model with Curie-Weiss-type interactions. These interactions can be considered as the limiting case of long-range interactions since every particle interacts with the other particles at equal strength. For the model in this study the canonical results were previously known, but for the microcanonical it was not. The analytic calculation of the microcanonical entropy as a function of energy and magnetization for anti-ferromagnetic and ferromagnetic cases is shown. The results indicate that the model has a non-concave entropy and therefore non-equivalence of ensembles for certain cases and thermodynamic equivalence to the Ising and Heisenberg models in other cases.

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## Dynamics of entanglement between photons and Pi-electrons in graphene

M. Ostovari†, R. Safaiee, M.M. Golshan

†Physics Department and Nanotechnology Research Institute, Shiraz University, Iran

Graphene, a two-dimensional sheet of graphite, because of its extraordinary properties, has recently attracted much interest. Such interest originates from the fact that in graphene the so-called Pi-electrons are almost free with long mean free path, phase coherence length, etc. [1]. These properties have made it feasible to use graphene, or layers of it, to develop spintronic devices [2]. Furthermore, nanoscaled structures (wires, dots, etc.) made of graphene have been proposed for use in the development of quantum computers, quantum communication channels and so forth [3]. A major role in these applications is played by the quantum entangled states [4]. Neglecting the spin state, the state of Pi-electron in graphene near the Dirac points, can be described by the eigenstates of single-particle Hamiltonian,  $H = v(\sigma P)$ , where  $v$ ,  $\sigma$  and  $P$  denote the Fermi speed, pseudo-spin and the momentum, respectively, forming the bases of joint Hilbert space. It is well established that near the Dirac points the dispersion relation is linear in electron wave number. In this article we consider a Pi-electron in graphene interacting with a one-mode quantized electromagnetic field and report the dynamical behavior of entanglement between pseudo-spin (conduction and valance bands) and photonic states. To this end, we add the interaction Hamiltonian to that of Pi-electron's and proceed to calculate the time-evolution operator (TEO). Using the so calculated TEO, we then determine the von Neumann entropy, a measure of entanglement, as a function of time. We thus show that the degree of entanglement between Pi-electron's states and photons in graphene strongly depends upon the photonic frequency and its polarization. From an analysis of the von Neumann entropy, we demonstrate that by adjusting the photonic frequency and its polarization, one can increase (decrease) the degree of entanglement. In fact, when the photonic frequency is in resonance with inter-band transitions, the entanglement is drastically enhanced. We further show that in graphene the entanglement is stronger than its ordinary counter parts, such as the entanglement of two-level atoms and photons [5]. Moreover, when the electronic momentum is along the photonic polarization the entanglement vanishes, while in the perpendicular case it is maximal.

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## Transport and pattern formation in internally actively driven systems

I. Pagonabarraga†

†Universitat de Barcelona

Active systems generate motion due to energy consumption, usually associated to their internal metabolism or to appropriate, localized, interfacial chemical reactivity. As a result, these systems are intrinsically out of equilibrium and their collective properties result as a balance between their direct interactions and the indirect coupling to the medium in which they displace. Therefore, a dynamical approach is required to analyze their evolution and quantify their selfassembly and ability to generate intermediate and large scale stable structures. I will discuss how physical mechanisms associated to the coupling of the active entities through the embedding medium can lead to cooperative motion and pattern formation even in the absence of additional, specific interactions between the particles. I will introduce effective, mesoscopic models which capture the dynamic coupling of the particles to their environment on the natural time and length scales in which they evolve, disregarding the details of the medium. I will analyze the peculiarities of collective transport of molecular motors along biofilaments, and the patterns they give rise to due to their coupling to the embedding fluid medium in which they displace. I will also describe how bacteria colonies develop patterns as a result of the relative hindrance to individual motility. In both cases I will discuss the relevant physical mechanisms underlying the specific properties of their collective behavior. By focusing on simplified models, it is then possible to identify the relevant parameters which control such behavior. Understanding the mechanical principles which determine cooperatively will help to clarify the role of physical coupling in active materials and understand how to combine them with biochemical interactions to control their properties and behavior. Since these systems are out of equilibrium, their mechanical balance is very sensitive to external disturbances. Therefore, once the basic principles which control the intrinsic activity of these systems has been clarified, it is interesting to analyze how do they react to externally applied fields or to geometric confinement. I will discuss the effect of these disturbances in the two examples mentioned above and will characterize the emergence of new transport modes and unsteady phases.

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## Rotated multifractal network generator

G. Palla<sup>†</sup>, P. Pollner, T. Vicsek

<sup>†</sup>Statistical and Biophysical Research Group of HAS, Eotvos University, Budapest, Hungary

Network models have been always playing a crucial role in complex network theory. On the one hand they enable singling out the simplest aspects of the studied systems and, thus, are extremely useful in understanding the underlying principles. On the other hand they can also help testing hypotheses about measured data. Although many important and successful models have been introduced over the past decade for interpreting the different aspects of the studied networks, most of these models explain only a particular aspect of the given system and for each newly discovered feature a new model had to be constructed. Due to this proliferation of network models, the concept of general network models and methods for generating graphs with desired properties has attracted great interest lately. A novel approach along this line is the Multifractal Network Generator [1], which was inspired by the recent discovery of the intimate connection between the limiting objects of graph sequences and 2d functions [2-3]. This approach is based on drawing link probabilities from a 2d multifractal measure obtained from a very simple generating measure in a recursive multiplicative process [1]. A key ingredient of the method is to go to the infinite limit of the singular measure and the size of the corresponding graph simultaneously. A very unique feature of this construction is that with the increasing systems size the generated graphs become topologically more structured. By changing the parameters of the generating measure the obtained random graphs can show diverse features (e.g., the degree- or clustering coefficient distributions can take up very different forms). To generate random graphs with prescribed statistical features (e.g., a given degree distribution) the optimal parameters of the generating measure can be determined from a simple simulated annealing process [1]. Although the described multifractal network generator has been shown to provide a simple and very flexible tool for creating random graphs, a slight disadvantage is that it can also lead to isolated nodes, whose number is relatively low for realistic cases, but may become dominant in the limiting case of infinitely large network sizes. Here we discuss the relation between this effect and the information dimension for the 1d projection of the link probability measure, and argue that the node isolation can be avoided by a simple transformation of the link probability measure based on rotation [4].

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## Spatio-temporal Anomalous Diffusion by NMR

M. Palombo<sup>†</sup>, A. Gabrielli, S. de Santis, C. Cametti, G. Ruocco, S. Capuani

<sup>†</sup>Physics Department of Sapienza University of Rome, Italy

Anomalous diffusion (AD), which predicts the mean-square displacement of a diffusing particle to grow nonlinearly in time,  $[x(t) - x(0)]^2 \sim t^\nu$  (with  $\nu = 1$ ), is a property of many complex systems and its related phenomena have been observed in various physical fields [1,2]. Features of AD can be described using fractional calculus as shown by Metzler and coworkers [3]. According to these Authors, molecular AD in media can be described by defining the motion propagator (MP) as the solution of fractional diffusion equations, which arise from the continuous time random walk (CTRW) model. These equations involve two fractional exponents, alpha and mu, which are the time and space derivatives fractional orders respectively [4]. However, to our knowledge, it has never been carried out an experimental alpha vs mu phase diagram showing the competition between superdiffusion and subdiffusion of diffusing molecules in heterogeneous media. In this work [5] we experimentally challenged the Metzler et al. prediction of the CTRW theory measuring the fractional exponents alpha and mu by NMR methods, based on Pulse Field Gradient (PFG) technique, and providing their interplay for the first time. Diffusion phase diagram, predicted by CTRW model, was experimentally obtained for the first time. alpha and mu were simultaneously measured by PFG NMR technique in samples of micro-beads dispersed in aqueous solution. Fractional exponent alpha was measured by collecting the PFG signal attenuation as a function of diffusion times compared to the asymptotic expression of FT of the MP for subdiffusive regime, obtained from [3]. Moreover, mu was extracted from the PFG signal attenuation as a function of gradients strength fitted to the expression of FT of the MP for superdiffusive regime, obtained from [3]. PFG NMR experiments were performed on controlled porous media characterized by packed polystyrene micro-beads in water of various sizes. Mono-dispersed samples (ordered systems) and poly-dispersed specimens (disordered systems) were used to check the potential ability of alpha and mu parameters to detect different degrees of systems disorder. Moreover, internal magnetic field gradients measurements were performed to investigate the influence of the magnetic susceptibility differences on measured alpha and mu parameters. We found that alpha depends on the disorder degree of the system. Conversely, mu depends on both beads sizes and magnetic susceptibility differences within samples. Experimental results fully match with the CTRW predictions.

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### Dynamic of the secondary processes in a mean-field exactly solvable model glass

M. Paoluzzi†, A. Crisanti, L. Leuzzi

†Roma Tre, Italy

The interrelation of dynamic processes active on separated time-scales in glasses and viscous liquids is investigated. Both a thermodynamic (static) and a dynamic approach are implemented on an exactly solvable spin model with quenched disordered interactions, developed to study the nature of polymorphism and amorphous-to-amorphous transitions. On the static front, the analysis is carried out within the framework of Replica Symmetry Breaking (RSB) theory, leading to the identification of low temperature glass phases of different kinds. A typical feature of the mean-field kind of models we investigate is that the configurational entropy can be computed at the static level, since glassy metastable states have infinite life-time in the thermodynamic limit. Moving to the study of dynamics this allows for predictions on the system relaxation above the mode coupling (MC) temperature, that are compared with the outcomes of the equations of motion directly derived within the Mode Coupling Theory (MCT) for under-cooled viscous liquids. The model under probe is a spherical  $s + p$  multi-spin interaction model. It displays the feature of having both usual "one-step" RSB solutions, known to reproduce all basic properties of structural glasses, and a physically consistent "two-step" solution. Since the dynamic counterpart of a RSB is known to be a time-scale bifurcation, providing a leading model to probe the behavior of characteristic time-scales in presence of secondary processes and the different mechanisms in which they can arise. Along certain cooling paths in the phase diagram, determined by analysis of the static thermodynamic properties, approaching the tricritical point MCT applied to the  $s + p$  model yields a time-correlation function with two plateaus at different correlation values. A first plateau occurs for  $t > t^*$  and a second one on the characteristic time-scale at which the  $\tau$  secondary relaxation occurs. We, thus, study the behavior in  $T$  of the characteristic relaxation times for processes on different time-scales and their functional interrelation. The hierarchical nesting implicit in the present approach hints that fast processes have a relevant influence on slow processes, even though taking place on well separated time-scales. This observation naturally stimulates a comparison with Ngai's Coupling Mode.

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### Fractal intermittency in integrated neural dynamics

P. Paradisi†, P. Allegrini, A. Gemignani, D. Menicucci

†Istituto di Scienza e Tecnologie dell'Informazione "A. Faedo" (ISTI-CNR), Pisa, Italy

In this work we discuss the crucial role of events in complex systems and we investigate brain activity and information processing as a paradigm of complexity. Recent literature supports the hypothesis that the brain, but also many other complex systems, obeys universal mechanisms of emergent complex phenomena, which are evident in dynamical systems poised near a critical point. [see[1] for a review of recent theoretical and empirical results supporting the notion of brain as a complex system or network in a criticality condition]. A universal, still overlooked, mechanism of critical systems is the emergence of crucial events, labelling the intermittent passage between metastable states, with a fractal time statistics [2,3]. The term "fractal" follows from the scale invariance of the Probability Density Function (PDF) of the Waiting Times (WT), the WTs being the times between two events. In fact, in cooperative systems, the WT-PDF displays a typical inverse power-law tail in the range of large WTs. The sequence of events is a reduced description of the system's evolution in terms of a birth/death process of cooperation, i.e. system elements being recruited into collective self-organized states emerging from the dynamics. These collective states or modes often dominate the system's dynamics. When the events locally reset the memory, the system becomes is described by a renewal, which is defined by the statistical independence of the events. If renewal assumption is satisfied, the WT-PDF plays a central role. In particular, for fractal intermittency, the power-law index in the WT-PDF is a measure of the system's complexity, which we define as "complexity index". However, a direct proof of the renewal hypothesis is often hard to obtain in experimental time series, mainly due to the presence of noise superposed to a genuine fractal intermittent process [2,4]. In this work we introduce a robust method to estimate the complexity index and the noise level, independently from the physical origin of the noise itself, internal or external to the system. Our method is based on representing the signal as the superposition of a genuine Non-Poisson renewal process and a Poisson noise. An application to brain activity by means of the analysis of ElectroEncephaloGrams (EEG) is shown. We discuss the interpretation of the results in terms of emerging properties of integrated neural dynamics, supporting the idea that brain activity behaves dynamically as a complex system in a critical state.

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## Theoretical advances in off-equilibrium behavior

G. Parisi†

†Dipartimento di Fisica, Roma, Italy

In this talk I will present a theory for the behaviour of large-scale dynamical heterogeneities in glasses. I will show that we can write a (physically motivated) simple stochastic differential equation that summarize our perturbative knowledge on large scale dynamical heterogeneities in glasses. It turns out that this behaviour is in the same universality class of the dynamics near the endpoint of a metastable phase in a disordered system, as far as reparametrization invariant quantities are concerned. Large scale dynamical heterogeneities in glasses have many points in contact with the Barkhausen noise. Numerical verifications of this theory have not yet done, but they are quite possible. I will base my presentation on the recent comprehensive study of the dynamics near the mode coupling transition that is at the basis of most of the analytic conclusions. A very important role in the arguments will be played by the results that have been obtained in on the numerical simulations of glasses forming mixtures and by the extreme large scale simulations of spin glasses that have been done using the Janus machine. It is refreshing to see how many different pieces of the puzzle fit together and how this new picture helps us to answer to old questions and to formulate new questions. As we shall see many progresses can be done if we concentrate our attention on reparametrization invariant quantities, i.e. on the relations among quantities that are obtained by eliminating the time parametrically. I present here a complete theoretical approach that is able to predict many of the observed quantities. More than twenty years ago it was recognized that a certain class of mean-field spin-glass models displays the same dynamical behaviour predicted by the Mode-Coupling Theory (MCT) of glasses. This has motivated a great deal of research and many believe that there is an intrinsic analogy between these two classes of systems. Lowering the temperature, these models display two transitions. At the dynamical transition temperature  $T_d$  the paramagnetic equilibrium state abruptly splits into a number of states that is exponentially large in the size of the system. Correspondingly the equilibrium dynamics displays the well known MCT phenomenology. MCT offers a good description of the early stages of the dynamical slowing down and in this talk I will discuss finite-size and finite-dimensional effects at  $T_d$ , i.e. at the Mode-Coupling temperature.

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## Carbon nanoelectronics - The super M conductivity and more

Y.W. Park†

†Seoul National University

Carbon nanoelectronics is a future electronics complementary to the present silicon technology. It opens new paradigm of science and technology. Carbon is a lighter element than Si or Ge. Diamond, graphite and graphene, C60 fullerene, carbon nanotubes are the proto types of carbon. The conducting polymers, organic single crystals, single molecules and bio-molecules can be regarded as extended carbon based materials with hydrogens, nitrogens and another elements attached to the carbon backbone. Fundamental properties and applicability of the carbon based materials and their composites are investigated. In particular the vanishing magneto resistance (VMR) observed in polyacetylene nanofibers [1,2] is a seminal discovery. The zero resistance change ( $R(H) = 0$ ) in high electric field of polyacetylene nanofibers is named as the super M conductivity in analogy to the zero resistance ( $R = 0$ ) of superconductivity. The possibility of superconductivity in carbon systems has now been realized as the super M conductivity in quasi-one dimensional polymer nanofibers. An electric field modulated high magnetic field switching device can be developed with the polyacetylene nanofibers as a potential application for more stable high speed magnetic levitation trains. Recent works on the electrospun CNT filled polymer composites [3] and coaxial carbon nanofibers with NiO core [4], graphene nanorings [5], perchlorate-doped TTF-diamide nanofibers with double and triple helix structures [6] are envisaged. In particular, the CNT based nonvolatile electromechanical memory [7] is a novel nonvolatile memory design based on the electromechanical motion of a cantilever to provide fast charging and discharging of a floating gate electrode. The operation is demonstrated by using an electromechanical metal cantilever to charge a floating gate that controls the charge transport through a carbon nanotube field effect transistor. The set and reset current persist unchanged for more than 11 hours constant operation. Over 500 repeated programming and erasing cycles were demonstrated under atmospheric conditions at room temperature without degradation. Multinary bit programming can be achieved by varying the voltage on the cantilever. The operation speed of the device is 1000 times faster than a conventional flash memory and the power consumption is much less than conventional Si based flash memory devices.

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## Dissipation and irreversibility in stationary trajectories

J.Mr. Parrondo†, E. Roldán

†Universidad Complutense de Madrid

The irreversibility of a stationary trajectory can be measured by comparing the trajectory with its time reverse. This comparison can be made quantitatively by using a concept from information theory, the Kulback-Leibler distance or relative entropy between two stochastic processes. The relative entropy between two probability distributions measures how difficult is to distinguish one from the other, i.e., it is a quantitative measure of distinguishability. Recent results based on fluctuation theorems show that the relative entropy between a trajectory and its time reverse is a lower bound to the entropy production or dissipation along the trajectory. This quantitative relationship between irreversibility and dissipation is remarkable, since the relative entropy depends only on the trajectory and not on the physical process that generates the trajectory. We can therefore estimate (or bound) the dissipation required to generate a time series of data without knowing the physical mechanism that has generated these data. We are developing techniques to estimate the relative entropy and obtain lower bounds to dissipation from a generic time series. In the talk, we will discuss these techniques applied to discrete systems. We estimate the relative entropy in a toy example, a discrete flashing ratchet with an external force or load. The ratchet consists of a random walker with an internal state, which exhibits a current in a given direction. This current decreases and eventually vanishes when the external force reaches certain value, usually termed stall force. Our technique is able to estimate the entropy production in different stationary regimes from trajectories where both the internal state and the position of the particle are available. More interestingly, we can still predict dissipation or entropy production when only the position of the particle is given as a time series. More interestingly, the relative entropy allows us to detect dissipation even at stall force, i.e., in the absence of flows in the available data. Since the standard methods to calculate entropy production rely in the presence of flows, our technique clearly goes beyond those methods and could be relevant to estimate dissipation from experimental data in many different fields. In particular, it could be used to discern between active and passive processes in biological systems, among other applications.

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## Freezing of mixed submonolayer films of Argon and Krypton on graphite

A. Patrykiewicz†, S. Sokolowski, M. Borówko

†Department for the Modelling of Physico-Chemical Processes, MCS University, Lublin, Poland

The adsorption of Argon and Krypton on graphite has been extensively studied for several decades and phase diagrams of monolayer films for both systems are now rather well established [1]. It is known that at low temperatures, Argon forms incommensurate, floating, solid, while Krypton orders into the commensurate phase. Moreover, Argon exhibits the usual gas-liquid and liquid-solid phase transitions. On the other hand, Krypton does not show the gas-liquid coexistence, but rather the gas-commensurate solid coexistence. The differences in the structure and phase behavior of Argon and Krypton monolayer films on graphite are expected to considerably affect the phase behavior of mixed films. Unlike the mixed Ar-Xe and Kr-Xe films on graphite [2,3], the Ar-Kr mixed films on graphite have not been experimentally studied yet. The only experimental study of submonolayers formed by Ar-Kr mixture on well defined solid surface was performed by Zeppenfeld et al. [4]. Using Monte Carlo method in the canonical ensemble, we study the freezing and low temperature structures of mixed submonolayer films of Argon and Krypton adsorbed on graphite basal plane. It is shown that both adsorbates form a mixed liquid phase, independently of the film composition, and that the liquid freezes into also mixed solid phase. The structure of the solid depends upon the film composition and the temperature. At the temperatures just below the melting point, we observe the formation of incommensurate (Ar-like) solid for small Krypton concentrations and the mixed commensurate (Kr-like) solid for sufficiently large Krypton concentrations. Upon the decrease of temperature, the film undergoes a phase separation and its structure depends upon the film composition. For very low Krypton concentrations, the incommensurate (Ar-like) structure is preserved and the Krypton atoms are dissolved in the Argon film. The only effect of Krypton on the structure of solid-like phase is the reduction of epitaxial rotation, which is known to be present in Argon films. When the Krypton concentration exceeds a certain threshold value the film exhibits a domain-wall structure. The commensurate domains consist of Krypton atoms, while Argon forms the incommensurate walls. For very high Krypton concentrations, Argon atoms are predominantly located at the boundaries of the commensurate Krypton film, thus Argon does not dissolve in Krypton-rich films at sufficiently low temperatures.

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## Nonlinear response theory of non-ideal systems

G. Pavlov†

†Institute of problems of chemical physics RAS, Chernogolovka, Russia

The nonlinear response theory approaches are developed to study the nonlinear phenomena and nonlinear transport in non-ideal Coulomb systems. The nonlinear phenomena: plasma wave echo and waves transformation have been investigated under non-ideal Coulomb system conditions based on a variant of the nonlinear response theory. Some general restrictions on the values of nonlinear response functions are considered. The model for the determination of quadratic response functions is presented. The conditions for experimental realization of the mentioned phenomena in non-ideal plasma are examined. It is shown that ultra-short field pulses can induce the phenomena. The theory of non-linear transport is elaborated to determine the Burnett transport properties of non-ideal multi-element matter. The procedure of the comparison of the phenomenological conservation equations of a continuous charged and neutral medium and the microscopic equations for dynamical variables is used for the definition of these properties. The Mori's algorithm is developed to derive the equations of motion of dynamical value operators of a non-ideal system in the form of the generalized nonlinear Langevin equations. In consequence, the correlation function expressions of kinetic coefficients corresponding to second order thermal disturbances (temperature, mass velocity, etc) have been found in the long-wavelength and low frequency limits. To establish a link between the results of the performed investigations and hydrodynamic problems the properties are discussed of the matrix of coefficients at highest derivatives in the set of conservation equations in the linearized Burnett approximation. The method for calculation of Burnett kinetic coefficients is developed. This method is based on the investigations of long-wavelength limits of correlation functions which determine the coefficients. For the determinations of long-wavelength limits of correlation functions corresponding kinetic equations are used. The calculations are provided for a model system. The report shows the difference and similarity between the variants of non-linear response theory: the description of nonlinear phenomena and nonlinear transport in non-ideal charged matter. The approach for non-linear transport properties definitions can be used for different dense matter: one and two-component Coulomb systems, electrolytes, liquid metals, nuclear matter etc and for dense neutral isotropic matter. It is important also to provide the calculation of Burnett kinetic coefficients of non-ideal matter by computer modeling.

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## Fisher information and semiclassical treatments

F. Pennini†, A. Plastino, G.L. Ferri

†Universidad Catlica del Norte, Antofagasta, Chile

Quantal phase-space distributions constitute a subject of permanent interest, with applications in statistical mechanics, quantum chemistry, quantum optics, and classical optics. Also for signal analysis in diverse fields such as electrical engineering, seismology, biology, speech processing, and engine design. We present here the difference between quantum statistical treatments and semiclassical ones, using as the main concomitant tool a semiclassical, shift-invariant Fisher information measure built up with Husimi distributions. Its semiclassical character notwithstanding, this measure also contains abundant information of a purely quantal nature. The paradigmatic semiclassical concept we appeal to is that of Wehrl's entropy, a useful measure of localization in phase-space that is built up using coherent states. Also, the semi-classical Husimi probability distribution refers to a special type of probability: that for simultaneous but approximate location of position and momentum in phase space. Such a tool allows us to refine the celebrated Lieb bound for Wehrl entropies and to discover thermodynamic-like relations that involve the degree of delocalization. Fisher-related thermal uncertainty relations are developed and the degree of purity of canonical distributions, regarded as mixed states, is connected to this Fisher measure as well [1-4]. Our emphasis will be placed on the study of the differences between (i) statistical treatments of a purely quantal nature, on the one hand, and (ii) semiclassical ones, on the other. We will show that these differences can be neatly expressed entirely in terms of a special version of Fisher's information measure: the so-called shift-invariant Fisher one, associated to phase space. In addition, we illustrate our procedures with reference to the harmonic oscillator instance. This is such an important system constitutes much more than a mere example. Nowadays it is of particular interest for the dynamics of bosonic or fermionic atoms contained in magnetic traps as well as for any system that exhibits an equidistant level spacing in the vicinity of the ground state, like nuclei or Luttinger liquids. Among many other examples one may mention that it is possible to describe relevant quantum effects in some bio-systems by approximating a group of proteins and its environment by a set of coupled harmonic oscillators [5].

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## Superstatistics in tick-by-tick financial data

J. Perelló†, M. Gutiérrez-Roig

†Departament de Física Fonamental. Universitat de Barcelona

Econophysics has been focussing for a long time in price fluctuations, also called returns. Despite there are no consensus about its distribution it is widely known that distribution of the returns in a short period of time have a fat tail, even in some cases can be fitted by a power-law. Here we change the approach of time scale and try to consider the return as the logarithmic variation of the price caused by just one single transaction. We discuss the differences and similarities between the time-based scale and single-event scale distributions. Since return distribution in the single-event scale has a fat tail the return distribution in the time-based scale must have a fat tail as well. In order to shed more light on the origin of these fat tails we develop a theoretical framework based in Superstatistics tools. In our model we consider two variables with two different "velocities". The returns, which corresponds to a faster one. And the slow fluctuating one, which corresponds to the volatility. We suppose an original Laplace distribution for the returns, but conditioned by the volatility, which follows an Inverse Gamma distribution. The coupling of these two distributions causes the power-law behaviour in the event scale return distribution. We prove that through the fact that distribution of the returns rescaled by the corresponding volatility loose its fat tail. All commented results are tested empirically with high-frequency financial data. Specifically, we use German DAX index, American SP500 index, Spanish IBEX index, Dow Jones Index and Dollar-Mark future markets financial series. The length of the data time-period goes from 5 months, in case of DAX, to almost 5 years, in SP500 and USDollar-Mark data series. All these data series contain at least 1 million of price records each one depending on the liquidity of each market. We process the data with non-parametric statistics to determine the shape of the tail in single transaction price change. This kind of study can be included inside the Continuous Time Random Walk formalism which aims to describe the continuous time price fluctuations through transaction to transaction behaviour. Joined with Perelló et al. (2008) completes both interevent times and continuous time price fluctuation of CTRW description. The model including the two descriptions will be discussed soon.

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## Epidemics in soil networks

F.J. Perez-Reche†, S.N. Taraskin, W. Otten, C.A. Gilligan, L. da F. Costa, M.P. Viana

†SIMBIOS Centre, University of Abertay, Dundee, UK

Understanding how microorganisms spread in soil is essential for multiple disciplines, including ecology, epidemiology, and environmental sciences. There is evidence that the heterogeneity of soil structure affects the distribution of soil microorganisms [1]. However, no systematic theoretical framework has been proposed so far to investigate if the soil structural heterogeneity has a significant impact on the ability of microorganisms to invade large regions of the soil pore structure. In this work, we use a network representation for the soil pore space and propose a model for spread of microbes in such networks that reveals the prominent effect of soil structural heterogeneity on invasion. Soil networks were extracted from the 3D digital images of six soil samples derived from an X-ray micro-tomography system. Images were mapped onto networks whose nodes represent the branching points of the pore space and edges are the channels linking such branching points. Typical values for the number of nodes and edges are,  $N = 5 \cdot 10^4$  nodes and  $E = 7 \cdot 10^4$ , respectively. The layout of nodes and links in these networks contains topological information about the pore space that is expected to be important for biological invasion. In addition, the microbial colonisation is known to be influenced by the structure of channels [1]. In order to investigate the role of the channel structure on invasion, the network links contain information on the channel arc-length,  $L$ , and variable section,  $S(x)$ , along the arc-coordinate  $x$ . The soil networks are characterised by a restricted node degree, a relatively high clustering coefficient, and a mean separation between pairs of nodes that increases as a power-law with  $N$  (i.e. soil networks exhibit the fractal-small-world phenomenon [2,3]). Microbial invasion of soil is described in terms of a model that can be mapped to the well-known SIR epidemiological model [4]. The pathogen is randomly passed from an infected node to a susceptible neighbour with probability  $T$  (transmissibility) that depends on the channel characteristics,  $L$  and  $S(x)$ . The transmissibility is heterogeneous as a consequence of the heterogeneity in  $L$  and  $S(x)$ . Our numerical simulations show that heterogeneity in transmission typically enhances invasion. This effect is associated with the possibility for microorganisms to invade some long channels in the soil network that act as shortcuts for spread (i.e. a mechanism reminiscent of the small-world phenomenon [5]). To conclude, our study suggests that neglecting the role of structural soil heterogeneity may lead to undesired situations in which the probability of an invasion is underestimated.

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## Synergy in complex spreading processes on networks

F.J. Perez-Reche<sup>†</sup>, J.J. Ludlam, S.N. Taraskin, C.A. Gilligan

<sup>†</sup>SIMBIOS Centre, University of Abertay, Dundee, UK

A common assumption in many models for spreading processes is that the transmission of the spreading agent between any pair of individuals in a population is not affected by the presence of other individuals in the system. However, there are evidences that this assumption is not satisfied by numerous spreading agents in both biological (e.g. fungal, bacterial, tumoral, and animal invasion) and social (e.g. spread of rumours or opinion) systems. Models for contagion processes [1,2], opinion dynamics [3], and animal invasion [4] have considered some constructive synergistic effects such as, e.g. population pressure and social impact. However, these effects are too simple to capture, for instance, possible changes in the foraging strategies of spreading agents that can significantly affect important features of invasions such as their size and time scales. We have recently proposed a model for spread of infection in networks which incorporates synergistic effects that allow the infectivity and/or susceptibility of hosts to be dependent on the number of infected neighbours [5]. The model is an extension of a basic spatial model for the SIR epidemic process in which hosts (sites) can be in one of the three states [4]: susceptible (S), infected (I) or removed/recovered and fully immune to further infection (R). In order to account for synergy in transmission, we assume that infection is transmitted randomly from an infected host, I, to a susceptible neighbour, S, with a rate that depends linearly on the number of infected hosts in the neighbourhood of the I-S pair. Motivated by synergistic effects observed in biological spreading processes, we have analysed the role on invasion of both constructive and interfering synergy. For populations of hosts arranged on a square lattice, we have obtained that constructive synergy leads to an exploitative regime which may result in a rapid invasion that infects a large number of hosts. In contrast, interfering synergy leads to an explorative foraging strategy characterised by slower and sparser invasion able of traversing larger distances by infecting fewer hosts. Preliminary analysis of synergistic agents spreading on more complex networks reveals a large variety of spreading behaviours arising from the interplay between synergy and topological properties of networks.

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## BioLCCC: polypeptide chromatography from a physicist's point of view

T. Perlova<sup>†</sup>, A.A. Goloborodko, I.A. Tarasova, A.R. Ivanov, A.V. Gorshkov, V.V. Evreinov, M.V. Gorshkov

<sup>†</sup>Russian Academy of Science, Institute for Energy Problems of Chemical Physics

For the last several decades, liquid chromatography (LC) has been an essential tool for studying peptides and proteins structures. Its importance even increased with the rise of mass spectrometry (MS) as a method of choice for polypeptide analysis [1]. Despite the fact that a typical mass spectrum contains much more information than a chromatogram, LC remains useful, since the chromatographic separation is based on the chemical properties of a molecule which are not observed by MS. There is still no general predictive theory of liquid chromatography of polypeptides. The classic treatment of the LC separation was developed for small molecules and barely applicable to large polymeric substances. Several empirical approaches were developed to predict the results of a chromatographic experiment on a polypeptide mixture [2-3]. However, most of them if not all based on extensive fitting of experimental results and have no proper theoretical grounding. Moreover, these models are applicable only to a narrow set of experimental conditions and fail to predict several observed effects, such as size-exclusion or inversion of elution order in gradient chromatography. In this work we present a summary of the model of Liquid Chromatography of Biomacromolecules at Critical Conditions (BioLCCC) [4]. Based on the theory of LC at critical conditions, BioLCCC extends its predecessor to the realm of short heteropolymers, an intermediate region between small molecules and large homopolymers. BioLCCC operates both on the micro- and macroscopic levels. Physical modelling at the micro level allows determination of the polypeptide distribution coefficient in a given composition of mobile phase. The computation procedure is based on the lattice model by DiMarzio and Rubin [5], which is replaced by a more accurate continuous model as the ratio of a Kuhn length to the total polypeptide length approaches unity. The obtained distribution coefficient function is then used at the macro level to describe the movement of the substance band through the chromatography column and to predict the retention time observed in the experiment. The model presented has been validated on numerous experimental datasets and is already used in proteomics data analysis. It explains most of the experimentally observed features of polypeptide chromatography and may be used both for the educational and practical purposes.

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### External Noise Effects in doped semiconductors operating under sub-THz signals

D. Persano Adorno<sup>†</sup>, N. Pizzolato, D. Valenti and B. Spagnolo

<sup>†</sup>Group of Interdisciplinary Physics, Dipartimento di Fisica, Università di Palermo

Semiconductor based devices are always imbedded into a noisy environment that could strongly affect their performance. For this reason, to fully understand the complex scenario of the nonlinear phenomena involved in the devices response, the analysis of the noise-induced effects on the electron transport dynamics in low-doped n-type samples is very relevant. Previous investigations on the constructive aspects of noise in different non-linear systems have shown that the addition of external fluctuations to systems, characterized by the presence of intrinsic noise, may affect the dynamics of the system in a counterintuitive way, resulting in a less noisy response [1]. The possibility to suppress the intrinsic noise in n-type GaAs bulk driven by a static electric field has been theoretically found when a gaussian noise is added to the external field [2]. Detailed studies of the electron transport dynamics in a GaAs bulk, working under cyclostationary conditions, has revealed that the addition of a correlated source of noise to the forcing sub-terahertz oscillating electric field can reduce the total noise power [3, 4]. This result is explained in terms of the noise enhanced stability phenomenon [5], arising from the fact that the transport dynamics of electrons in the semiconductor receives a benefit by the constructive interplay between the fluctuating electric field and the intrinsic noise of the system [4]. Furthermore, a nonlinear behavior of the integrated spectral density (ISD) with both the noise intensity [3-4] and the correlation time of the external source of noise [4] has been found. In the present work we investigate the effects of different types of external source of noise on the intrinsic carrier noise in low-doped n-type semiconductor structures operating under periodic conditions. The electron dynamics is simulated by a Monte Carlo procedure which keeps into account all the possible scattering phenomena of the hot electrons in the medium. The modifications caused by the addition of an external source of noise are investigated both by computing the velocity fluctuations correlation function and the integrated spectral density and directly calculating the variance of the electron velocity fluctuations, as a function of the parameters characterizing the kind of the external noise. Our findings confirm that the diffusion noise in low-doped semiconductors can be reduced by the addition of a fluctuating component to the driving electric field, but this effect critically depends both on the intensity and the type of the external noise source.

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### Monte Carlo investigation of electron spin relaxation in GaAs crystals during low-field transport

D. Persano Adorno<sup>†</sup>, S. Spezia, N. Pizzolato and B. Spagnolo

<sup>†</sup>Group of Interdisciplinary Physics, Dipartimento di Fisica, Università di Palermo

A great emerging interest within the condensed matter physics is the use of electron spin in semiconductor-based spintronic devices to perform both logic operations, communication and storage. In order to make spintronics a feasible technology, sufficiently long spin lifetimes and the possibility to manipulate, control and detect the spin polarization are required. The loss of spin polarization before, during and after the necessary operations is a crucial problem into spin device design; thus, a full understanding of the role played by the lattice temperature, the doping density and the amplitude of the applied electric field on the electron spin dynamics in semiconductors is essential for the design and fabrication of spintronic devices. In recent years many experiments have been carried out with the aim of study the influence of transport conditions on relaxation of electron spins in semiconductors, but they are mainly focused on the analysis of spin lifetimes at very low temperatures ( $T < 10K$ ). Despite a lot of theoretical or simulative works have been devoted to the investigations of non-equilibrium spin relaxation, till today, to the best of our knowledge, an investigation of influence of lattice temperature on the spin depolarization time up to room temperature in the presence of driving electric fields in semiconductor bulk structures, for different values of the doping density, is still missing. Aim of this work is to analyze the influence of the lattice temperature on electron spin lifetimes in n-type GaAs crystals driven by low-amplitude electric fields. The transport of electrons is simulated by using a semiclassical Monte Carlo approach [1], which takes into account the intravalley scattering mechanisms of warm electrons in the semiconductor material and includes the spin polarization vector [2-3]. Spin relaxation is considered through the D'yakonov-Perel mechanism [4], which is dominant in the investigated range of temperature ( $40 < T < 300K$ ) [5]. The evolution of spin polarization is studied by analyzing the computed lifetimes as a function of the doping density in the range  $10^{13} - 10^{16}cm^{-3}$  (non-degenerate regime), in the presence of a static electric field with amplitudes in the range  $0.05 - 0.5kV/cm$ . Our findings show that the electron spin lifetime is not marginally influenced by the intensity of the driving electric field, the lattice temperature and the impurity density, which hence represent key parameters into the depolarization process.

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## Quasistatic heat processes in mesoscopic non-equilibrium systems

J. Pešek†, K. Netočný)

†Charles University in Prague, Faculty of Mathematics and Physics, Praha, Czech Republic

In recent years there has been renewed interest in processes connecting non-equilibrium stationary states [1,2]. Among them quasistatic processes play an important role in building up the thermodynamic formalism, both in equilibrium and beyond. However a basic difficulty in the non-equilibrium case is the divergence of energy flows between system and its environment in the quasistatic limit. We discuss various approaches how to overcome this problem by constructing finite (excess) parts of these energy flows. It has also been recently proposed [4] that for weakly non-equilibrium processes the excess heat (approximately) satisfies thermodynamic relations extending those for equilibrium processes, with a generalized entropy function. In order to systematically analyze these propositions, we first derive non-perturbative formulas for quasistatic excess path quantities in Markovian systems such as overdamped diffusion and Markov jump processes, and we propose a method how to possibly access them experimentally, which is demonstrated by numerical simulation. By extending the method of McLennan [3], we perturbatively construct the excess work and heat for both overdamped diffusions and jump processes driven by small non-gradient forces. It is shown that the excess heat always satisfies the (generalized) Clausius relation up to the first order in non-equilibrium driving, whereas this is no longer true in general when beyond the leading order. In the latter case, the excess path quantities do not derive from (generalized) thermodynamic potentials but they require new non-potential corrections. We also discuss the possible meaning of those new correction and show that in some particular cases they could be described in terms of a suitable effective temperature. In more general, we ask whether the new terms emerging in the quasistatic heat relations can be associated with some small enough collection of (macroscopic) observables together with corresponding Lagrange multipliers. As an application of our approach, we re-examine the concept of heat capacity for non-equilibrium systems. We compare several natural definitions and indicate how they are mutually related and how the non-equilibrium driving enters these relations. The latter are made explicit up to the leading perturbative order.

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## Quantifying career growth and career longevity in academia

A. Petersen†, H.E. Stanley, S. Succi

†IMT Institute for Advanced Studies Lucca, Boston University

A scientist's career is subject over time to a myriad of decisions. As a result, the path to success can be difficult to analyze, since there are potentially many factors, e.g. individual / mentor, institutional, collective factors, to be accounted for in a statistical analysis. Since scientists in academia are subject to competitive forces, one motivating question in this talk is: "What are the statistical regularities in career achievement across an entire cohort of competitors?" We use concepts and methods from statistical physics to describe statistical regularities in the career output of individual scientists [1,2,3] which exist despite the idiosyncratic details of scientific achievement. In this talk, I motivate a stochastic model for career development [2] that relies on two key ingredients, random progress within the career and random stopping times terminating the career. This model is exactly solvable and predicts the probability density function (pdf) of career longevity, which is characterized by two parameters,  $\alpha$  and  $x_c$ . The parameter  $\alpha$  quantifies the power-law scaling of the pdf, where the scaling regime is truncated by an exponential cutoff after the crossover value  $x_c$ . This model highlights the importance of early career development, showing that many careers are stunted by the relative disadvantage associated with inexperience. There have been many recent studies which propose measures for scientific impact (e.g. the h-index) and the distribution of impact measures among scientists [1]. However, being just a single number, the h-index cannot account for the full impact information contained in an author's set of publications. In this talk I will also discuss an alternative approach, which is to analyze the fundamental properties of the entire rank-ordered rank-citation profile  $c_i(r)$  [3]. Using the complete publication careers of 200 highly-cited physicists and 100 assistant professors, I will show remarkable statistical regularity in the functional form of  $c_i(r)$  for each physicist  $i = 1..300$ . We find that  $c_i(r)$  can be approximated by a discrete generalized beta distribution (DGBD) over the entire range of ranks  $r$ , which allows for the characterization and comparison of  $c_i(r)$  using a common framework. Since two scientists can have equivalent  $h_i$  values while having different  $c_i(r)$ , our results demonstrate the utility of a scaling parameter,  $\beta_i$ , in conjunction with  $h_i$ , to quantify a scientist's publication impact. We believe that our findings can be used in the development of a descriptive understanding of the forces behind career progress, and to discriminate between various models for career development.

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### Investigation of avalanche recurrence interval statistics in fiber bundle models and connections with earthquake recurrence times

M. Petrakis<sup>†</sup>, D. Hristopoulos

<sup>†</sup>Dept. of Mineral Resources Engineering, Technical University of Crete, Chania, Greece

Fiber bundle models (FBM) are statistical models of fracture [1] currently utilized in many areas of research [2], ranging from the study of composite materials to earthquake dynamics and seismology [3]. The FBM provide a simple, yet rich model for modelling breakdown phenomena. A FBM consists of an arrangement of parallel elastic fibers subject to an external load. The fibers have random strength thresholds that represent the heterogeneity of the medium. As the external load increases, the weakest fibers rupture and are removed from the bundle. The ensuing redistribution of the load to the surviving fibers may trigger an avalanche of breaks. Typically, the fiber rupture thresholds follow the Weibull distribution. Several studies in the seismological and physics literature use the Weibull probability distribution as a model for the recurrence intervals of large earthquakes [4-5]. To our knowledge, the evidence supporting the Weibull distribution is based on fits of the seismological data, but an understanding in terms of a simple or complex physical model is lacking. Our goal is to investigate the statistical properties of the FBM fiber burst avalanches with respect to the statistical and spatial distribution of the strength thresholds and the load transfer rules. Our focus is the distribution of recurrence intervals for avalanches that release an amount of energy exceeding a specific threshold. In particular, we aim to determine to what extent the statistics of the break avalanches can capture seismological laws (e.g., Omori-Utsu, Gutenberg-Richter), and whether the FBM can provide some insight into the statistics of recurrence intervals of earthquakes. We investigate the impact of the distributional properties of the fiber strength thresholds on the number of low-energy avalanches and the respective recurrence intervals distribution. We consider both unimodal and bimodal Weibull strength distributions. Bimodal distributions are used to simulate the inclusion of low-strength inhomogeneities (e.g., corresponding to a fault system) in the FBM. We show that for unimodal distribution of thresholds with a low Weibull modulus (implying a broad fiber failure threshold distribution) the avalanche recurrence intervals follow the Weibull distribution. In contrast, for unimodal distributions with high Weibull modulus and for bimodal threshold distributions, the recurrence intervals exhibit deviations from the Weibull function in the tails of the distribution.

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### Effects of multiplicative noise on instabilities

F. Pétrélis<sup>†</sup>, S. Aumaitre, K. Mallick, A. Alexakis and S. Fauve

<sup>†</sup>Laboratoire de Physique Statistique, Ecole Normale Supérieure, Paris

Close to the onset of an instability, fluctuations are expected to affect the dynamics of the unstable mode. In several cases, e.g. the dynamo instability, fluctuations appear as multiplicative noise in the evolution equation of the unstable field. After a brief presentation of some effects of the noise on the onset of instability, we describe its effect on the nonlinear dynamics of the field: what is called on-off intermittency. Considering several colored noises, we will identify which property of the noise controls on-off intermittency and generalize the result to several other out of equilibrium situations. This is my abstract but it seems to short to be validated by the system so I will repeat it: Close to the onset of an instability, fluctuations are expected to affect the dynamics of the unstable mode. In several cases, e.g. the dynamo instability, fluctuations appear as multiplicative noise in the evolution equation of the unstable field. After a brief presentation of some effects of the noise on the onset of instability, we describe its effect on the nonlinear dynamics of the field: what is called on-off intermittency. Considering several colored noises, we will identify which property of the noise controls on-off intermittency and generalize the result to several other out of equilibrium situations. Close to the onset of an instability, fluctuations are expected to affect the dynamics of the unstable mode. In several cases, e.g. the dynamo instability, fluctuations appear as multiplicative noise in the evolution equation of the unstable field. After a brief presentation of some effects of the noise on the onset of instability, we describe its effect on the nonlinear dynamics of the field: what is called on-off intermittency. Considering several colored noises, we will identify which property of the noise controls on-off intermittency and generalize the result to several other out of equilibrium situations. After a brief presentation of some effects of the noise on the onset of instability, we describe its effect on the nonlinear dynamics of the field: what is called on-off intermittency. Considering several colored noises, we will identify which property of the noise controls on-off intermittency and generalize the result to several other out of equilibrium situations. Close to the onset of an instability, fluctuations are expected to affect the dynamics of the unstable mode. In several cases, e.g. the dynamo instability, fluctuations appear as multiplicative noise in the evolution equation of the unstable field.

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## On the nesting of Painleve hierarchies

A. Pickering<sup>†</sup>

<sup>†</sup>Department of Applied Mathematics, Universidad Rey Juan Carlos, Spain

There is a well-known relationship between completely integrable partial differential equations and Painleve equations. This is a topic of research that continues to generate much interest. In recent years the attention of many researchers has turned to the study of Painleve hierarchies, related to the fact that completely integrable partial differential equations have their corresponding hierarchies. The questions of how to derive Painleve hierarchies, and of how to study their properties, for example, the existence of auto-Backlund transformations, are avenues of research that are proving to be very interesting. Here we discuss the phenomenon whereby solutions of members of one Painleve hierarchy are also solutions of members of a different Painleve hierarchy. Our interest is in using the properties and structures of the underlying completely integrable systems in order to explain results for their related Painleve hierarchies. We consider examples where two different Painleve hierarchies (continuous or discrete) are related to the same hierarchy of (at least) bi-Hamiltonian completely integrable equations (partial differential equations or lattice equations respectively). These Painleve hierarchies result from equations which depend on a parameter such that when this parameter is zero, inversion of one Hamiltonian operator defines one Painleve hierarchy, whereas when this parameter is nonzero, an alternative integration using the factorization of the second Hamiltonian operator defines a different Painleve hierarchy. We find that solutions of members of the first Painleve hierarchy give rise to solutions of higher-order members of the second Painleve hierarchy. Special integrals of a further Painleve hierarchy related by Backlund transformation to the second Painleve hierarchy can also be constructed. We thus explain the recent results of Sakka, who showed that solutions of members of a hierarchy based on the second Painleve equation give solutions of members of a hierarchy based on the fourth Painleve equation. We also give examples relating solutions of members of an alternative discrete first Painleve hierarchy and an asymmetric discrete first Painleve hierarchy, and solutions of a generalized discrete thirty-fourth Painleve hierarchy and a discrete first Painleve hierarchy. Special integrals of further discrete Painleve hierarchies are also given.

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## Complexity from natural to socio-economic sciences

L. Pietronero, M. Cristelli, A. Tacchella, A. Zaccaria

†ISC-CNR and Univ. Sapienza, Roma, Italy

We discuss how complexity ideas can have an important impact in socio-economic disciplines with two examples. The critics to the standard model of economics has now reached the main institutions as mentioned by the president of the ECB J.C. Trichet in a recent speech [1]. He suggests the introduction of concepts like finite rationality and strong interactions among agents, intrinsic instabilities and self-organized critical states which are typical of complexity models. On the other hand the strong fluctuations of the number of agents imply new ideas which are not contained in the physical models. We have a large variety of Agent Based Models which are all able to reproduce the main anomalies of the market fluctuations. In order to progress it is therefore necessary to increase the number and quality of the Stylized Facts. We describe the state of the art of the Agent Based Models and outline the possible developments and their implications for the problem of systemic risk [2]. We also discuss a recent new approach to the complexity of countries and products in the spirit of the recent papers by Hidalgo and Hausmann. The basic information is represented by the matrix of countries and exported products. The standard economic analysis is essentially based on the GDP but the diversification of this into a series of different products provides an additional element of fitness in the spirit of biodiversification in a fluctuating environment. In fact the idea that specialization of countries towards certain specific products is considered as optimal in the standard analysis, but this could only be valid in a static situation. The strongly dynamical situation of the world market suggests that flexibility and adaptability are also important elements. The basic idea is to introduce a Fitness parameter for each country which is able to take into consideration this effect. Such an analysis, selfconsistently also leads to a ranking of the Quality of the products. These concepts are implemented with the use of statistical concepts inspired to the page rank (Google) problem may lead to a novel classification for the fitness of the countries and the quality of products which adds new information with respect to the standard economic analysis. This information can be used in various ways. The direct comparison of the Fitness with the country GDP gives an assessment of the non expressed potential of the country. Also for each country it is possible to define the quality of the products exported and how competitive is this country with respect to the other countries which produce the same product. Finally it is possible to make a planning for the optimal development of a country by considering its potential for adding a new product.

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## Binary bets revisited: physicist point of view

E. Piotrowski<sup>†</sup>, J. Luczka, J. Sladkowski

<sup>†</sup>Institute of Mathematics, University of Białystok, Białystok, Poland

Boolean bets have profound theoretical and practical applications but often lead to intractable computational models [1]. We extend the projective covariant bookmaker's bet model to the forecasting gamblers case [2]. Let  $p_{1AB}$  be the probability of correctness of the gambler forecast for the events AB, and let  $p_{2AB}$  be the probability of her/his mistake. The gambler's knowledge is measured by the rates of return  $r_{AB} = p_{1AB} - p_{2AB}$ . Gambler's knowledge  $r_{AB}$  transforms the two element space of events into the four element space (according to the game results and correctness of the bets). The probability of correctness of forecasts (success) shifts probabilities of branching. The formula for these shifts of probabilities leads to an addition rule for the "logarithmic rates of return" of the form of the special theory of relativity velocity addition. Assuming that the rational gambler bets such stakes that her/his expected profit rate is maximal we can find some optimal allocations of capital (wagers) and the expected profit rates. In the absence of information about other bookmaker's wagers stochastic logarithmic rates completely determine the optimal stakes of the forecasting gambler. The the maximal expected profit can be decomposed into two parts the profit on due to the (un)popularity of the winning bet and the one due to the Boltzmann/Shannon entropy  $-S_{AC}$  of the forecast. The difference between these two contributions does not depend on the the sums of wagers coming from all the gamblers. The presented formal description of bookmaker bets can be used in the case of branching of events that might be created hierarchically as a binary tree with the leafs - elementary events in full analogy with the construction of tree-shaped key to compressing/decompressing Huffman code (the elementary event might be of compound boolean form) . The approach has also an interesting interpretation from the point of view of thermodynamics and special theory of relativity (econophysics). The model is universal in the sense that many kinds of financial decisions can be described as the systems based on a hierarchy of formal binary bets used, for example to reduce the number of financial instruments required to support an optimal allocation of risk [3] (e. g. trading in compound securities, the Arrow-Debreu-McKenzie model).

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## Frustration and quantum phase transition in the anisotropic antiferromagnet on a square lattice

A. Pires<sup>†</sup>

<sup>†</sup>Universidade Federal de Minas Gerais

In the last decade there has been a great interest in the physics of the two-dimensional frustrated spin-1/2 Heisenberg antiferromagnet, commonly referred to as J1-J2 model. Frustration and competition between interactions is a highly debated issue in the field of quantum magnetism. In the absence of frustration it is known that the ground state is antiferromagnetically ordered at zero temperatures. One method of enhancing fluctuations is to add diagonal bond frustration. The next nearest neighbor interaction induces frustration and break the antiferromagnetic order. The system undergoes a second order quantum phase transition at a certain value of  $J2/J1$  from the Neel to a pin liquid state. For large values of  $J2/J1$  one has a collinear ordered state, where the neighbors spins align ferromagnetically along one axis of the square lattice and antiferromagnetically along the other. Recent interest in this model comes from the discovery of superconductivity in the pnictides, and a two-dimensional Heisenberg antiferromagnet with spin  $S = 1$  and next near neighbor interaction has been used to study magnetic excitations in those compounds. New physical features, such as quantum phase transitions, due to additional terms, as for instance single ion anisotropy, are possible when  $S = 1$ . A very interesting model, in this context, is the antiferromagnet with exchange and single ion anisotropies. This model has been studied theoretically and experimentally in two and three dimensions in connection with Bose- Einstein condensation of magnons for large values of the single ion anisotropy. In our calculation we use the bond operator formalism to study quantum phase transitions in the Heisenberg antiferromagnet with exchange and single ion anisotropies on a square lattice, with nearest and nearest neighbors interaction. We calculate the staggered magnetization at zero temperature in the Neel and collinear phases as a function of the frustration  $J2/J1$ , for several values of the exchange anisotropy. We calculate the phase diagram at  $T = 0$ , showing the ordered Neel and the collinear phases and the gapped disordered phase. The disordered ground state is most likely the dimer state and can be studied using the bond operator formalism. We also present results for the ground state energy per site as a function of the anisotropy parameter  $D$ , for different values of the frustration. In the isotropic limit our calculation for the staggered magnetization is consistent with previous results.

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## Algebraic statistics and information geometry of reversible Markov chains

G. Pistone<sup>†</sup>

<sup>†</sup>Collegio Carlo Alberto, Moncalieri

The detailed balance condition and the equivalent Kolmogorov's cycle condition for Markov matrices have both an algebraic form, i.e. are homogeneous binomials in the indeterminates given by the invariant probability and the transition probabilities. For example, the detailed balance condition says that the polynomial  $\kappa(x)p_{x,y} - \kappa(y)p_{y,x}$  in the ring of polynomials with rational coefficients and indeterminates  $\kappa(x)$ ,  $P_{x,y}$ , is zero. The ideal of polynomials in these indeterminates which are zero on the algebraic variety defined by such binomials is actually a toric ideal and hence has a monomial parameterization  $P = t^A$ , where  $t$  is a set of parameters and  $A$  is a matrix with integer entries, see Pistone and Rogantin (2011) and reference therein. In this paper a Graver basis of the toric ideal is constructed, i.e. a finite (big) set of binomials, which in turn generate in the ring all elements of the ideal. A Graver basis is a universal Grbner basis, therefore it is suitable to characterize the quotient ring of polynomials. Moreover, the exponents of such a basis form a Markov basis which connects all elements of the integer kernel of  $A$  with potential combinatoric applications and applications to simulation as it was done in the seminal paper by Diaconis and Sturmfels (1998). Reversible Markov chains are a subclass of those stationary Markov chains having the property that a trajectory with positive probability has a reversed trajectory of positive probability. This is precisely the class where the entropy production is computed with the divergence between the forward and the backward probability, according to the definition used in Lebowitz and Spohn (1999). The reversible case is characterized by the zero entropy production. The information-geometric description of the relative position of the sub-manifold of reversible Markov chains in the larger class is of interest, e.g. in computing the information projection from the larger class to the restricted one. An other related issue is the use of a parameterization of the class of reversible matrices when the class of invariant probabilities is restricted. This is of interest in MCMC, e.g. the zero-variance principle in MCMC by Assaraf and Caffarel, see the review paper by Mira et al. (2010) and references therein. This is in large part a joint work with Maria Piera Rogantin, U. Genoa.

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## Chronic Myeloid Leukemia: Stochastic modeling of cancer progression under intermittent therapies

N. Pizzolato<sup>†</sup>, D. Valenti, D. Persano Adorno and B. Spagnolo

<sup>†</sup>Dipartimento di Fisica, Group of Interdisciplinary Physics, Universit di Palermo and CNISM

Chronic Myeloid Leukemia (CML) is a slowly progressing cancer of blood cells. In patients affected by CML, too many blood stem cells develop into abnormal white blood cells (leukemic cells), which build up in the blood and strongly limit the functioning of healthy white blood cells, red blood cells, and platelets. In recent years, a promising alternative to chemotherapy and radiation has emerged for the treatment of CML, with therapies based on the continuous administration of drugs, namely imatinib (Gleevec) or, more recently, dasatinib or nilotinib, which inhibit specific protein targets which are essential for the proliferation of cancer cells. Although these inhibitors are highly effective and represents the first example of a successful molecular targeted therapy, the appearance of resistance is observed in a proportion of patients. The evolutionary dynamics of cancer initiation and progression has been theoretically approached with mathematical deterministic equations [1] or with stochastic models [2,3], both using the basic idea that cancer arises when a single non-differentiated cell experiences multiple mutations [4]. In previous work we studied the appearance of resistance during continuous administration strategies of targeted drugs [5], finding that the occurrence of resistance to the therapy can be related to a progressive increase of deleterious mutations, as confirmed by clinical studies. In the present paper we build upon that framework to study the effect of time-varying dosing schedules on the evolutionary dynamics of leukemic cells. Cancer progression is modeled by applying a Monte Carlo method to simulate the stochastic behaviour of cell reproduction and death in a population of blood cells which can experience genetic mutations. The evolutionary dynamics of normal, first-mutant and cancerous cell populations is simulated as birth-death process in which the drug concentration affects the birth and death rates of both the sensitive and resistant cell populations. The patient response to the therapy is investigated by simulating a drug administration following a continuous or pulsed time scheduling. Several scenarios of the evolutionary dynamics of imatinib-treated leukemic cells are described as a consequence of the efficacy of the different modelled therapies. A permanent disappearance of leukemic clones is achieved with a continuous therapy. However, our findings demonstrate that an intermittent therapy could represent a valid alternative in patients with high risk of toxicity and that a suitable tuned pulsed therapy can also reduce the probability of developing resistance. Our results are in agreement with clinical observations.

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## Domain walls in helical magnets

V. Pokrovsky<sup>†</sup>, F. Li

<sup>†</sup>Texas A&M University

Topological defects are determined by topology of the degeneracy space of the order parameter, i.e. the manifold of its values possessing the same energy or proper thermodynamic potential [1]. Vortices are determined by topologically different curves in this space. Domain walls (are topological defects in ordered systems with violated discrete symmetry [1]. In ferromagnets domain walls separate domains with different values of magnetization pinned by anisotropy. Locally domain walls look as ferromagnets with magnetization changing slowly orientation but conserving its absolute value [3]. Domain walls in helical magnets [2] separate domains with different orientations of helices also pinned by anisotropy. However, the symmetry of order parameter allows not only discrete change of orientation, but also continuous change of phase. Therefore, the plane defects in helical magnets combine properties of domain walls (discrete symmetry) and the vortices (continuous symmetry). We demonstrate that for almost all orientations of the domain walls, they consist of chain of vortices. For few exceptional discrete orientations domain walls are free of vortices. These exceptional domain walls have much lower surface energy and grow faster than their vortex-loaded counterparts resulting in zig-zag domain wall structures. Zig-zag domain wall structures were observed recently in the experiment [4]. Even for the exceptional directions, the absolute value of the wave vector of helix is not conserved in the domain wall. The helical magnets are divided in two classes: centrosymmetric and non-centrosymmetric. In the first class the reason of the helicity is the non-nearest neighbor exchange forces (see A. Hubert in the Ref. 3. This situation is realized in several rare-earth metals (Ho for example). The helix wave vectors can be oriented along or opposite one direction. The exceptional orientation of the domain wall is perpendicular to this direction. Domain walls oriented differently consist of chains of equidistant vortices perpendicular to the wave vector direction. The reason of helicity in the non-centrosymmetric case is Dzyaloshinskii-Moria interaction. This situation is realized in several alloys like FeSi, FeGe, Fe<sub>1-x</sub>CoxSi. In crystals with cubic symmetry the wave vectors are directed either along the cube diagonals or along its four-fold axes. Exceptional orientations of the domain wall planes are bi-sectors of pairs of wave vectors, which it separates. At any other orientation domain wall consist of a chain of equidistant vortex lines parallel to the plane of the pair of wave vectors.

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## On quantum evolution equations for correlation operators

D. Polishchuk<sup>†</sup>, V.I. Gerasimenko

<sup>†</sup>Taras Shevchenko National University of Kiev

We consider the microscopic origin of non-equilibrium correlations of many-particle systems obeying quantum statistics. We deduce the von Neumann hierarchy for correlation operators that give an alternative approach to the description of the evolution of quantum states. We construct a solution of the Cauchy problem of the von Neumann hierarchy and prove an existence and uniqueness of a strong and a weak solution for corresponding initial data from the space of sequences of trace-class operators. In particular case of the absence of correlations at initial time it is established that the correlations generated by dynamics of systems are governed by the corresponding-order cumulant (semi-invariant) of the groups of operators of the von Neumann equations for density operators. Usually the evolution of infinite-particle quantum systems is described within the framework of the evolution of states by the quantum BBGKY hierarchy for marginal density operators. An equivalent approach is given in terms of the marginal correlation operators by the nonlinear quantum BBGKY hierarchy. The nonlinear quantum BBGKY hierarchy for the marginal correlation operators is rigorously derived from dynamics of correlations governed by the von Neumann hierarchy for correlation density operators. The non-perturbative solution of the Cauchy problem of the nonlinear quantum BBGKY hierarchy is constructed in case of chaos initial data and for general initial data. We prove an existence theorem of a strong and a weak solution for corresponding initial data from the space of sequences of trace-class operators. The non-perturbative hierarchy solution is represented in the form of an expansion over the particle clusters which evolution is governed by the corresponding-order cumulant of the groups of operators of the von Neumann equations. The mean field asymptotic behavior of stated solution of the nonlinear quantum BBGKY hierarchy is established. The constructed asymptotics is governed by the nonlinear quantum Vlasov hierarchy. The relationships of constructed solution of the von Neumann hierarchy both with the non-perturbative solution of the Cauchy problem of the nonlinear quantum BBGKY hierarchy for marginal correlation operators and with the non-perturbative solution of the Cauchy problem of the quantum BBGKY hierarchy for marginal density operators are established.

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### Effect of Rashba spin-orbit coupling on the entanglement between electronic spin and its spatial states in graphene

F. Pouresmaeeli†, M.M. Golshan

†physics department, Shiraz university, Iran

Graphene, a two-dimensional sheet of graphite, recently attracted much interest. Such interest originates from the fact that in graphene the so-called Pi-electrons are almost free with long mean free path, phase coherence length, etc. [1]. These properties have made it feasible to use graphene, or layers of it, to develop spintronic devices [2]. Furthermore, nanoscaled structures (wires, dots, etc.) made of graphene have been proposed for use in the development of quantum computers, quantum communication channels and so forth [3]. A major role in these applications is played by the quantum entangled states [4]. It is well established that for applications in quantum information processing, it is required that physical information contained in a composite system be either local or distributed amongst the subsystems, giving rise to entangled states. In fact, a bipartite system, in a pure ensemble, is entangled if and only if the state of the total system is unfactorizable. The entropy of the subsystems is a measure of the missing statistical information about subsystems. Since statistical information is contained in the entropy and since an increase in the entanglement would reduce the local information, it is logical to use the von Neumann entropy. It is then obvious that the entanglement of electronic spin and its spatial degrees of freedom states forms a vivid candidate for the realization and practical use of entanglement. In the present work we consider a layer of graphene in the  $x - y$  plane to which a uniform magnetic field in the  $z$ -direction is applied. Because of the fact that the Pi-electron in graphene experiences a Rashba spin-orbit coupling, the pseudospin and spin states are mixed, while the internal and pseudospin states are mixed through the magnetic field. It is then expected that the spin and the union of pseudospin and internal states become entangled. Diagonalizing the matrix representation of Hamiltonian with respect to the bare states, one obtains eight eigenvalues along with the corresponding eigenvectors. Using these results enables us to calculate the time evolution operator and consequently the density matrix as a function of time. Partial tracing of density matrix over spin states leads to the reduced density matrix for the spatial states. The degree of entanglement between the spin states and the internal excitations is then obtained with the computation of von Neumann entropy. The results indicate that the maxima in the entanglement increase as the Rashba spin-orbit coupling parameter is increased. These results are particularly useful in controlling the degree of entanglement between the Pi-electron's states.

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### Measurement error, expectations and fluctuations in macroeconomic output

S. Raghavendra†, P. Piironen

†J.E. Cairnes School of Business and Economics, National University of Ireland, Galway

The analysis of the role of expectations within the context of endogenous and exogenous business cycle models is one of the central concerns in Macroeconomics. In the literature various authors have specified different expectation rules and studied their impact on the variability of output. The resulting dynamics in these models are produced by the errors in forming the expectations about the steady-state values or steady-state growth rates. There exists a huge literature on these kinds of learning rules, where the agents with quasi, semi and fully bounded rationality try to estimate the steady-state values, and the resulting errors in forming expectations lead to fluctuations in output. In this paper we take a different approach - here in this paper, instead of using the agents' behavioral error in learning the steady-state values we consider the impact of the measurement error in the GDP estimates and its influence on output dynamics. That is, we study the effect of the measurement errors, which feeds into future expectations, on the variability in the macroeconomic output. Such an approach introduces a temporal variation in the steady-state value (a stochastic process), which we think is a useful way to model fluctuating growth in the context of a business cycle model models. Central Statistical Organizations in many countries bring out output growth estimates and revise their projections up to 4-6 quarters (Quarterly National Accounts in case of Ireland). Revisions are made to estimates of the components of GDP for a number of reasons including: availability of firmer and more complete data, changes in methodology, correction of errors in the source data and updating the base period used for constant price estimates. In the literature, there are very few attempts to systematically analyze the revisions of the estimates of GDP, not just in Ireland but also at an international level. In other words, what is the macroeconomic cost of revisions to the expected growth rates by CSO and the policy institutes? Surely, the revisions change human agents' expectations about the future, no matter how they form those expectations. One such analysis was done recently by Patrick Quill (2008) and the objective of his paper is to show that the '...the mean of total revision to GDP growth rate is positive but not significantly different from zero'. In this paper, in contrast to this statistical debate, we are interested in analyzing how the revisions to the macroeconomic statistics could permeate into agent's future expectations and cause fluctuations in output. We also discuss possible implications of our model to the 'emergent macroeconomics' literature.

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## Integrable Maps from classical Gaudin models

O. Ragnisco<sup>†</sup>, F. Zullo

<sup>†</sup>Universit Roma Tre, Italy

The Gaudin models describe completely integrable long range spin-spin systems, both at the classical and at the quantum level. They were first introduced by Gaudin [1] as the anisotropic, integrable generalization of what today is called the xxx, or isotropic, Gaudin model; the results of the analysis were the construction of the xxz model for the partially anisotropic case, and of the xyz model for the fully anisotropic case. The Lax matrices depend on the spectral parameter respectively through rational, trigonometric and elliptic functions. The Poisson structure of the models can be specified in the framework of the r-matrix approach; Sklyanin showed how to obtain both the Lax matrices and the r-matrices of the models by a limiting procedure on the lattice Heisenberg magnet. Obviously the integrability structures of the trigonometric and rational cases can be also obtained as special cases of the corresponding structures for the more general elliptic model. From the point of view of separation of variables, functional Bethe ansatz and quantum inverse scattering method, a lot of results appeared in the late 80s-early 90s. On the other hand, a large number of works have pointed out very interesting connections between the Gaudin models and various branches of physics. For a recent list of the subjects involved the reader can see for example [2]. In [3] the authors gave the Backlund transformations for the rational Gaudin model in the framework of a research program started some years before by Kuznetsov and Sklyanin on the study of such transformation for finite dimensional integrable systems. It was shown how the two-parameter family of transformations therein obtained could be seen as a time discretization of a family of flows and that the interpolating Hamiltonian flow encompasses all the commuting flows of the model. The properties of the transformations found there are noteworthy: in fact the maps are (i) explicit, (ii) symplectic, (iii) they preserve all the integrals of the continuous model, (iv) they possess the so called spectrality property. Furthermore they enjoy the so-called permutability property. An analogue construction for the trigonometric case is reported in [4,5]. In the present talk we complete the picture describing the Backlund transformations and the corresponding time-discretization for the elliptic Gaudin model. After revising the main features and the integrability structures of the continuous-time model, we construct the dressing matrix, give the explicit form of the transformations, that can be viewed as bi-rational maps for fixed initial conditions, and discuss their symplectic nature.

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## Diffusion eigensystems of stochastic processes

F. Raischel<sup>†</sup>

<sup>†</sup>CFTC, Universidade de Lisboa

We present a novel method for evaluating the stochastic properties of multi-dimensional Langevin-like systems. Applications to environmental research will be given. Many physical processes have to be modeled taking into account stochastic contributions, which usually encompass all contributions to the system that cannot be resolved and independently measured. An adequate description of these systems consisting of both a deterministic and a stochastic term is by means of a Langevin equation of the stochastic process, or the Fokker-Planck equation for the corresponding PDF. We employ standard methods for the extraction of drift and diffusion coefficients of these equations from measured time series, which have already been applied to a wide range of physical and statistical problems, such as turbulence, financial indices, surface profiles and climate research.[1] An interesting question arises if a multi-dimensional process exists, which can be described by a system of coupled Langevin-like equations. When addressing stochastic multi-dimensional systems it is typically difficult to ascertain which are the best variables to describe their evolution. Will it be possible to increase the predictability of the system and to extract more information from the system than from the sum of its constituents? We confirm this possibility by presenting a transform of the original process using the eigensystem of the diffusion matrix. It will be shown that if the transform is applicable, at least one of the coordinates has reduced stochasticity in the new set of coordinates. An analogy exists to the so-called Principal Component Analysis (PCA), which however exclusively considers the variance of the process. We will present two examples of a successful application of this method[2,3]. First, we investigate a system of concentration measurements in two nearby locations for which only a very vague deterministic contribution is visible. Decoupling of the measurements by means of the stochastic eigenvectors reduces the stochasticity and increases the visibility of the deterministic contribution. As another example, we present an analysis of data generated through simulation of a Hopf bifurcation overlaid with a delta-correlated Gaussian noise. We extract the information defining the Langevin process, determine the stochastic eigensystem and find a transform that enables us to learn more about the principal directions of dynamical noise involved.

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## Statistical mechanics of simplicial complexes: Combinatorial laplacian and spectral entropy

M. Rajković†, S. Maletic

†Institute of Nuclear Sciences VInca, Belgrade

Topological, algebraic and combinatorial properties of simplicial complexes which are constructed from networks (graphs) are examined from the aspect of statistical mechanics. We show that basic statistical features of random networks, scale free networks and networks with exponential degree distribution are preserved by topological invariants of simplicial complexes. Combinatorial invariants (Q-vector, f-vector, h-vector etc.) preserve scaling properties of the networks and the corresponding simplicial complexes. New topological invariants are introduced as persistent homology and persistent cohomology reflecting homological and cohomological properties respectively. Simplicial complexes may be constructed from undirected or directed graphs (digraphs) in several different ways. Here we consider two of them: the neighborhood and the clique complex. We show how a new branch of statistical mechanics may be developed which we call statistical mechanics of simplicial complexes. We also explore the topological properties of independent sets corresponding to several generic types of complex network (a set is independent if it is a clique and hence a simplex of graph's complement so that it is actually a set of vertices such that for every two vertices in this set there is no edge connecting the two) and their statistical and scaling features. Construction and analysis of weighted simplicial complexes obtained from weighted graphs (networks) is also presented with special emphasis on the method for estimation of optimal slicing parameter. Of special interest are the properties of eigenvalues and eigenfunctions of non-normalized combinatorial Laplacian. Combinatorial Laplacian of simplicial complex, in contrast to graph Laplacian, is defined in higher dimensions thus enabling extraction of more information from the underlying graph (network) and additional invariants in higher dimensions. Spectral entropy, obtained from the combinatorial Laplacian, reflects many important properties of a simplicial complex (complex network) and thus has important practical applications. We also derive an entropy of a simplicial complex from a purely combinatorial aspect, called combinatorial entropy, and explore its properties. In order to illustrate the advantages of simplicial complex approach over standard graph (networks) approach we present results of the analysis of several biological and social networks. Relationship between dynamics and structure on simplicial complexes is further explored both in the context of classical and quantum formalism and its relationship with exterior calculus is presented.

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## Shared information in stationary states at criticality

V. Rittenberg, F.C. Alcaraz

†Bonn University

In recent years a lot of attention has been paid to the understanding of the entanglement properties of ground-state wavefunctions of hermitian Hamiltonians describing one-dimensional quantum chain spin systems. For a pure state if  $C = A+B$  is a bipartition, the von Neumann entanglement entropy  $SvN$  is defined as:  $SvN(A) = SvN(B) = Tr(ZA \ln ZA)$ , where  $ZA = TrB(Z)$ ,  $Z$  is the density matrix related to the ground-state wavefunction. If  $C$ ,  $A$  and  $B$  have  $L$ ,  $l$  and respectively  $L/l$  sites, for a large system and large subsystems,  $S_q$  converges to a constant if the system is not critical. If the system is gapless and conformal invariant, one gets  $SvN(l, L) = a \ln l + C$  ( $L \gg l \gg 1$ ), and the finite-size scaling behavior  $SvN(l, L) = a \ln \sim LC + C$ ,  $\sim LC = L \sin(\pi l/L)/\pi$ , where  $a = c/6$  for an open system ( $c$  is the central charge of the Virasoro algebra).  $C$  is a nonuniversal constant. We consider a bipartition of a large one-dimensional interacting classical system  $C$  and study the shared information between the subsystems  $A$  and  $B$  due to large scale correlations. Such systems can be described by probability distribution functions (PDF) of stationary states of stochastic models. They are ket ground-states wavefunctions written in a special basis, of non hermitian and in general, nonlocal quantum chain Hamiltonians which give the time evolution of Markov processes. The essential difference between the classical and the quantum cases is that in the former case the components of the ket wavefunctions are non-negative real numbers which, when properly normalized, represent the probabilities to find in the stationary state, various configurations of the system. We present several estimators of shared information which have neat properties. They vanish if the subsystems are separated. Their values increase if the shared information increases. This happens if the systems are more constrained. It turns out that, in many cases, the estimators have a behavior which copies the one of the entanglement entropy. For example, if the PDF describes a system with a finite correlation length, the estimators stay finite for large systems. The area law is respected. If the system is critical one gets, in general, logarithmic corrections to the area law. We are going to discuss in detail these corrections. In stationary states, the existence of divergent correlation lengths does not imply conformal invariance. Therefore one can have a larger variety of critical systems. We are going to show how the estimators behave in several cases. The configuration space in which the estimators are defined is given by Dyck paths. Dyck paths appear in different contexts. They are used in polymer modelling. They can also represent a one-dimensional interface.

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### Stationary states of the Hamiltonian mean-field model and a renewal stochastic process analog with a phase transition along time evolution

A. Robledo<sup>†</sup>, F. Baldovin, J. Velázquez

<sup>†</sup>Instituto de Física, Universidad Nacional Autónoma de México, Mexico City, Mexico

The Hamiltonian mean field (HMF) model is a key reference in the study of statistical-mechanical and non-equilibrium properties of long-ranged interacting systems [1]. This model combines simplicity in its definition and unhindered access to many of its properties via analytical and numerical simulation methods, yet it exhibits puzzling behaviors that have remained only partially explained. Prominent amongst these is the unusually long lifetime of metastable states occurring below the critical temperature that separates uniform and nonuniform (clustered) phases. To address this issue we obtain the stationary properties for the HMF model and learn about the underlying cause of its enduring subcritical-temperature uniform metastable states. Our study makes use of two complementary methods, Widom's particle insertion [2] and free energy density functionals, to determine analytically the spatial structure and stability of the entire family of stationary states of the model and the requisites that the thermal fluctuations about them must have to be capable of terminating these. These constraints are mainly of a geometrical nature and distinguish the model's kinetics of phase change from ordinary spinodal decomposition. Then, as an illustration of interdisciplinary applications of statistical physics, we consider a renewal stochastic process with correlated events analogous to the HMF model that is representative of agents that perform a repeated task with an associated outcome, such as an opinion poll [3]. We approach the analysis of renewal stochastic processes generated by non-independent events from the perspective that their basic distribution and associated generating functions obey the statistical-mechanical structure of systems with interacting degrees of freedom. Based on this fact we observe that the density distribution for the occurrence of the  $n$ -th event at time  $t$  is analogous to a partition function, of a "microcanonical" type for  $n$  "degrees of freedom" at fixed "energy"  $t$ . We obtain a set of four partition functions of which that for the generating function described by  $z$ -transform and Laplace transform variables, conjugate to  $n$  and  $t$ , respectively, plays a central role. These partition functions relate to each other in the customary way and in accordance to the precepts of large deviations theory, while the entropy, or Massieu potential, derived from the  $n$ -th event distribution satisfies an Euler relation. When a sequence of many events takes place in a sufficiently short time the process exhibits clustering of the outcome, but for larger times the process resembles that of independent events. The two regimes are separated by a second order transition.

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### A generalization of the cumulant expansion. Application to a scale-invariant probabilistic model

A. Rodríguez <sup>†</sup>, C. Tsallis

<sup>†</sup>Universidad Politécnica de Madrid

As well known, cumulant expansion is an alternative way to moment expansion to fully characterize probability distributions provided all the moments exist. The reason why cumulants are important within a thermodynamical context relies of the fact that, contrary to moments, they preserve extensivity. For instance, the total specific heat, which has to be extensive, appears as a second-order cumulant (a specific combination of first and second order of the moments of the hamiltonian). Nevertheless, a variety of physical systems exist for which not all the moments of their probability distributions are defined, so the cumulant expansion is no longer valid. For this cases, the so-called escort mean values (or  $q$ -moments) have been recently proposed to characterize such probability densities [1]. The purpose of this contribution is to introduce a new mathematical object, namely, the  $q$ -cumulants, which, in analogy to the cumulants, provide an alternative characterization to that of the  $q$ -moments for the probability densities.  $q$ -cumulants work within the frame of nonextensive statistical mechanics and are obtained making use of the so-called  $q$ -exponential and  $q$ -logarithm functions, defined in the aforementioned theory. They are related with the so-called  $q$ -characteristic function, which in turn is the  $q$ -Fourier transform [2] of the probability distribution. To illustrate the technical details of the procedure, we apply this new scheme to further study a recently proposed family of scale-invariant discrete probabilistic models [3,4], having  $q$ -Gaussians as limiting probability distributions. The model consists of a set of  $N$  equal, long-range-correlated binary random variables satisfying that the functional form of the marginal distribution of the  $N$  variables system coincides with corresponding joint probability distribution of the  $N-1$  variables system (scale invariance condition). Despite the presence of strong correlations, the appropriate entropy to describe this models is the Boltzmann entropy. We obtain the scaling laws of the  $q$ -moments as well as the  $q$ -cumulants of this family of models and express the corresponding scaling exponents as a function of  $q$ . We believe that this generalization of the cumulant expansion can be of interest for analytically discussing long-range-interacting systems for which moments above a given order typically diverge.

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## Direct algebraic decoration transformation for spin models

O. Rojas<sup>†</sup>, S.M. de Souza

<sup>†</sup>Departamento de Ciências Exatas, Universidade Federal de Lavras

Exactly solvable models is one of the most challenging topics in statistical physics and mathematical physics. Statistical physics models in general cannot be solved analytically, but only numerically. For example, Ising models with spin-1/2 or higher under external magnetic field are challenging current issues. Exact solutions were obtained only for a very limited cases. After the Onsager solution for the two-dimensional Ising model, several attempts to solve other similar models were performed. On the other hand, several decorated Ising models have been solved using the well-known decoration transformation presented in reference [1], which has been recently generalized in reference [2] for arbitrary spin and for any mechanical spin, such as the classical-quantum spin model. This transformation has been widely used in the literature and, in some cases, it has been applied in several steps that introduce a number of intermediate parameters. The decoration transformation can also be applied to classical-quantum (hybrid) spin models, i.e. Ising-Heisenberg models. Several quasi-one dimensional models such as the diamond-like chain have been widely investigated, as well as two-dimensional lattice spin models by using the decoration transformation approach[3]. Another interesting application of decoration transformation was also investigated when considered a delocalized interstitial electrons on diamond-like chain and also investigated the magnetocaloric effect in a kinetically frustrated diamond chain. Recently, the decoration transformation approach has been also applied to spinless interacting particles, thus showing the possibility of application to interacting electron models[4]. In this work we present a direct generalized transformation for a mixed or decorated spin model onto a uniform spin model, in which the main difference to the aforementioned generalized transformation[1,2] is that there is no step by step transformation. The seminal idea of decorated spin model transformation of type star-star already was emphasized and used in a particular case by Baxter [5]. In order to introduce a direct transformation of decorated spin model onto a uniform spin-1/2 model, we will follow the basic idea used by Baxter [5]. The advantage of this transformation is to perform a direct mapping of a decorated spin model onto another effective spin thus simplifying algebraic computations by avoiding the proliferation of unnecessary iterative transformations and parameters that might otherwise lead to transcendental equations.

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## A first principle derivation of the stress tensor of a discrete system

G. Rossi<sup>†</sup>, S. Morante, M. Testa

<sup>†</sup>University of Roma, Tor Vergata

Numerical simulations represent today an invaluable tool of investigation in many research areas ranging from Field Theory and Quantum Chemistry to applications in turbulence and in the study of models of biologically interesting systems. The key feature of the numerical approach, which makes it so powerful, lies in the fact that it allows a formulation of the physical problem in terms of the fundamental degrees of freedom of the system under consideration. In many important cases like spin systems, QCD, fluid dynamics and a fairly large number of important biological systems (e.g. cell membranes, micelles, short peptides, etc.) numerical simulations have gone a long way and quite accurate results are nowadays available. There are, however, a few key aspects of any simulation strategy that need to be discussed and carefully dealt with as they largely affect the reliability of the system model description and the way physical quantities are computed. These are the choice of the fundamental degrees of freedom that are used to describe the system at the microscopic level, the nature (classical or quantum mechanical) of their specific interactions and the kind of boundary conditions assigned to the system. In view of all these subtleties and technical problems it is of the utmost importance to have general guiding principles, allowing a clear-cut resolution of possible ambiguities and leading to consistent formulae valid in different simulation settings. Our point of view is that such guiding principles must be found in the methods of Statistical Mechanics. In this talk we wish to illustrate the power of this philosophy by providing a rigorous construction of the stress tensor of a discrete system. We present a simple and general derivation of the formula which gives the local expression of the stress tensor of a system, with arbitrary (short-range) interactions and boundary conditions, in terms of its microscopic degrees of freedom. We prove that the stress tensor components can be interpreted as the set of Lagrangian multipliers which need be introduced to enforce the conditions relating point particle displacements to the body local deformation tensor. Our construction works in the canonical as well as the micro-canonical ensemble and it is applicable both in a classical and a quantum mechanical setting. We also prove that the conservation of the stress tensor, expressing the system local equilibrium condition, is a consequence of the invariance of the partition function under local canonical transformations leaving invariant the phase-space measure (Noether theorem). The old question of the uniqueness of the stress tensor formula is positively answered.

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### Self-consistent steady states in Hamiltonian mean field dynamics

S. Ruffo†, P. de Buyl, D. Mukamel

†Florence University and ENS Lyon

Systems with long-range interactions, like gravitational, charged and dipolar systems, can be made extensive, but are intrinsically non additive. The violation of this basic property of thermodynamics is the origin of many intriguing phenomena at statistical equilibrium. However, even more interesting is the process of relaxation to equilibrium, which is characterized by time-scales that diverge with system size. It has been shown for specific models that the system remains trapped in Quasi-Stationary States (QSS) that persist on times that diverge algebraically with the number of particles  $N$ . Different theoretical approaches have been developed to justify the presence of these states, all of them based on kinetic equations where mean-field effects play a relevant role, well known is the approach based on Lynden-Bell's entropy. We introduce a model of uncoupled pendula, which mimics the dynamical behavior of the Hamiltonian Mean Field (HMF) model. As for the HMF model, this simplified integrable model is found to obey the Vlasov equation and to exhibit QSS, which arise after a collisionless relaxation process. Both the magnetization and the single particle distribution function in these QSS can be predicted using Lynden-Bell's theory. The existence of an extra conserved quantity for this model, the energy distribution function, allows us to understand the origin of some discrepancies of the theory with numerical experiments. It also suggests us an improvement of Lynden-Bell's theory, which we fully implement for the zero field case. Studies of the relaxation time in the HMF model, have shown that it diverges as  $N^\gamma$  for large  $N$ , with  $\gamma \simeq 1.7$  for some initial conditions with homogeneously distributed particles. We propose a method for identifying exact inhomogeneous steady states in the thermodynamic limit, based on analysing models of uncoupled particles moving in an external field. For the HMF model, we show numerically that the relaxation time of these states diverges with  $N$  with the exponent  $\gamma \simeq 1$ . The method, applicable to other models with globally coupled particles, also allows us to evaluate exactly the stability limit of homogeneous steady states and to approximately relate the initial distribution to its correspond.

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### Time-evolving statistics of chaotic orbits of conservative maps

G. Ruiz†, T. Bountis, C. Tsallis

†Universidad Politecnica de Madrid

As is well-known, invariant closed curves of area-preserving maps present complete barriers to orbits evolving inside resonance islands in the two-dimensional phase space. Outside these regions, there exist families of smaller islands and invariant Cantor sets, to which chaotic orbits are observed to “stick” for very long times. Thus, at the boundaries of these islands, an ‘edge of chaos’ develops with vanishing or very small Lyapunov exponents, where trajectories yield quasi-stationary states (QSS) that are often very long-lived [1]. We study chaotic orbits of conservative low-dimensional maps and present numerical results showing that the probability density functions (pdfs) of the sum of  $N$  iterates characterized by non-vanishing Lyapunov exponents, aiming to understand the connection between their intricate phase space dynamics and their time-evolving statistics. Our pdfs do not converge to a single shape but pass through several ones. It is the purpose of this work to explore the statistics of regions where the motion explores domains with intricate invariant sets and the maximal (positive) Lyapunov exponent is very small. In some cases where the chaotic layers are thin and the (positive) maximal Lyapunov exponent is small, long-lasting quasi-stationary states (QSS) are found, whose pdfs appear to converge to q-Gaussians associated with nonextensive statistical mechanics. More generally, however, as  $N$  increases, the pdfs describe a sequence of QSS that pass from a q-Gaussian to an exponential shape and ultimately tend to a true Gaussian, as orbits diffuse to larger chaotic domains and the phase space dynamics becomes more uniformly ergodic. We have found, in several examples of the Ikeda [2] and McMillan [3] area preserving maps, as well as one case of a 4-dimensional symplectic accelerator map [4], that q-Gaussians approximate well quasi-stationary states (QSS), which are surprisingly long-lived, especially when the orbits evolve in complicated chaotic domains surrounding many islands. This may be attributed to the fact that the maximal Lyapunov exponent in these regions is small and the dynamics occurs close to the so-called “edge of chaos” where stickiness effects are important near the boundaries of these islands. On the other hand, in simpler-looking chaotic domains (surrounding e.g. only two major islands) the observed QSS passes, as time evolves, from a q-Gaussian to an exponential pdf and may in fact become Gaussian, as the number of iterations becomes arbitrarily large. Even in these cases, however, the successive QSS are particularly long-lasting, so that the Gaussians expected from uniformly ergodic motion are practically unobservable.

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## Two types of auto-Backlund transformation for integrable partial differential equations

P. Ruiz Gordoa<sup>†</sup>, A. Pickering, J.M. Conde

<sup>†</sup>Department of Applied Mathematics, Universidad Rey Juan Carlos, Spain

The existence of auto-Backlund transformations is generally understood as being a characteristic feature of integrable equations. The nature of such transformations changes depending on whether the equation under consideration is a partial differential equation (PDE) or an ordinary differential equation (ODE). Two differences between PDE and ODE auto-Backlund transformations are the dependence of the former on the spectral parameter and the fact that in the latter changes in the coefficients of the equation are allowed for. We have recently been interested in making use of properties of completely integrable systems in order to investigate properties of integrable ODEs, and in particular, of Painleve hierarchies. In the work presented here the idea is to lift these techniques back up to the level of completely integrable systems in order to investigate the properties of these last. For a particular PDE that, in many respects, is similar to the second Painleve equation, we are able to construct auto-Backlund transformations of both ODE and PDE type. We believe this is the first time that such an ODE type auto-Backlund transformation has been given for a PDE. Our derivation of ODE type auto-Backlund transformations is in fact valid for a wide class of equations that include both integrable and nonintegrable ODEs and PDEs. It is this fact that allows us to arrive at some general conclusions about the relationship between the existence of ODE type auto-Backlund transformations and integrability. The application of our results also allows us to reduce the order of the so called KdV6 equation which has recently attracted much interest. For the equation under study we further investigate its integrability properties. Its Lax pair and Darboux transformation are presented, together with a PDE type auto-Backlund transformation. The nonlinear superposition formula is constructed; it consists of an algebraic equation that allows us to build a new solution of the equation in terms of three previously known solutions. One and two soliton solutions are constructed whose velocity depends on the functions of  $t$  that appear as coefficients in the equation, a fact that allows a rich variety of exact solutions. The Lie symmetry analysis of the equation is also performed and the results for the travelling wave solutions are compared to those obtained from the Lax pair and the Darboux transformation.

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## Cooling by heating

B. Rutten<sup>†</sup>, B. Cleuren, C. Van den Broeck

<sup>†</sup>U Hasselt

The conversion of solar energy into electrical power is very common nowadays. However, it is less obvious that solar radiation can be employed for refrigerating purposes. Based upon previous work by Sols et al. [1], we present a minimal model of a refrigerator, which is operated by direct illumination with solar radiation. The model consists of two contacts (electron reservoirs, labeled "left" and "right"), which are connected to each other by two quantum dots. These quantum dots allow for an exchange of electrons and heat between the two contacts. The electron movement between the two quantum dots is directly mediated by the solar energy, which is the basic driving force. The basic principle of the refrigerator can be explained as follows: extracting (inserting) electrons above (below) the Fermi level results in an effective cooling of the contact. The purpose of the quantum dots is then precisely to select the appropriate electrons. The extracted (inserted) electrons are transported to the other contact, which is heated (cooled). Based on previously obtained analytic results for the steady-state current in a two-level dot [2], it is possible to calculate several interesting quantities for the refrigerator. In order to prevent charging of the contacts, which would hinder operation, the chemical potential (Fermi level) is chosen such that the electric current is zero. By this method we calculated the steady-state electric currents, heat currents and the resulting coefficient of performance, optimizing the energy levels of the quantum dots to achieve the maximal cooling effect. The model is also analyzed within a thermodynamical framework. The familiar bilinear form for the entropy is recovered and in the linear regime, the Onsager symmetry was checked. When the determinant of the Onsager matrix becomes zero, the two acting heat currents ( $Q_{SUN}$ ,  $Q_{RES}$ ) become linearly dependent, which is known as the strong coupling condition. This resulted in a coefficient of performance determined solely by the energy levels of the device, irrespective of temperature. An arbitrarily large coefficient of operation can thus be obtained in principle. But choosing higher efficient energy levels will correspond with a loss of current, resulting in an upper limit in practical operation. An interesting question is the maximal temperature difference that can be maintained between the contacts. In order to get a realistic estimate the Stefan-Boltzmann law has to be included. For this purpose we took the contacts to be cubes with a side of 10 nm. This led to a maximal achievable temperature difference of about  $8 \cdot 10^{-6} K$  at an ambient temperature of 10K.

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## On functionals for mean values of observables in scaling limits

T. Ryabukha†

†University of Warsaw, Poland and Institute for Mathematics of NAS of Ukraine

Significant progress in studying the dynamics of infinitely many particles has been achieved in the last decades [1, 2, 3]. It is described either by the BBGKY hierarchy of integral differential equations for states or by the dual BBGKY hierarchy of PDEs for observables. Till now the rigorous results concerning with existence of solutions of these hierarchies are known for particular instances of interaction potentials. A solution of the initial value problem for the BBGKY hierarchy in the form of a series expansion as the result of integrating the perturbation series in time variables was first constructed by D. Petrina for a one-dimensional system of particles interacting via a short-range hard-core potential for initial data close to the equilibriums. The divergence of integrals over the configuration variables in every term of the expansion is a typical problem that complicates constructing the solution for in finite-particle systems. The method of an interaction region was proposed by D. Petrina as one approach for circumventing this obstacle. The further results of the authors C. Cercignani, R. Illner, M. Pulvirenti, H. Spohn, D. Petrina and V. Gerasimenko for three-dimensional elastic-sphere systems were based on constructing a solution of the BBGKY hierarchy in the form of a perturbation series. The representation of a solution of the initial value problem for the BBGKY hierarchy was recently obtained in the form of an expansion over particle clusters whose evolution is determined in terms of the corresponding order cumulant of the evolution operators for finitely many particles. The method of an interaction region for this representation was applied as well [4]. In addition we constructed a solution of the dual hierarchy for observables in suitable functional spaces [5]. We used the obtained results for proving existence of functionals determining the mean values of observables for such systems. These results can also be applied in the case of the so-called intensive thermodynamic observables (e.g., the number of particles). We study some scaling limits of constructed functionals.

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## Core-softened system with attraction: Trajectory dependence of anomalous behavior

V. Ryzhov†, Yu.D. Fomin, V.N. Ryzhov, E.N. Tsiok

†Institute for High Pressure Physics, Russian Academy of Sciences, Moscow Region, Russia

It is well known that some liquids (for example, water, silica, silicon, carbon, phosphorus, and some biological systems) show an anomalous behavior in the vicinity of their freezing lines. The water phase diagrams have regions where a thermal expansion coefficient is negative (density anomaly), self-diffusivity increases upon compression (diffusion anomaly), and the structural order of the system decreases with increasing pressure (structural anomaly). It is reasonable to relate this kind of behavior to the orientational anisotropy of the potentials, however, a number of studies demonstrate water-like anomalies in fluids that interact through spherically symmetric potentials. As it was discussed in many works (see, for example, the review [1]) the presence of two length scales in the core-softening potential may be the origin of water-like anomalies. In those thermodynamic regimes where the two length scales are both partially effective, a system of particles behaves, in many respects, as a mixture of two species of different sizes [2]. This leads to the existence of two competing local structures. The evolution of these structures under changing the thermodynamic conditions can result in the anomalous behavior. In the present talk we report a molecular dynamics study of the core-softened system and show that the existence of the water-like anomalies in this system [2-5] depends on the trajectory in  $P-\rho-T$  space along which the behavior of the system is studied. For example, diffusion and structural anomalies are visible along isotherms, but disappears along the isochores and isobars, while density anomaly exists along isochors. It should be noted that it may be no signature of a particular anomaly along a particular trajectory, but the anomalous region for that particular anomaly can be defined when all possible trajectories in the same space are examined (for example, signature of diffusion anomaly is evident through the crossing of different isochores. However, there is no signature of diffusion anomaly along a particular isochor). It is shown that if the potential has an attractive part, the diffusion anomaly shifts to higher temperatures. Moreover, in contrast to the case of the purely repulsive potential, the diffusion anomaly appears for the density dependence of the diffusion along isobars. It is shown that the validity of the Rosenfeld entropy scaling relation for the diffusion coefficient also depends on the trajectory in the  $P-\rho-T$  space along which the kinetic coefficients and the excess entropy are calculated.

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## Nanoscale structures and giant Nernst effect below the pseudogap in under-doped high- $T_c$ superconductors

M. Saarela<sup>†</sup>, F.V. Kusmartsev

<sup>†</sup>University of Oulu

Observations of a large Nernst signal in an extended region above the critical temperature  $T_c$  in hole-doped cuprates provides an accurate tool to study fluctuations in the pseudogap region. We show that the nanoscale structures in these layered superconductors may be caused by localized negative charges, which can collect a cluster of charge carriers around them. Our results support the scenario that superfluidity vanishes because long-range phase coherence is destroyed by a charge density wave instability induced by localized charges and the over-screening of the long-ranged part of the Coulomb interaction, which is enhanced due to decreasing carrier density. [1] When the carrier density is low enough localized charges begin to trap hole charge carriers and form bound states of clusters of charge carriers, which we call Coulomb bubbles. These bubbles are embedded inside the superconductor and form nuclei of the new insulating state. The growth of a bubble is terminated by the Coulomb force and each of them has a quantized charge and a fluctuating phase. When clusters first appear they are covered by superfluid liquid due to the proximity effect and invisible. However when the carrier density decreases the size of bubbles increases and the superconducting proximity inside them vanishes. The insulating state arises via a percolation of these insulating islands, which form a giant percolating cluster that prevents the flow of the electrical supercurrent through the system. We present an effective Hamiltonian, which takes into account the Jahn-Teller distortion of the apical oxygen and provides two bands, the band of charge carriers and the localized impurity band causing the clustering. Our results are consistent with the two-component picture for cuprates deduced earlier by Gorkov and Teitelbaum [2] from the analysis of the Hall effect data and ARPES spectra. In that picture density of charge carriers increases by doping and when the temperature increases by activation as trapped holes are released from the Coulomb bubbles. The pseudogap temperature is defined as the temperature where the number of activated holes becomes larger than the number of doped holes. Coulomb bubbles are also observed as inhomogeneities in Scanning Tunneling Microscope (STM) experiments [3].

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## 2d Criticality in a 3d Ising model.

A.A. Saberi<sup>†</sup>

<sup>†</sup>Institute for Research in Fundamental Sciences (IPM), Iran.

The fractal structure and scaling properties of a 2d slice of the 3d Ising model is studied using Monte Carlo techniques. The percolation transition of geometric spin (GS) clusters is found to occur at the Curie point, reflecting the critical behavior of the 3d model. The fractal dimension and the winding angle statistics of the perimeter and external perimeter of the geometric spin clusters at the critical point suggest that, if conformally invariant in the scaling limit, they can be described by the theory of Schramm-Löwner evolution ( $SLE_\kappa$ ) with diffusivity of  $\kappa = 5$  and  $16/5$ , respectively, putting them in the same universality class as the interfaces in 2d tricritical Ising model. It is also found that the Fortuin-Kasteleyn (FK) clusters associated with the cross sections undergo a nontrivial percolation transition, in the same universality class as the ordinary 2d critical percolation. The fractal structure and scaling properties of a 2d slice of the 3d Ising model is studied using Monte Carlo techniques. The percolation transition of geometric spin (GS) clusters is found to occur at the Curie point, reflecting the critical behavior of the 3d model. The fractal dimension and the winding angle statistics of the perimeter and external perimeter of the geometric spin clusters at the critical point suggest that, if conformally invariant in the scaling limit, they can be described by the theory of Schramm-Löwner evolution ( $SLE_\kappa$ ) with diffusivity of  $\kappa = 5$  and  $16/5$ , respectively, putting them in the same universality class as the interfaces in 2d tricritical Ising model. It is also found that the Fortuin-Kasteleyn (FK) clusters associated with the cross sections undergo a nontrivial percolation transition, in the same universality class as the ordinary 2d critical percolation. The fractal structure and scaling properties of a 2d slice of the 3d Ising model is studied using Monte Carlo techniques. The percolation transition of geometric spin (GS) clusters is found to occur at the Curie point, reflecting the critical behavior of the 3d model. The fractal dimension and the winding angle statistics of the perimeter and external perimeter of the geometric spin clusters at the critical point suggest that, if conformally invariant in the scaling limit, they can be described by the theory of Schramm-Löwner evolution ( $SLE_\kappa$ ) with diffusivity of  $\kappa = 5$  and  $16/5$ , respectively, putting them in the same universality class as the interface.

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### Quantization of nonequilibrium nonstationary system

M. Saikhanov†

†Complex Research Institute of the Russian Academy of Sciences

The possibility of theoretical description, including states far from equilibrium, based on quantization of irreversible processes in nonequilibrium systems is demonstrated. The theoretical model of nonequilibrium system is constructed based on analysis of energy spectrum having discrete-continuous (layered) structure [2]. The combining of particles of nonequilibrium system in groups with close energy states (energy layers) is realized through their selective interaction and has a local character. The simultaneous macroscopic description on local (quasi-equilibrium subsystem) and global (all nonequilibrium system) scale is achieved by constructing functional of the total production of entropy. (1) It contains the local parameters of deviations of thermodynamic forces and rates of their changes; - numbers irreversible process and the quasi-equilibrium subsystems; - the equilibrium values of thermodynamic forces. These parameters are entered as statistical averages over the quantum energy states of local-equilibrium subsystem, the energy spectrum of which is identified as a quasi-continuous. The introduction of the second group of parameters allows adequate considerate the nonstationary aspect of nonequilibrium system, which cannot be carried out under local-equilibrium thermodynamics [1]. The kinetic processes in a strongly nonequilibrium system are realized on a level of transitions between discrete energy layers. The stability and persistence of nonstationary equilibrium with the help of functional total production of entropy (1) is researched and formulated the variation principle of its evolution. The minimizing of the changes of functional (1) on considered time interval leads to variation principle, which is a generalization of the principle of minimum entropy production in nonstationary case: (2) Here and - the moments of beginning and the end of the nonstationary process - the rate of change of entropy production due to the change of local parameters of thermodynamic forces and the rates of their changes. The directive equation (2) should be applied taking into account the limiting conditions for the above parameters by considering the concrete nonstationary systems. The application of this principle for the theoretical description of the magnetic spin-lattice relaxation time and thermal relaxation in a two-temperature plasma is considered. The derived results coincide with experimental and theoretical data. In the second case the formulas of temporal dependence of temperature of superheated electrons for quasi-stationary and nonstationary plasma on a base of introduced approach is deduced on a base of introduced approach [1].

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### Modelling the effect of magnetic dots on the transport and magnetic properties of superconducting films

A. Sanchez†, A. Sanchez, C. Navau, N. Del-Valle and D.-X. Chen

†Universitat Autònoma Barcelona

There is a very large interest in the effect that magnetic dots and other types of magnetic-field sources create in the physical properties of superconducting systems. One of the systems in which this interaction is most studied is in thin superconducting films. Ferromagnetic parts of hard type -with a large coercivity- and of soft type - small coercivity- have been used in many experiments to modify and tune the superconducting properties of the films. Many different geometries have been used, such as magnetic inclusions in the superconductor, columns traversing the thickness of the films or even magnetic substrates covering the whole width and length of the films. Although many experimental works have been devoted to the study of these properties, there is a lack of theoretical explanation for some of the effects arising from the ferromagnetic-superconductor interaction. In this work, we provide some of the theoretical tools to analyze ferromagnetic-superconductor hybrids in two cases of practical interest: when a transport current is fed in the SC by an external source -transport case-, and when a current is induced in the SC by an externally applied magnetic field -magnetic case. Calculations are based on assuming the critical-state model in the superconductor, with a field dependent critical-current density. We will present a general framework for modelling these systems, under some approximations and illustrate the capabilities of the modelling with some examples. Among them, we shall study the case in which a transport current is fed in a thin superconducting tape with a soft ferromagnetic substrate - as in the case of some actual coated conductors-, to show that the transport critical-current density decreases at non-zero applied fields because of the effect of the magnetic substrate, and this reduction varies depending on the magnetic substrate thickness. Also, we will study the effect that an array of magnetic dots can have in the shape of the hysteresis loops of superconducting films, and, in particular, in the position of the magnetization peaks. Since the width of the loop is related to the critical-current density, tailoring the arrays of dots can be used to tune the critical-current density to desired values. The theoretical results will be compared with existing experimental results.

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### Are the virial coefficients and the close-packing of hard disks and hard spheres related?

A. Santos<sup>†</sup>, M.A.G. Maestre, M. Robles, M. López de Haro

<sup>†</sup>Department of Physics, University of Extremadura, Badajoz, Spain

The virial expansion of the equation of state of a hard-core fluid expresses the compressibility factor as a series in powers of the packing fraction, the coefficients being the (reduced) virial coefficients. So far, only values up to the tenth virial coefficient have been reported. There are a number of controversial and open issues related to the virial expansion of hard-disk and hard-sphere fluids. To begin with, even in the case that many more virial coefficients for these systems were known, the truncated virial series for the corresponding compressibility factors would not be useful in principle for packing fractions higher than the one corresponding to the radius of convergence of the whole series. Moreover, even the character of the virial expansion (either alternating or not) is still unknown. So far, all the available virial coefficients for these systems are positive, but results from higher dimensions suggest that this feature might not be true for higher virial coefficients. Since the compressibility factor of hard-disk and hard-sphere fluids, both for the stable and metastable fluid phases, is a monotonically increasing function of the packing fraction, one may reasonably wonder at which singular packing fraction the analytical continuation of the compressibility factor diverges. Clearly, this value may not be bigger than the close-packing value, i.e., the maximum packing fraction that is geometrically possible (the so-called Kepler's problem). The aim of this work is to address the question as to whether, using the available information on virial coefficients, one can get a reasonable estimate of the singular packing fraction and whether there may be a systematic method to improve the estimation as more virial coefficients become available. We have checked that the use of the direct Padé approximants of the compressibility factor is not reliable for our purpose. As an alternative approach, and following an idea of Sanchez, we may formally invert the virial series and express the packing fraction as a series expansion in powers of the pressure. We have constructed (diagonal) Padé approximants of the latter series from the knowledge of the first two, four, six, eight, and ten virial coefficients for hard disks and hard spheres and have obtained the corresponding estimates for the singular packing fraction. In addition, making use of the values of the eleventh to sixteenth virial coefficients estimated by Clisby and McCoy, we have also considered further estimates. The results are fully consistent with the conjecture stating that the singular packing fraction coincides with the close-packing value. Nevertheless, full confirmation must await the availability of higher virial coefficients.

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### Velocity cumulants and correlations in a granular gas of rough spheres

A. Santos<sup>†</sup>, G.M. Kremer

<sup>†</sup>Department of Physics, University of Extremadura, Badajoz, Spain

A granular fluid is usually modeled as a system of identical, inelastic smooth hard spheres with a constant coefficient of normal restitution. Despite its simplicity, this model has been useful to capture the basic properties of granular flows. On the other hand, the model can be made closer to reality by introducing more ingredients, such as coefficients of normal restitution depending on the impact velocity, presence of an interstitial fluid, non-spherical shapes, polydispersity, or roughness, to name just a few. In this work we focus on roughness and consider a dilute granular gas of hard spheres colliding inelastically with constant coefficients of normal and tangential restitution. Roughness is especially relevant, not only because beads and grains are unavoidably rough, but also because this ingredient unveils an inherent breakdown of energy equipartition in granular fluids, even in homogeneous and isotropic states. The basic quantities characterizing the distribution function of linear and angular velocities are the second-degree moments defining the translational and rotational temperatures. The deviation of the velocity distribution function from the Maxwellian parameterized by both temperatures can be measured by the cumulants and velocity correlation functions associated with the fourth-degree moments. Two of those parameters measure the kurtosis of the translational and rotational distribution functions, while the translational-rotational correlations are measured by other two parameters. We evaluate the collisional rates of change of these second- and fourth-degree moments by means of a Sonine approximation where the velocity distribution function is approximated by the two-temperature Maxwellian times a truncated polynomial expansion. The results are subsequently applied to two paradigmatic states: the homogeneous cooling state and the homogeneous steady state driven by a white-noise stochastic thermostat. It is found in both cases that the Maxwellian approximation for the temperature ratio does not deviate much from the Sonine prediction. On the other hand, non-Maxwellian properties measured by the cumulants and velocity correlation functions cannot be ignored, especially in the homogeneous cooling state for medium and small roughness. In that state, moreover, some quantities differ in the quasi-smooth limit from those of pure smooth spheres. This singular behavior is directly related to the unsteady character of the homogeneous cooling state and thus it is absent in the stochastic thermostat case.

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## Superstatistics in models of space-time foam in string theory

S. Sarkar

†King's College London

D-particles (either elementary or D3-branes compactified along 3-cycles) in the bulk form the constituents of space-time foam. D-particles capture closed and opens strings in type II theory. After the capture, the interaction (scattering) can be considered within string perturbation theory. D-particles are dynamical and recoil. A stochastic Langevin equation for the velocity recoil of D-particles can be derived from the pinched approximation for a sum over genera in the calculation of the partition function of a bosonic string in the presence of heavy D-particles. The string coupling  $g_s$  in standard perturbation theory is related to the exponential of the expectation of the dilaton. Inclusion of fluctuations of the dilaton itself and uncertainties in the string background will then necessitate fluctuations in  $g_s$ . The fluctuation in the string coupling in the sum over genera typically leads to a generic structure of the Langevin equation where the coefficient of the noise term, dependent on the string coupling, fluctuates. The positivity of  $g_s$  leads naturally to a stochastic modelling of its distribution with a  $\chi^2$ -distribution. This then rigorously implies a Tsallis type non-extensive or, more generally, a superstatistics distribution for the velocity recoil of D-particles. As a concrete and physically interesting application, we provide a rigorous estimate of an entanglement effect, pertinent to CPT Violating modifications of the Einstein-Podolsky-Rosen correlators in entangled states of neutral Kaons [1,2]. In the case of D-particle foam fluctuations, which respect the Lorentz symmetry of the vacuum on average, we find that the -effect is significantly suppressed, compared to naive, dimensional-analysis based, previous estimates, so that its detection in future meson factories presents a major experimental challenge. We discuss how such a type of stochastic space-time foam can lead to cosmological effects similar to those induced by modifications of particle distributions [3,4] within the framework of Tsallis entropies. The restrictions placed on the free parameters of the Finsler type metric are obtained from solving the Boltzmann equation in this background for relic abundances of a Lightest Supersymmetric Particle (LSP) dark matter candidate. It is demonstrated that the D-foam acts as a source for particle production in the Boltzmann equation, thereby leading to enhanced thermal LSP relic abundances relative to those in the Standard CDM Cosmology. For D-particle masses of order TeV, such effects may be relevant for dark matter searches at colliders. The latter constraints complement those coming from high energy gamma-ray astronomy on the induced vacuum refractive index that D-foam models entail.

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## The simulation of three dimensional spin-3/2 Ising model on a cellular automata with an external magnetic field

G. Sezgin†, G. Sezgin and N. Seferoglu

†Gazi University, Institute of Science and Technology, Advanced Technologies, Ankara, Turkey

In recent years, much attention has been directed to the spin-3/2 Ising systems in which initially introduced to give a qualitative description of phase transition observed in the compound DyVO4 and also to study tricritical properties in ternary mixtures [1]. The spin-3/2 Ising model that contains a single-ion anisotropy parameter ( $D$ ) or crystal field interaction in addition to the both bilinear( $J$ ) and a biquadratic ( $K$ ) exchange interactions is known as the spin-3/2 Blume-Emery-Griffiths (BEG) model. The model is also known Blume-Capel model without  $D$  parameter. There has been a number of theoretical studies to obtain the phase diagrams and critical and multicritical behavior of the model. In spite of these studies, further studies using alternative methods are desirable to obtain the new phase diagrams and the critical behavior of the model. Most of previous studies undertaken within the spin-3/2 BEG model have only considered some portions of the phase diagram in zero external field. In our recent studies, the BEG model and BC model are investigated on a cellular automaton and the phase diagrams of the model are constructed in the  $(D/J, kT/J)$  and  $(K/J, kT/J)$  plane [2,3]. Moreover, the static critical exponents of the BEG model are estimated within the framework of the finite-size scaling [2]. The purpose of the present study is to obtain the phase diagram of the BEG model in the presence of external field using cellular automaton. For this purpose, the model on the simple cubic lattice with nearest-neighbour ferromagnetic bilinear interaction ( $J > 0$ ) in the presence of an external magnetic field is simulated on a cellular automaton by using the cooling algorithm improved from the Creutz cellular automaton[4]. The simulations are carried on  $LxLxL$  simple cubic lattice with  $L = 12, 14, 16$ , and 18. The periodic boundary conditions are applied in all directions. In calculations, the temperature dependence of the order parameters and associated fluctuations are obtained at various of the model parameters in an external magnetic field. The phase diagrams of the model are constructed in the  $(K/J, kT/J)$  plane for  $D/J = 3$  and in the  $(D/J, kT/J)$  plane for  $K/J = 0.5$  at  $0 < |h| < 1$  parameter region. To produce the phase diagrams, the finite critical temperatures are estimated from the maxima of the fluctuations of the order parameters on the lattice with  $L = 16$ . In addition, the magnetic exponent is estimated within the framework of the finite-size scaling at selected  $K/J$  and  $D/J$  values for the  $0 < |h| < 1$  parameter region.

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## Superconductor-insulator magneto-oscillations in superconducting strips

E. Shimshoni<sup>†</sup>

<sup>†</sup>Bar-Ilan University

Thin superconducting strips of width comparable to the coherence length were found experimentally to exhibit magneto-resistance oscillations, when subject to a perpendicular magnetic field  $B$  at low temperatures  $T$  [1]. Most interestingly, the transition from a superconductor to insulator at a critical field  $B_c$  is preempted by several consecutive transitions at lower fields, from a superconductor to insulator and vice versa alternately. To investigate this phenomenon, we have studied a quasi one-dimensional model for the quantum dynamics of vortices in a line-junction between coupled parallel superconducting wires, at parameters close to their superconductor-insulator transition [2]. In this regime, where quantum fluctuations of the collective superconducting phase field are maximized, the coupling between the wires is dominated by a competition between the Josephson coupling and the inter-wire Coulomb interactions. A Refermionization transformation is applied to the model, which maps the vortices onto nearly free one-dimensional Fermions, at a chemical potential dictated by  $B$ . As a consequence of this mapping, we find that a quantum phase transition of the type characteristic to the Ising chain in a transverse field occurs at critical values of the vortex filling, between a superconducting phase close to integer filling and an insulator close to  $1/2$ -filling. The quantum critical points are characterized by the vanishing of a gap, which changes sign alternately, indicating transitions between an ordered and disordered Ising phases. The disordered Ising phase is identified as superconducting, suggesting that the order Ising field physically represents phase-slips. The ordered Ising phase hence corresponds to the insulator, in which phase-slips (associated with vortex hopping across the strip edges) proliferate. The duality relating the order and disorder Ising fields is therefore intimately related to vortex-charge duality. To study the resulting transport properties of the system, we introduce a scattering potential due to random impurities. The resistance of the double-wire system is then evaluated as a function of  $B$  and  $T$ . The calculation yields magneto-resistance oscillations similar to the experimental observation in strip-like samples. In particular, in the superconducting regions the resistance is exponentially suppressed at low  $T$ , with an activation gap given by the Ising gap; this indicates thermal activation of phase-slips. In the insulating regimes, the resistance diverges in the limit of low  $T$ . Finally, we study non-linear current-voltage characteristics [3], which also reflect the alternating superconductor and insulating behaviors.

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## Names, terror, wealth and politics: the role of fortune in complex systems

N. Shnerb<sup>†</sup>, Y. Maruvka, D. Kessler

<sup>†</sup>Department of Physics, Bar-Ilan University, Israel

Out of ten pairs of pants bought a year ago, the survivors are perhaps those made of a better material; if wineglasses are considered, persistence is mainly a matter of luck. In the absence of prior knowledge, statistics must be used in order to identify the role of fortune: wineglass life expectancy, for example, is described by an exponential distribution. Strong deviations from this statistics indicate to what extent "death" is a result of accumulated wear, rather than from uncorrelated random events. Can we implement the same technique to tell apart the fittest from the luckiest in complex systems? Here the situation is more intricate, as many social and economic systems involve strong multiplicative noise that yields fat-tailed distributions. As a result, huge differences in abundance (or wealth or political success) may be a result of stochasticity only. Take surname statistics as an example: nobody really thinks that the Smiths and the Johnsons are more fertile than the Shnerbs, right? The frequency differences are a simply the outcome of a "neutral" birth-death-mutation process. Following former works [1] we have recently solved for the distribution of surnames in a growing society, showing that it converges to a universal limit [2]. It turns out that many social and economic phenomena yield the same distribution. These include the number of terror attacks with certain number of casualties, number of WWW sites with certain degree of links as a function of the degree, number of nodes with  $m$  links (for the internet physical connections) vs.  $m$ , word occurrence in texts and number of firms with  $m$  employees [3]. This suggest that the role of signal-to-noise ratio in complex systems of this kind may be very small, and put in question the usefulness of a large scale planning in these fields. Previous neutral theories, like the Yule-Simon Birth-Mutation process [4], have focused on the power-law tail of the distribution. This tail is, in most cases, brutish, noisy and short, and some recent works have pointed out the technical difficulties in the fit and the inference of the slope [5]. Our theory economically and accurately explains the entire distribution, and thus it allows for an accurate and reliable inference of the process parameters. We thus suggest the BDM distribution as a standard neutral model: effects of fitness and selection are to be identified by substantial deviations from it.

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### Diffusion in a tilted periodic potential. Specific properties of motion in the underdamped limit

A. Shushin†

†Institute Of Chemical Physics, Russian Academy of Sciences, Moscow, Russia

This work concerns the analysis of the effect of external force ( $F$ ) on the activated motion of Brownian particles on the periodic potential  $U(x)$  in the underdamped limit, described by the Langevin equation. The analysis is made within two models: the model discrete activated jumps [1,2] and continuum model [3]. The first model is well known. It treats activated migration of particles as a set of discrete jumps between potential wells. As for the continuum model [3], it is based on the two-state approximation, in which the kinetics of the processes under study is assumed to be determined by transitions between two continuum states (media) in the phase space: the overbarrier and underbarrier ones. In the overbarrier state the effect of the potential  $U(x)$  is neglected, i.e. the process is approximated by the free stochastic motion in the presence of the external force, while in the underbarrier state this effect is modeled by fast relaxation of the direction of the particle velocity. Despite strong simplifications, assumed in the two-state approximation, it is expected to be quite accurate [3,4]. In our work we have demonstrated the equivalence of these two models in the underdamped limit. The equivalence manifests itself in the same mathematical form of the most important characteristics of the process, the probability distribution function (PDF) of trapping, predicted by both models. This equivalence is important, because it allows one to safely apply the simple continuum model, making it possible to describe the characteristic features of the locked-to-running transition for any values of forces as well as the specific properties of the transient kinetics. The analysis revealed the strongly non-exponential behavior of the PDF of Levy-flights type for values of the force  $F$ , close to the threshold of appearance of the running state. Special attention is paid to the analysis of specific features of  $F$ -dependence the PDF in the region of the locked-to-running transition. The continuum model is shown to be able to describe time dependent properties of the process, some of which are briefly discussed. Note the proposed analysis can be modified to describe anomalously slow velocity relaxation, (described by the subdiffusion equation in the velocity space), resulting in the superdiffusion type motion of particles.

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### T=0 phase diagram of the 1D Hubbard model with magnetic interactions in the narrow-band limit

G. Sica†, F. Mancini, E. Plekhanov

†Department of Physics "E.R. Caianiello", University of Salerno, Italy

In this paper we study a generalization of the Hubbard model by considering spin-spin interactions described by an exchange constant  $J$ . An external magnetic field  $h$  is also taken into account. In the narrow-band limit and for the 1D case, we present the exact solution obtained in the framework of the Green's functions formalism, using the Composite Operator Method [1-2]. We show that, by means of algebraic relations, the hierarchy of the equations of motion closes and the Green's functions can be expressed in terms of a finite number of parameters to be determined self-consistently [3]. We have studied some relevant features like double occupancy, magnetization, spin-spin and charge-charge correlation functions and derived a phase diagram for both ferro ( $J > 0$ ) and anti-ferro ( $J < 0$ ) coupling in the limit of zero temperature. We observe that, in presence of a ferromagnetic coupling, for positive values of the local Coulomb potential  $U$  a ferromagnetic ( $F$ ) order is established with finite values of the magnetization and of the spin-spin correlation functions. For  $U < 0$  instead, there is a critical line that separates the ferromagnetic phase from a non-magnetic (NM) one characterized by the absence of correlations and long range orders. In the presence of an anti-ferromagnetic coupling several phases are observed. For low values of the external magnetic field, NM-phase still appears for negative values of  $U$  while for  $U > 0$  an anti-ferromagnetic (AF) order is established in which we have finite values of spin-spin correlation functions without magnetization. For both positive and negative values of  $U$ , when the external magnetic field is strong enough to dominate respect to  $J$ , a ferromagnetic behavior is observed despite of the presence of an anti-ferromagnetic coupling. In this phase (called  $F1$ ), the effect of the magnetic field induces the spins to be all aligned and the magnetization reaches its saturation value. Nearest neighbor spin-spin and charge-charge correlation functions remain zero until we reach quarter filling. For 0.5

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## Probability analysis of unzipping of dsDNA

N. Singh<sup>†</sup>, S. Srivastava

<sup>†</sup>Department of Physics, Birla Institute of Technology and Science, Rajasthan, India

In recent years, single molecule force spectroscopy (SMFS) experiments using optical tweezers, atomic force microscope etc., have directly measured these forces and provided unprecedented insight into the mechanism involved in the process of DNA separation and its stability. Simple models like Poland Scheraga or Peyrard Bishop Dauxious models have described some of the essential macroscopic features of the melting profile of dsDNA quite effectively and predicted that the force induced melting transition is a first order transition. However, semi-microscopic information about the opening such as whether a dsDNA opens from the end or interior of the chain, distribution of partially opened regions in the form of bubbles in the chain are some of the intriguing issues in these studies. Moreover, in all SMFS experiments, the experimental setup puts an extra constraint on the ends of the chain, which makes force-induced melting different from the thermal melting. These constraints do not matter in the thermodynamic limit. However, all SMFS experiments involve a finite length of the strand and hence these constraints may affect the melting profile of the chain. We have studied the effects of these constraints on the melting profile of heterosequence of finite length single molecule experiments and provide semi microscopic information about the formation of bubbles in the form of partially opened regions during the opening of a dsDNA. More precisely, by employing the probability analysis of opening of individual bases, we delineate the mechanism involved in the separation of a dsDNA. Using PBD model, we have calculated the probability of opening of the entire sequence to see how the chain opens. Motivated by recent SMFS experiments, we constrained one end of the chain and showed that the mechanism of opening of dsDNA not only depends on the sequence of base pairs, but also on which end has been constrained by the experimental setup. It is also surprising to note that in the study of the force induced unzipping using PBD model, the bubbles do not play a significant role. This could be due to fact that in this model, opening of a bead is restricted to move in one dimension only and thus entropy of the loop has been underestimated. It would be nice to repeat these experiments by attaching different tags at the weaker junction and to see how the chain opens during thermal melting and force induced unzipping.

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## Generalization of the Aoki-Yoshikawa sectoral productivity model

J. Sladkowski<sup>†</sup>, E.W. Piotrowski, J. Syska

<sup>†</sup>Institute of Physics, University of Silesia, Katowice, Poland

Recently, M. Aoki and H. Yoshikawa put forward a statistical physics inspired model for economy of a country [1,2] ( $A - Y$  model). In their model, there are  $g$  economics sectors and each sector is characterized by the amount of production  $n(i)$  and by the level of productivity  $a(i)$ ,  $i = 1, 2, \dots, g$ . The total endowment of production is fixed by exogenous conditions to be equal to  $n$ , the sum of all  $n(i)$  (a global constraint). The production of the  $i$ -th sector is given by  $Y(i) = a(i)n(i)$  and the sum of all  $Y(i)$  is interpreted as the Gross Domestic Product (GDP). Aoki and Yoshikawa tried to find the probability distribution of production factors, say workers, among the sectors. With some simplifying assumptions (discreteness of the sectorial productivity of the form  $a(i) = ia$ ,  $g$  tending to infinity), the problem can be solved by methods of statistical physics giving approximate solutions for the most probable occupation  $N(i) \sim (n/r)\exp(-i/r)$  and the probability of finding a randomly selected worker in the  $i$ -th sector  $P(y = i|n) = N(i)/n \sim (na(i)/iY)\exp(-(na(i)/Y)$ , where  $Y$  is the GDP of the economy in question and  $r$  is the aggregate demand per agent divided by the smallest productivity ( $r \gg 1$ ). We would like to present a generalization of the model, that can be analysed and solved exactly with help of the Extreme Physical Information Principle (EPI), [3,4]. We introduce additive fluctuations to the productivity that, on the basis EFI and information capacity principle [3,4], allows for writing the following second order differential equation for the probability amplitudes  $q$  of the productivity fluctuations:  $q''(x_a) = Cq(x_a)$ , where  $C$  is a positive constant and  $a$  is the productivity real parameter. In particular, we get that the productivity distribution density function takes the form  $p(a) = (1/(Y/n - a_0))\exp(-(a - a_0)/(Y/n - a_0))$  for  $a > a_0$  and  $p(a) = 0$  if  $a > 1$ . In the limit of low productivity both methods slightly differ:  $P(y = i|n)_AY \sim (1/r + 1/r^2)\exp((-i/r)$  versus  $P(y = i|n)_{our} \sim (1/R + 1/2R^2)\exp((-i/R)$ . Note that the discretization of the productivity parameter  $a$  is only a technical trick -  $a$  is actually a continuous parameter. Therefore our model is more realistic (we also have finite  $g$ ). Our method can be applied in other systems in statistical equilibrium and the dynamics can be analysed along the path suggested in [2].

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## Long-range orientational correlations and rotational anisotropy in random packings of hard colloidal spheres

E. Sloutskin<sup>†</sup>, A.V. Butenko

<sup>†</sup>Physics Department, Bar-Ilan University, Israel

Random close-packed materials are abundant in nature and industry, yet these materials are still poorly understood. Formed by random forces, these materials are often naively assumed to be disordered, such that only short-range correlations are present and all spatial directions are equivalent. Yet, the mechanical stability of random close-packed materials under gravity implies that a network of mechanical forces percolates through a macroscopic sample. Such network may give rise to long-range correlations, and the rotational symmetry may be broken. During the last decades, significant theoretical efforts, mostly based on computer simulations [1,2], were devoted in order to detect and fully understand the possible long-range correlations in random packings of spheres. Unfortunately, while large-scale simulations are very difficult, systems with long-range correlations are usually sensitive to the sample size. Consequently, our current understanding of these systems is very poor; in particular, the type, and the existence, of long -range correlations in randomly packed materials are still under debate [2].

We employ experimental confocal microscopy to directly measure the positions of hard colloidal spheres, which form truly macroscopic, random colloidal packings [3]. These packings are formed by centrifugation of a dense fluid suspension; a critical time  $t_0$  exists, such that the system is fully jammed at  $t > t_0$ . The positional correlations between our particles, both in the fluid state ( $t < t_0$ ) and in the jammed state ( $t > t_0$ ), decay to zero, as the separation between them reaches just a couple of particle diameters; this indicates that only short-range positional order exists in our system. Strikingly, while the positions of the centers of mass of our spheres are disordered, the orientations of the bonds between the nearest neighbors (NN) are correlated, for  $t > t_0$ , throughout the system[4]. This is demonstrated by the bond-orientational correlation function  $g_6(r)$ , which does not vanish, in the jammed state, even for very large separation values  $r$ . These extended orientational correlations break the global rotational symmetry in our system. Instead of being rotationally invariant, the probability to have a NN bond in a certain direction  $P(\theta)$  exhibits a six-fold symmetry[4]. Moreover, the six-fold symmetry of  $P(\theta)$  is correlated with the direction of gravity; this suggests that the percolation of a mechanical network of forces in our system may be responsible for the observed rotational anisotropy. An alternative scenario, relating the observed effects to an hexatic-type phase transition, which may be occurring in the metastable fluid upon jamming, will be discussed as well.

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## Destroying and Creating Optimal Paths

J. Soares de Andrade Jr.<sup>†</sup>

<sup>†</sup>Departamento de Física, Universidade Federal do Ceara

How does the transport through a disordered medium fail? Typically transport in disordered systems is dominated by a few channels. In fact, the optimal path through a random energy landscape plays an important role in physics to explain problems ranging from flow through disordered porous media to navigation in transport networks. Suppose that in a typical city a large fraction of the working population move by car during every day from a residential zone to the industrial pole. In order to reach their work, they have to cross the city through an intricate network of streets. If we now assume that one such trajectory is a unique optimal path and can accommodate a certain (finite) flux of cars, we expect a traffic jam somewhere along this route. As a consequence, many people, on their way to work and others, will then try to follow a possible next-to-optimal path. Another traffic jam will then be generated and so on, and so forth. Two questions naturally arise: (i) how and when the transportation network will eventually collapse and (ii) how the topology and heterogeneity of this network affects its performance. In the first part of this talk, we present a minimal model for this complex problem that can still capture its essential features and give some physical insight about the possible answers to these questions. We also show that the optimal path fracture resulting from this new paradigm has very close connections with the lines dividing adjacent drainage basins (watersheds), the hulls of explosive percolation clusters, and the critical behaviour of the recently proposed bridge percolation model. In the second part, we investigate the navigation problem in lattices with long-range connections and subject to a cost constraint. Our network is built from a regular two-dimensional ( $d=2$ ) square lattice to be improved by adding long-range connections (shortcuts) with probability,  $P_{ij} \sim r_{ij}^{-\alpha}$ , where  $r_{ij}$  is the Manhattan distance between sites  $i$  and  $j$ , and alpha is a variable exponent. We introduce a cost constraint on the total length of the additional links and find optimal transport in the system for  $\alpha = d + 1$  established here for  $d = 1$  and  $d = 2$ . Remarkably, this condition remains optimal, regardless of the strategy used for navigation, being based on local or global knowledge of the network structure, in sharp contrast with the results obtained for unconstrained navigation using global or local information, where the optimal conditions are  $\alpha = 0$  and  $\alpha = d$ , respectively. The validity of our results is supported by data on the U.S. airport network.

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## Nanostructures in a binary mixture confined in slit-like pores with walls decorated with tethered polymer brushes in the form of stripes. Dissipative particle dynamics study

S. Sokolowski<sup>†</sup>, J.M. Ilnytskyi

<sup>†</sup>Department for the Modelling of Physico-Chemical Processes, MCS Lublin, Poland

We have employed the Dissipative Particle Dynamics simulation technique [1,2] to study the structure of a binary mixture inside pores modified by stripes [3] of tethered polymer brushes. The solvent has been composed of two types of beads. The beads A have been identical to the beads of chain particles, while the beads of the second sort, B, have been selected in such a way that the bulk A-B mixture exhibits partial mixing. All the calculations have been carried out assuming that the ratio of the total number of A beads to the number of B beads is equal to one. The main aim of our calculations has been to investigate the morphologies that appear inside the pore, depending on the pore size  $d$  and on the width of the stripes,  $w$ . We have constructed a sketch of the diagram of morphologies and have found that they are governed by (i) the characteristic length-scale  $L$  that is the length of the chains, and by (ii) the fraction of the A-type molecules in the polymer-free regions. For the values of  $w$  close to  $L/2$  and for  $d$  greater than  $d/2$  lamellar morphology was found, while for  $w \lesssim L/4$  and  $L/2 \lesssim d \lesssim L$  we observe the formation of pillars. Beyond the above regions either droplets or droplets plus bulk-type morphologies have been observed. The self-aggregation within polymer-free stripes seems to be governed by the size of the stripes and by the fraction of A beads within these regions. When this fraction is close to  $1/3$  cylinders inside polymer-free regions have been formed, while for the fraction close to  $1/2$  lamellae have developed. In order to characterize the observed morphologies we have calculated several parameters, as the radii of gyration, local densities and histograms of the minimum distances between polymers located in different (within the same plane, as well as at opposing walls) stripes. It would be of interest to check how the morphologies evolve if the ratio of A to B beads will change and how the morphologies formed under static condition will be preserved - or changed - if the fluid will undergo a pressure-driven flow along the pore axis. The last aspect seems to be particularly, and even important from practical point of view, because the formation of some structures inside the pore "locks" the pore. It would be therefore of interest to check what pressure will be necessary to break the "dams" inside the pore and to open it.

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## Economic complexity, crisis and recovery

S. Solomon<sup>†</sup>

<sup>†</sup>Hebrew University of Jerusalem

Phil Anderson proposed the concept of complexity in order to describe the emergence and growth of macroscopic collective patterns out of the simple interactions of many microscopic agents. In the physical sciences this paradigm was implemented systematically and confirmed repeatedly by successful confrontation with reality. In the social sciences however, the possibilities to stage experiments to validate it are limited. Recessions and shocks caused by dramatic political and economic events provide such opportunities. We exploited these empirical data to validate a simple agent based solvable model. The empirical data confirmed the theoretical prediction that the dynamics is dominated by singular rare events which insure the resilience and adaptability of the entire system: growth is led by few singular "growth centers / sectors". Immediately after the shock, these very limited sub-systems develop at a tremendous rate while the rest of the economy decays. Only in a second stage, growth propagates by diffusion to sectors/regions bringing eventually the entire economy to a common uniform growth rate. The model leads to a simple and intuitive three-parameter effective equation that fits remarkably well the evolution of the gross domestic product (GDP) of many countries during times of recession and recovery. We showed that this equation can be used to detect large and small shocks, including those which do not lead to a recession. We demonstrated how the severity and length of recessions depends on the policy of resources transfer between the growing and failing parts of the economy. The above framework was extended further to the study of interactions between economic sectors, countries and blocks. The confrontation with the new data confirmed the theoretical predictions robustly in a very wide range of conditions in the growth rates evolution of various sub-systems. The J-curve phenomenon received more empirical support. In particular we confirmed the connection between the economic minimum and the crossover of the new emergent leading sector with the old decaying one. We describe in detail the 'Growth Alignment Effect' (GAE), its theoretical basis and demonstrate it empirically for numerous cases in the inter-national and intra-national economies. The GAE is the concept that in steady state the growth rates of the GDP per capita of the various system components align. We differentiate the GAE predictions from the usual convergence or divergence conceptual framework. Further investigations of GAE and subsidiaries are suggested and possible uses are proposed. Due to its simple and robust nature, the method can be used as a tool for economic decisions and policy making.

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## Conductance, shot noise, and Klein tunneling in ballistic graphene

E. Sonin<sup>†</sup>

<sup>†</sup>Racah Institute of Physics, Hebrew University of Jerusalem

Conductance and shot noise in a ballistic graphene have been intensively investigated theoretically and experimentally [1-5]. The talk will address coherent and incoherent ballistic transport. At high voltages the difference between coherent and incoherent transport is not essential. But at low voltages conductance and Fano-factor dependence for incoherent transport becomes non-monotonous so that the conductance has a minimum and the Fano factor has a maximum at non-zero voltage bias. The talk will also focus on the effect of Klein tunneling on conductance and shot noise. Analyzing conductance and shot noise in a ballistic graphene sheet, a commonly accepted assumption was that under electrodes the graphene sample is strongly doped. A further assumption, which simplified a theoretical analysis, was that the level of doping changed abruptly. This led to a rectangular potential barrier for electrons in a graphene sheet. The model of the rectangular barrier could not explain experimental investigations of transport through tunable potential barriers [4,5], which revealed asymmetry of the dependence of resistance on gate voltage with respect to the sign of gate voltage measured from the electrostatic potential of the Dirac point in the sheet center. One may expect that in reality the doping level in the contacts varies continuously. The electrostatic potential in the graphene sheet near the contacts and inside the graphene sheet can be modeled by a trapezoid barrier, which allows to use the exact solution of the Dirac equation in a uniform electric field in the slope areas (the two lateral sides of the trapezoid). The theory shows that asymmetry manifests the difference between the Klein tunneling and the over-barrier transmission. The phase coherence between Klein tunneling events in the slope areas (p-n transitions) leads to conductance and Fano-factor oscillation at high negative gate voltages (Fabry-Perot interference). In the limit of a very steep slope, when the trapezoid barrier transforms to a rectangular one, the probability of overbarrier reflection exactly coincides with the reflection from the band boundary, and oscillations and asymmetry with respect to the gate voltage sign vanish. So experimental detection of asymmetry provides evidence of the Klein tunneling. The experimental and theoretical dependence of the oscillation period from the charge density is in reasonable agreement. The comparison of the developed theory with the experiment supports the conclusion that the Klein tunneling was revealed experimentally.

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## Relaxation phenomena in classical and quantum systems

B. Spagnolo<sup>†</sup>, P. Caldara, A. La Cognata, D. Valenti, A. Fiasconaro

<sup>†</sup>Dipartimento di Fisica, Group of Interdisciplinary Physics, Università di Palermo

Bistable systems often play the role of archetypal models to understand the dynamical behavior of complex systems. Examples range from microphysics to macrophysics, biology, chemistry and also econophysics. Relaxation in many natural systems proceeds through metastable states, often observed in condensed matter physics, and also in various other fields, from cosmology to biology and high-energy physics. In spite of such ubiquity, the microscopic understanding of metastability still raises fundamental questions. Moreover the statistical mechanics is essential to study the physical properties of complex systems and to investigate stochastic systems in which the microscopic degrees of freedom behave collectively over large scales. In this work we will consider relaxation phenomena in three different bistable systems. (i) As a first classical system, we will consider an asymmetric bistable system and the role of the multiplicative and additive noise in the life time of the metastable state is investigated. This model is useful to describe the dynamical behavior of an out of equilibrium Ising spin system. A nonmonotonic behavior of the lifetime as a function of both additive and multiplicative noise source intensities is found, revealing the phenomenon of noise enhanced stability (NES) [1]. (ii) The dynamics of two competing species in the presence of Lévy noise sources is analyzed in the second classical system. By using the Lotka-Volterra model in the presence of two symmetrical non-Gaussian alpha stable noise sources, we analyzed the time behavior of this ecosystem interacting with the surrounding environment. The interaction parameter between the species is a random process which obeys a stochastic differential equation with a generalized bistable potential in the presence both of a periodic driving term and an additive alpha stable Lévy noise [2]. The role of the two non-Gaussian noise sources in the exclusion and coexistence regimes is analyzed. Quasiperiodic oscillations and stochastic resonance phenomenon in the dynamics of the competing species are found. (iii) Finally the dynamics of a quantum particle subject to an asymmetric bistable potential and interacting with a thermal reservoir, that is the environmental noise, is investigated. In order to analyze the evolution of this quantum system we use the Caldeira-Leggett model, which allows to derive a quantum mechanical analogue of the generalized Langevin equation. The study is performed by using the approach of the Feynman-Vernon functional in discrete variable representation (DVR) [3]. We obtain the time evolution of the population distributions in both energy and position eigenstates of the particle, for different values of the coupling strength with the thermal bath.

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## Fluctuations and statistical physics: Quantifying extremely rare events with applications to the present worldwide crisis

G. Stanley<sup>†</sup>

<sup>†</sup>Boston University

Recent analysis of truly huge quantities of empirical data suggests that classic economic theories not only fail for a few outliers, but that there occur similar outliers of every possible size. In fact, if one analyzes only a small data set (say data points), then outliers appear to occur as “rare events.” However, when we analyze orders of magnitude more data (200 million data points!), we find orders of magnitude more outliers—so ignoring them is not a responsible option, and studying their properties becomes a realistic goal. We find that the statistical properties of these “outliers” are identical to the statistical properties of everyday fluctuations. For an intriguing variety of switching processes in nature, the underlying complex system abruptly changes at a specific “phase transition” point from one state to another in a highly discontinuous fashion. Examples of phase transitions range from magnetism in statistical physics to physiology and macroscopic social phenomena. Financial market fluctuations are characterized by many abrupt switchings on very short time scales from increasing “microtrends” to decreasing “microtrends”—and vice versa. We ask whether these ubiquitous switching processes have quantifiable features analogous to those present in phase transitions, and surprisingly we find striking scale-free behavior not only after, but also before the switching occurs. Moreover, we find that the same laws govern the formation and bursting of large bubbles as tiny bubbles, over a factor of 1,000,000,000 in time scale. We interpret our findings as being consistent with time-dependent collective behavior of financial market participants [1]. Finally, we describe our very recent work [2] on the novel effects that occur when networks are coupled, as is the case in financial networks. The key concept is that systems comprised of more than one network are vastly more susceptible to failure cascades than isolated networks. This work was carried out in collaboration with a number of colleagues, chief among whom are T. Preis (Mainz), J. J. Schneider (Mainz), S. Havlin and R. Parshani (Bar-Ilan), S. V. Buldyrev (Yeshiva U), and G. Paul, P. Gopikrishnan and V. Plerou (BU).

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## Nematic order of interfaces in systems with competing interactions

D.A. Stariolo<sup>†</sup>, D.G. Barci

<sup>†</sup>Universidade Federal do Rio Grande do Sul, Porto Alegre, Brazil

Phase separating systems often self-organize to form patterns with different kinds of order. Commonly observed patterns are stripes or bubbles in quasi two dimensional systems, formed as a consequence of competing interactions at different scales [1]. This gives rise to a complex phase diagram, with both orientational order of interfaces and positional order of the microscopic units [2,3]. The usual approach to the description of the possible phases and their stability in such systems is through classical elasticity theory [4], applied to the ground state or low temperature structures. We will describe a Ginzburg-Landau model for a generic system with isotropic competing interactions in two dimensions [5]. From pure symmetry considerations we will show that a nematic order parameter appears naturally in the Ginzburg-Landau expansion for the density or magnetization. This model supports a nematic-isotropic phase transition which is second order at mean field level but of the KT type upon considerations of fluctuations. At lower temperatures, a nematic-stripe crystal is possible, although this phase is unstable in the thermodynamic limit, for purely isotropic interactions. Then, we will describe a canonical statistical mechanics approach to the description of phase transitions in systems with competing interactions, going beyond the GL approach. We introduce a suitable order parameter for the orientation of interfaces of lattice models, which allows a unified computation of positional and orientational order in microscopic models. The applicability of the method is quite general. In the continuum limit we make contact with the previously described Ginzburg-Landau effective theory. In this talk we show results of a mean field approximation for the phase diagrams of two stripe forming Ising systems: an Isotropic Next Nearest Neighbor Ising model (INNNI) and the Dipolar Frustrated Ising Ferromagnet (DFIF). In the first case the onset of orientational order coincides with that of positional one through an isotropic-stripe crystal phase transition. In the later model there is a purely nematic phase at intermediate temperatures, with orientational but no positional order. The origin of the different behavior in both models will be discussed together with possible extensions beyond mean field theory.

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## How to model asset dynamics on the basis of dependence, anomalous scaling, and non-stationarity

A. Stella<sup>†</sup>

<sup>†</sup>Dipartimento di Fisica e Sezione INFN, Padova

Independence and normal scaling play an important role in quantitative finance. In spite of this, the analysis of financial time series reveals that anomalous scaling and dependence need to be considered in order to account for relevant stylized facts. Following a line of thought which recently led to formulate limit theorems for sums of dependent random variables, we show that simple generalizations of the Gaussian stability allow to construct discrete time stochastic processes with features suitable to describe the dynamics of returns in finance. These processes produce sequences of strongly dependent random variables, whose sums have probability density functions obeying a time-inhomogeneous form of anomalous scaling. This is particularly interesting for financial applications. Indeed, in that context the presence of non-stationarity can be suspected to underlie the time evolution of indices and is explicitly revealed by phenomena like the return sequences after a major crash, which resemble the aftershock regimes after a main earthquake in seismology. The difficulty to validate models embodying non-stationarity is due to the fact that in general we deal with single historical time series and stationarity assumptions are often mandatory in order to collect adequate statistics for quantities like the empirical probability density function of asset returns. A first opportunity of applying our non-stationary scaling process is offered by whole ensembles of histories of successive returns one can extract on a daily basis in precise time windows from high frequency financial time series. We will show in these examples that the time correlations and time inhomogeneities of our model processes are consistent with the statistics of these ensembles. Turning to the case of single, long time series, we will outline a strategy by which our model processes can be used to generate alternative single histories obeying the same stylized facts within time scales on which empirical anomalous scaling is obeyed. This strategy is based on an auto-regressive scheme of the kind used, e.g., in ARCH simulations. The fact that the generated series obey the stylized facts provides already indirect evidence that indeed non-stationarity underlies the index time evolution. Our model reveals also adequate to describe the strong non-stationarities occurring after major financial crashes, and accounts for statistical features of the after-crashes not covered by the Omori law.

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## Nonlinear analysis of EEG from migraine brains

S. Stramaglia<sup>†</sup>, L. Angelini, M. Pellicoro

<sup>†</sup>INFN, Sezione di Bari, Italy

Synchronization phenomena have been recognized as a key feature for establishing the communication between different components of complex systems. An important application is the study of electroencephalographic (EEG) signals, by means of phase synchronization or analyzing the mutual information between regions of the brain. Mutual information can also be regarded as a Kullback-Leibler entropy, measuring the similarity between two probability distributions. Important information on the structure of complex systems can be also obtained by measuring the causal relationships between modules of a complex system: transfer entropy and Granger causality have emerged in the last years as leading statistical techniques to accomplish these tasks. Kernel algorithms work by embedding data into a Hilbert space, and searching for linear relations in that space. The embedding is performed implicitly, by specifying the inner product between pairs of points. A kernel method to provide nonlinear measures of Granger causality has been recently proposed (Marinazzo et al., 2008), with the following two main features: (i) the non-linearity of the regression model can be controlled by choosing the kernel function; (ii) the problem of false-causalities, which arises as the complexity of the model increases, is addressed by a selection strategy of the eigenvectors of a reduced Gram matrix whose range represents the additional features due to the second time series. Migraine, an incapacitating disorder of neurovascular origin, consists of attacks of headache, accompanied by autonomic and possibly neurological symptoms. This pathology affects a relevant fraction of the general population and represents a social problem. The study of phase synchronization in EEG rhythms, showed in Migraine without aura patients a pattern of alpha rhythm hyper-synchronization under repetitive flash stimulation, opposite to a desynchronization trend in nonmigraine subjects (Angelini et al, 2004) this nonlinear EEG pattern was found to be modulated by Anti Epileptic Drugs. We describe the application of these nonlinear techniques to EEG data from migraine patients with and without aura, and show that patients are characterized by an increased information flow on the cortex w.r.t. healthy subjects; we also discuss the neuro-physiological implication of our results.

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## Persistence in reactive-wetting interfaces

H. Taitelbaum<sup>†</sup>, Y. Efraim

<sup>†</sup>Department of Physics, Bar-Ilan University

We report on the first persistence results of reactive-wetting advancing interfaces. Our reactive-wetting system, which is the only known system in room temperature, consists of small mercury (Hg) droplets (150 microns in diameter) spreading on thin silver (Ag) films (2000 - 4000 Å). Using a recently developed method for reconstructing the dynamical three-dimensional shape of spreading droplets from a microscope top-view, we study the time evolution of the droplet radius and its contact angle. We find that the "bulk propagation" regime is controlled by chemical reaction on the surface, and a "fast flow" regime occurs within the metal film as well as on the surface, and consists of both linear (in time) and diffusive propagation. The transition time between these two main time regimes depends solely on the thickness of the Ag film. A reaction band with an intermetallic compound Ag<sub>4</sub>Hg<sub>3</sub> is formed in this process. We study this reaction band by means of the kinetic roughening characteristics of the top-viewed mercury-silver triple line, which is the statistical characterization of the morphology of the triple line. It is expressed in terms of the statistically calculated growth and roughness exponents. The latter are used to determine the universality class of the system. We discuss the growth and roughness exponents of the propagating interface, the temporal interface width fluctuations during a single growth process, and the lateral correlation length along the triple line - all as a function of the silver substrate roughness and the temperature of the system. The kinetic roughening studies revealed limited information on the spatio-temporal behavior of the system. However, by calculating the persistence exponent, we were able to identify two distinct kinetic time regimes in this process. In the first one, while the interface is moving, but its width is not yet growing, the persistence exponent is  $\theta = 0.55 \pm 0.05$ , which is typical for a random, noisy behavior. In the second regime there is an effective growth of the interface width, with growth exponent  $\beta = 0.67 \pm 0.06$ , followed by saturation, according to the Family-Vicsek description of interface growth. The persistence exponent in this regime is  $\theta = 0.37 \pm 0.05$ , which indicates that the relation  $\theta = 1 - \beta$ , seems to hold even for this non-linear experimental system.

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## Temperature dependence of consumption behaviors and optimization of sales strategy

H. Takayasu<sup>†</sup>, Y. Goto, M. Takayasu

<sup>†</sup>Sony CSL

It is said that about 70% of the whole economic activities are influenced by climate. Considering the effect of predicted future abnormal weather it is an important basic step to clarify temperature dependence of human consumption behaviors based on detailed sales data. Summer of 2010 was extraordinary hot in Japan and ice creams and ice candies were sold well. In order to quantitatively analyze this phenomenon we analyzed Point-Of-Sales data of about 300 convenience stores in Tokyo area for about a year comparing with the temperature data provided by the Meteorological Agency [1]. By plotting daily sales number of each product against the mean temperature, it is found that the mean number of sales for each product is well approximated by an exponential function of the mean temperature. From this result it is possible to measure the characteristic temperature for each product, for examples,  $T = 10$  for an ice candy and  $T = 28$  for a high quality ice cream. This  $T$  means that the sales number increases  $e = 2.718$  times when the temperature increases  $T$  degrees. The sales number of ice candy increases much more rapidly than that for ice creams when the temperature goes higher. There are products with negative values of characteristic temperature meaning that sales decrease with higher temperature such as hot drinks. After estimating the characteristic temperature for each product we consider optimization of sales strategy taking into account the effect of such temperature dependence. We use officially predicted tomorrow's temperature and estimate the best number of stocks for tomorrow's sale. It is shown that the prediction error of temperature plays an important role and we estimate the error width by observing from the temperature data. For products with short "consume by" date, large number of stocks can make a big loss by disposal [2,3]. We assume that the number of customers basically follows a Poisson distribution with the mean value given by an exponential function of predicted temperature and estimate the most appropriate number of stocks that maximize the profit. Checking the theoretical estimates with the real data of convenience stores, we find that there are many stores which underestimate the sales number increase caused by high temperature losing chances of big sales.

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## Empirical laws and mathematical models for the statistics and structure of business firm networks

M. Takayasu†

†Tokyo Institute of Technology

In this paper I will firstly review empirical laws found in the huge database of about 1 million Japanese business firms including information about business partners. From this data we can construct the whole network structure of Japanese business firms with directed links showing the B-to-B money flow direction. The link number distribution generally follows a power law with the exponent about 1.3 for the whole country, for each business category and for each prefecture. This law holds for in-links and for out-links, and for the sum of both links. Namely, the scale-free property is very universal [1]. Similarly universal strong power laws are confirmed for the distribution of sales which follows the so-called Zipf's law with exponent 1, the number distribution of employees follows a power law with exponent about 1.3, and the distribution of growth rate of annual sales roughly follows also a power law with an exponent close to 1. There are quantities which follows an exponential law. The distribution of age of business firms follows an exponential distribution implying that firms bankrupt nearly randomly. For each firm the number of links grows exponentially with the age on average. The distribution of link-length between any pair of firms decays exponentially with the mean value about 5 and the maximum value 21. It is confirmed that a young firm tends to have a link with an existing firm with probability proportional to its link numbers, showing the validity of so-called preferential attachment. However, simple preferential attachment is not enough and it is required to consider the effect of merger or coagulation to explain the link number distributions [2]. From the viewpoint of statistical physics the link distribution of business firm network has interesting similarity with aerosol systems. In the case of business networks new firms are injected at nearly a constant rate, firms grow slowly in most cases but sometimes grow discontinuously by merger, and a firm disappears by bankruptcy, while in the case of aerosols new particles are constantly injected, particles grow discontinuously by coagulation and they disappear by sedimentation, for example. In both cases statistically steady power law distributions realize even though each process is highly irreversible. Our recent study on this topic clarifies the way of estimating the money flow flux among firms [3], and development of network failure can now be simulated numerically finding a kind of dynamical phase transition under some conditions.

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## Numerical simulation of 2D granular particles and its analyses by means of the micropolar fluid model

K. Takechi†, K. Yoshida, T. Arimitsu

†University of Tsukuba

Collective motions of granular materials behave like fluid motions under appropriate conditions such as sparse powders. However, unlike the Newtonian fluids, the basic equations for the collective motions of granular particles have not been well established yet, because the characteristics specific to particles reveal themselves for dense powders. In order to check the validity or the applicability of the existing theories for granular flow, we performed a numerical simulation of two-dimensional granular particles under uniform shear and external body torque, and analyzed it [1] by means of the models of the micropolar fluid, a fluid with polar micro-structures such as spin. The microscopic properties of granular particles are reflected in the equations of motion or the macroscopic fields through the constitutive equations i.e., the relations between strains and stress. We focus on the constitutive equations of granular flow in which the spin field is not subordinate to the vorticity field. Uniform mean shear field and mean spin field, which are not subordinate to the vorticity field, are realized in the simulations with external torque. The spin of granular particles can be coupled to the dynamics of the macroscopic collective motions of granular particles. The estimates of stresses based on kinetic theory by Lun [2] are in good agreement with the simulation results for a low area fraction  $\nu = 0.1$  but the agreement becomes weaker as the area fraction gets higher. However, the estimates in the kinetic theory can be fitted to the simulation results up to  $\nu = 0.7$  by renormalizing the coefficient of roughness. For a relatively dense granular flow ( $\nu = 0.8$ ) near the jamming transition point, the simulation results are also compared with Kanatani's theory [3]. It is found that the dissipation function and its decomposition into the constitutive equations in Kanatani's theory are not consistent with the results obtained by our simulation. The present study suggests that there is a regime of relatively dense granular flows in which physical pictures based on neither kinetic theories nor Kanatani's theory is adequate. The regime is located near the jamming transition point. In order to understand the mechanism of the transition, it would be necessary to draw a new microscopic description of particle interaction that is different from mutually independent short-time collisions in the kinetic theory or long-time contacts with sustained velocity difference in Kanatani's theory.

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## Kinetics of a frictional granular motor

J. Talbot<sup>†</sup>, R.D. Wildman, P. Viot

<sup>†</sup>Université Pierre et Marie Curie, Paris, France

Recently, Eshuis et al. (2010), inspired by Smoluchowski's gedankenexperiment, constructed a macroscopic rotational motor consisting of four vanes immersed in a granular gas. When a soft coating was applied to one side of each vane, a ratchet effect was observed above a critical granular temperature. There are also several theoretical studies of granular motors, e.g. Cleuren and Eichhorn (2008), Costantini et al. (2008) and Talbot et al. (2010). All of the proposed devices share the common features of non-equilibrium conditions and spatial symmetry breaking. None of these studies, however, considered the effect of dry friction. Yet as the experiment of Eshuis et al. confirms, frictional forces between the granular rotor and the axis of rotation cannot be neglected. We consider a two-dimensional system where a heterogeneous chiral rotor with moment of inertia  $I$ , mass  $M$  and the length  $L$  immersed in a granular gas at density  $\rho$  with a velocity distribution  $\phi(v)$  characterized by a granular temperature  $T$ . The motor is composed of two materials with coefficients of restitution  $\alpha_+$  and  $\alpha_-$  so that collisions of the bath particles with the former (latter) exert a positive (negative) torque. At low bath temperature, the rotor effect diminishes rapidly because the torque due to thermal fluctuations becomes smaller than the frictional torque. The motor torque is the average of all impulses resulting from collisions between the rotor and the bath particles. For large rotor mass, one finds that the fluctuation torque is proportional to  $(\alpha_+ - \alpha_-)\rho L^2 T$ . The incorporation of friction in the existing models and its effect is non-trivial. We report results for the heterogeneous rotor in the presence of dry (Coulomb) friction within the framework of the Boltzmann-Lorentz equation. A time scale analysis leads us to consider two limits: for large friction, where the rotor comes to rest between collisions, we obtain an exact solution for the angular velocity distribution function. In the limit of large rotor mass and small friction, where sliding dominates the motor dynamics, we derive a Fokker-Planck equation for which the exact solution is also obtained. In both cases, the velocity distributions may display strongly non-Gaussian behavior.

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## Quantum fluctuation relations

P. Talkner<sup>†</sup>, M. Campisi, P. Hänggi

<sup>†</sup>Institute of Physics, Universitaet Augsburg, Germany

Fluctuation relations allow to extract equilibrium properties of a system from the statistics of work performed on the system by the action of an external force. The resulting perturbations of the system are neither restricted to quasi-static changes nor to the regime of linear response and may drive the system arbitrarily far from thermal equilibrium. Accordingly, the fluctuation relations have a much wider area of validity than linear response theory. Two fundamental ingredients play a decisive role in the foundation of fluctuation relations: the principle of micro-reversibility and the fact that initially, before the protocol starts to act, the system stays in thermal equilibrium that is described by the Gibbs canonical ensemble [1]. Fluctuation theorems for micro-canonical and grand-canonical initial states can also be obtained but will not be considered in the present talk. For the present purpose, the well-known principle of microscopic reversibility must be generalized to also allow for explicitly time-dependent driving in the framework of classical and quantum Hamiltonian mechanics. Moreover, both in classical and quantum statistical mechanics consistent definitions of work and energy have to be employed in order to avoid seeming contradictions [1,2]. In the present talk we will mainly concentrate on the quantum versions of fluctuation theorems. We will discuss their generalization to open quantum systems without making recourse to quantum master equations [3]. Most relevant in this context are the proper identification of work and the consistent statistical mechanical and thermodynamic description of an open system which may be in strong interaction with its environment. We show how this can be achieved such that the fluctuation relations continue to hold for the work performed on the open system and the free energy of the open system in formally the same way as for a closed system. We will also consider the impact of measurements on the underlying statistics of work. Even though projective measurements of any observable being performed during the realization of a force protocol do strongly influence the distribution of work applied by this force the fluctuation theorems of Tasaki-Crooks and Jarzynski stay intact [4]. This also continues to hold true in the limit of infinitely frequent measurements that totally suppress the coherent dynamics of the system and give rise to the quantum Zeno effect. We shall discuss these measurement-related issues on the example of a two-state model undergoing a Landau-Zener transition [5].

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### Molecular dynamics simulations tackle the species specificity of the complement inhibitor compstatin by designing transgenic animal proteins and de novo dual-specificity ligands

P. Tamamis<sup>†</sup>, A.L. de Victoria, R. Gorham Jr., C.A. Floudas, D. Morikis, G. Archontis

<sup>†</sup>University of California at Riverside, Princeton University and University of Cyprus

Overzealous or inappropriate activation of the complement system, the first line of defence against foreign pathogens, may cause or aggravate several pathological conditions or complement related diseases, such as macular degeneration and rejection of xenotransplantation. Therefore, the development of drugs that can regulate the activation of the complement system is of profound medical interest. Compstatin, a 13-residue peptide inhibits protein C3 in primate mammals. In the past, Molecular Dynamics (MD) simulations of compstatin in solution improved our understanding on the properties of compstatin and related analogues [1]. Despite compstatin's therapeutic potential against human complement activation, compstatin is inactive against non-primate species. Undoubtedly, the inhibition of non-primate species complement activation is vital to drug development, since it would allow testing of compstatin in non-primate animal disease models. Recent MD simulations have elucidated the species specificity of compstatin by investigating complexes between the most potent natural compstatin analog (W4A9) and human [2] or rat/mouse [2,3] C3. The human-complex simulations provided information on the relative contributions to stability of specific C3 and compstatin residues. In non-primate simulations, the protein underwent reproducible conformational changes, which eliminate or weaken specific interactions and reduce the complex stability. Based on these results, we searched for dual-specificity active inhibitors towards two different directions. In the first direction, we designed in-silico novel transgenic mouse C3 proteins, by incorporating specific human-like mutations, and investigated complexes between the "transgenic" mouse C3 proteins and W4A9 using MD simulations [3]. In the "transgenic" complexes the conformation remains closer to that of the human complex, the protein-ligand interactions are improved, and the affinity for compstatin becomes "human-like". This work creates new avenues for a compstatin-sensitive animal model and introduces a computational toolkit to design novel "transgenic" sequences [3]. In the second direction, we conducted a series of simulations of novel compstatin analogs in complex with human and rat/mouse C3, aiming at both increasing the potency against human C3 and inhibiting non-primate species complement activation. The simulations reveal promising high potency human and probable dual-specificity inhibitors; in line with this, a recent experimental work examined a specific category of the promising analogs speculated by our simulations and depicted improved-altered local interaction capabilities in complex with human C3 [4].

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### From molecules to network fluids: Condensation or percolation ?

M. Telo da Gama<sup>†</sup>

<sup>†</sup>Centro de Física Teórica e Computacional, Universidade de Lisboa

We address the influence of directional interactions, or bonding sites, on the structure and phase diagram of complex fluids. We investigate the interplay between the self-assembly process, driven by the bonding interactions, and phase transitions (condensation, ordering) for primitive models of patchy colloids, consisting of particles with a number of distinct bonding sites. The models are studied on and off-lattice in two and three spatial dimensions. In the off-lattice work we consider primitive models of patchy colloids in three dimensions consisting of hard spherical particles with identical or distinct short-ranged attractive sites (patches) on their surfaces and use Wertheim's thermodynamic perturbation theory and a generalization of the random-bond theory of percolation to: 1. Derive asymptotic expansions for the free energy close to the ideal limits of linear chains, hyperbranched polymers, and dimmers. 2. Study the interplay between condensation and cluster formation (percolation). 3. Propose an effective model for the cluster size distribution functions, in excellent agreement with computer simulations. 4. Investigate - by theory and simulation - the empty fluid regime and the re-entrant phase diagram of the primitive model of colloidal particles with distinct patches, in the limit of linear chains. 5. Investigate the empty fluid regime of primitive models of binary mixtures of patchy colloidal particles. In the lattice work we consider the interplay between the self-assembly process and the condensation and/or ordering transitions, for models consisting of particles with four and two bonding sites, on two-dimensional lattices. The theory is applied over a wide range of temperature and density and the results are compared with Monte Carlo simulations. Finally, extensive Monte Carlo simulations and finite-size scaling analyses were carried out to investigate the nature of the condensation and ordering transitions - on the square lattice. The transitions are found to be in the two-dimensional Ising universality class. For models with two bonding sites this results is in agreement with the results for models where the rods are monodisperse and in contrast with a recent claim that equilibrium polydispersity changes the nature of the phase transition in these models. We confirmed that polydispersity does not affect the nature of the ordering transition of this class of lattice models by investigating the same model on a triangular lattice.

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## Group Entropies

P. Tempesta<sup>†</sup>

<sup>†</sup>Departamento de Física Terica II, Facultad de Fsicas, Universidad Complutense de Madrid, Spain

In this talk, the notion of Group Entropy, recently introduced, is discussed. It enables to unify and generalize many different definitions of entropy known in the literature, as those of Boltzmann-Gibbs, Tsallis, Kaniadakis and Abe. The underlying idea is to extend the nonextensive approach in statistical mechanics in order to englobe a very general class of correlation laws among subsystems of a given system. Under suitable hypotheses, to each choice of the correlation law it corresponds an entropic functional and a related microscopical description of the dynamics. Our theory reposes on the notion of formal groups, proposed by Bchner with the aim of generalizing Lie groups and algebras. In the last decades, the theory of formal groups has been widely investigated for its crucial role in many branches of pure mathematics, like cobordism theory, theory of genera, homology theory and in the theory of elliptic curves. Precisely, we will show that, given a suitable formal group law, it is possible to associate with it in a natural way a universality class of statistical systems and an appropriate entropy of trace-form type. Due to the ubiquitous role of the notion of entropy in modern science, the possible applications of the proposed construction are manifold. For instance, apart the many physical contexts when the nonextensive statistical approach is required, a nontrivial application of our entropic functionals emerges in the context of Information Theory. We will show that these entropies provide a class of information measures, including the Shannon information measure as a particular case. Also, we propose a generalization of the Kullback-Leibler divergence, that allows us to define a new set of tests measuring the difference between two given probability distributions. Finally, we describe a possibly new connection between statistical mechanics and number theory. This fascinating topic has an intriguing history, dating back to the 70's with the works of Montgomery and Odlyzko, relating the Gaussian unitary ensemble with the zeros of the Riemann zeta function. Under appropriate hypotheses, with a given class of universality we can associate a zeta function, constructed by using the same realization of the Lazard group used to define the corresponding entropy. The first nontrivial case is the Tsallis class, which corresponds to the celebrated Riemann zeta function.

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## Instabilities of molecular motor assemblies: oscillations and motion reversal due to weak noise

G. Thomas<sup>†</sup>, T. Guérin, J. Prost, J.-F. Joanny

<sup>†</sup>CNRS

In many examples in intracellular transport, cargoes transported by molecular motors display a motion that is bidirectional rather than unidirectional. This type of motion, also observed in acto-myosin systems, typically appears in the presence of two groups of motors with opposite polarity, and enables an object to move efficiently in two directions. Bidirectional motion can be seen as a consequence of the collective behavior of molecular motors. We propose an analytic theory for the calculation of the reversal time in this system that lacks detailed balance. We consider the bidirectional motion as a first exit problem. We identify the noise strength by doing an expansion of a master equation and apply the Wentzell-Freidlin theory to define an effective nonequilibrium potential and provide analytical estimates of the reversal time. The reversal time follows an Arrhenius law. With high probability, a reversal event takes place along an "optimal trajectory" in the space of configurations. Our results match very well with the results of stochastic simulations. We discuss another collective effect (an oscillatory instability) appearing in the same model and compare our results to a recent experiment indicating that the acto-myosin system can become self-oscillatory when subjected to an elastic load. Instabilities due to molecular motors might be relevant for describing mechanical oscillations in a variety of biological systems, including muscles, sensory hair-cell bundles of the inner ear, and flagella. Finally, we compare two models of molecular motors: the crossbridge (or "powerstroke") and rigid models. We discuss the frequency and amplitude of the oscillations and determine whether motor assemblies can trigger oscillations with a period shorter than the chemical cycle time. We also compare the theoretical force-displacement relations to the tension-length curves measured in muscles. Finally, we introduce a more general theory, the "soft-motor model", which enables to explicitly link the crossbridge and the rigid models which we re previously considered as distinct in the literature. In this new model, the crossover parameter (the "pinning parameter") plays an important role in the single-state version of our model, which turns out to be a classical model of solid friction.

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## Entropy of non-ergodic systems - a derivation from first principles

S. Thurner†, R. Hanel

†Medical University of Vienna

In information theory the so-called 4 Shannon-Khinchin (SK) axioms single out Boltzmann Gibbs entropy,  $S \sim -\sum_i p_i \log p_i$ , as the unique possible entropy. Physics is different from information in the sense that physical systems can be non-ergodic. To characterize strongly interacting statistical non-ergodic systems with a thermodynamical framework - complex systems in particular - it might be necessary to introduce generalized entropies. A series of such entropies have been proposed in the past. Until now the understanding of the fundamental origin of these entropies and its deeper relations to complex systems is unclear. To clarify the situation we note that non-ergodicity explicitly violates the fourth SK axiom. We first show that by violating this axiom and keeping the other three intact there exists an explicit form of 'generalized' entropy,  $S \sim \sum_i \Gamma(d+1, 1 - c \log p_i)$ , where  $c$  and  $d$  are scaling exponents, uniquely describing a statistical system; Gamma is the incomplete Gamma function. All recently proposed entropies compatible with the first 3 SK axioms appear to be special cases. We next prove that each (!) statistical system is uniquely characterized by the pair of the two scaling exponents  $(c, d)$ , which define equivalence classes for all (!) interacting and non interacting systems. The corresponding distribution functions are special forms of Lambert-W exponentials containing - as special cases Boltzmann, stretched exponential and Tsallis distributions (power-laws) - all widely abundant in nature. This is, to our knowledge, the first ab initio justification of the existence of generalized entropies. Finally, we show how the phase space volume of a system is related to its generalized entropy. We give a concise criterion when this entropy is not of Boltzmann-Gibbs type but must assume a generalized form. We prove that generalized entropies can only exist when the dynamically (statistically) relevant fraction of degrees of freedom in the system vanishes in the thermodynamic limit. These are systems where the bulk of the degrees of freedom is frozen and is practically statistically inactive. Systems governed by generalized entropies are therefore systems whose phase space volume effectively collapses to a lower-dimensional 'surface'. We explicitly illustrate the situation for binomial processes and argue that generalized entropies could be relevant for self organized critical systems such as sand piles, for spin systems which form meta-structures such as vortices, domains, instantons, etc., and for problems associated with anomalous diffusion.

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## Multi-agent analysis of financial data

T. Tokar†, D. Horvath, M. Hnatic

†SORS Research a. s., Junior researcher

In the proposed work we deal with distributed prediction and economic activity data analysis. The agent-based parallelization of the elementary prediction methods is studied as a feasible tool for analysis of historical data of currency exchange rates. Each trend-following agent (TF) makes its prediction according to trend using individual time window. Correct/wrong predictions result in the increase/decrease of TF's "utility" - measure of agent's competence to make predictions. TF agents are born with certain, non-zero utility that is equal for each newly born agent. Once, utility of agent falls under the zero value, the agent is replaced by successor, with new randomly generated time scale which remains unchanged during the life. Six years tick quotation records of EUR/USD, USD/CHF and USD/JPY exchange rates were used to perform numeric analysis. Three different variants of prediction were examined, corresponding to three different prediction horizons - 1, 100, 1000 tick quotes ahead. Besides of population of non-interacting agents, we have studied several forms of the interaction between agent within population, including evolution-like selection and preferential reproduction of competent agents, and interaction through broadcasting of the information obtained by certain agent. We were interested in relationship between life-time of the agent and agent's time window. Another point of our interest were the transient characteristics of the population of agents within particular data environments. Our aim was to use this approach to understand process of creation of internal scales within data environments and their subsequent vanishing and replacement by new ones. Our results indicate that even so simple strategy as trend following may give its user chance to survive for surprisingly long time. We found data specific, non-trivial relationships between life-time of the agent and its scale. Study of the transient characteristics revealed existence of stages of continuous growth of the mean utility of population, where deaths occur only very rarely. These are interrupted by short epochs of massive extinctions. These observations conclude in hypothesis of repetitive formation of characteristic scales within price change trends of certain duration, that are subsequently replaced by newly formed scales.

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## Pesin Identity at the edge of chaos: Averaging on single trajectories vs ensemble averages

R. Tonelli†, M. Lissia, F. Ricci

†University of Cagliari, Italy

Recently an extension of the Pesin identity for dynamical systems in the chaotic state has been proposed by us [1] in the case of dynamical systems at the threshold of chaos, using the framework of non-extensive thermodynamics [2]. This identity in the case of chaotic systems is based on the definition of Kolmogorov-Sinai entropy evaluated along a single trajectory in the phase space, assuming this trajectory on the basin of attraction of some attractor onto which the dynamics evolves [3]. The phase space is then partitioned into boxes and the system state is thus observed at discrete intervals of time. This definition requires the evaluation of the joint probability that the trajectory visits a defined sequence of boxes in the limit that such a sequence be infinite. This may be computationally prohibitive and an approach based on the evolution of the divergence among two infinitely close trajectories is usually employed. This is also true in the case of the less critical divergence at the threshold of chaos, for systems weakly sensitive to initial conditions. In order to extend the Pesin identity, using the formalism of non-extensive thermodynamics, to the threshold of chaos, we have used more convenient ensemble averages to calculate the non-extensive entropies. Despite the result of obtaining a correct extension of the identity, our approach raised some criticism because of the possible differences that might exist among the ensemble averaged entropy and the entropy calculated using joint probabilities along a single trajectory. In order to solve this problem in this work we propose a new quantity related to the scaling of the correlation integral extended using the formalism of non-extensive thermodynamics. We evaluate an extension of the correlation integral [4][5] along a single trajectory in the phase space and use appropriated deformed exponentials/logarithms to analyze power-law behaviors. In the limit of usual exponentials/logarithms we recover the standard form for the KS entropy which is null at the border of chaos and is useless to characterize power-law sensitivity. We compare the usefulness of this formalism with results obtained through ensemble averages for deformed entropies.

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## Limitations of statistical mechanics: Hints from large deviation theory

H. Touchette†

†Queen Mary, University of London

The ensemble theory of equilibrium systems formulated by Boltzmann and Gibbs is not usually thought of as having theoretical limits, in the sense for example that we think of Newtonian mechanics as being limited to non-relativistic systems. Some might argue that such limits must exist - that equilibrium statistical mechanics, as any theory, must have boundaries beyond which it does not apply. But what exactly are these boundaries? Can they be defined mathematically? That is, can we state conditions that guarantee that a given many-body equilibrium system falls within the working realm of equilibrium statistical mechanics? Many approaches can be followed to answer these questions. On the one hand, one can define, as van Hove, Ruelle and Fisher did, classes of many-body systems for which the entropy and free energy can be shown to exist in the thermodynamic limit, and so for which equilibrium statistical mechanics give verifiable predictions. From there, one can then derive conditions on the entropy, as was done recently for long-range interacting systems, to decide when this function is the Legendre transform of the free energy and vice versa, and so to decide when the microcanonical and canonical ensembles give equivalent predictions, again in the thermodynamic limit. On the other hand, one can study more physical issues such as whether ergodicity is a necessary condition for equilibrium statistical mechanics or whether relativistic systems can be described by this theory. In this talk I will argue that the proper theoretical framework for answering questions about the conditions of validity or the limits of equilibrium statistical mechanics, taken as the ensemble theory of Boltzmann and Gibbs, is the framework of large deviation theory. The latter theory has been used with great success in the last 30 years or so to study the properties of equilibrium systems, and to put the foundations of equilibrium statistical mechanics on a clear and rigorous footing. This effort, as I will argue, is important for statistical mechanics: it means, beyond the obvious practical benefits, that the knowledge of the conditions of validity of large deviation theory can be used to derive conditions of validity for equilibrium statistical mechanics. Some examples drawn from large deviation theory will be used to illustrate what is meant by this. The more speculative problem of generalizing equilibrium statistical mechanics will also be discussed.

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## New phenomena at oxide interfaces

J.-M. Triscone<sup>†</sup>, A. Caviglia, N. Reyren, S. Gariglio, C. Cancellieri, A. Fête, M. Gabay, B. Sacépé, A. Morpurgo, J.-M. Triscone

<sup>†</sup>University of Geneva

Oxide materials display within the same family of compounds a variety of exciting electronic properties ranging from ferroelectricity to ferromagnetism and superconductivity. These systems are often characterized by strong electronic correlations, complex phase diagrams and competing ground states. This competition makes these materials electronic properties very sensitive to external parameters such as pressure or magnetic field. An interface, which naturally breaks inversion symmetry, is a major perturbation and one may thus expect that electronic systems with unusual properties can be generated at oxide interfaces. The numerous degrees of freedom in oxide materials (charge, spin, orbital, lattice) may lead at such interfaces to novel ordering phenomena and electronic properties different from the ones of the bulk compounds. A striking example is the interface between LaAlO<sub>3</sub> and SrTiO<sub>3</sub>, two good band insulating perovskites, which was found in 2004 to be conducting with a high mobility [1]. The ground state of this system is a superconducting condensate, with a critical temperature of about 200 mK [2]. The characteristics observed in the normal and superconducting states are consistent with a two-dimensional electronic system. The thickness of the electron gas is found to be a few nanometers at low temperatures. This electron gas with low electronic density, typically  $5 \cdot 10^{13} \text{ electrons/cm}^2$ , and naturally sandwiched between two insulators is ideal for performing electric field effect experiments allowing the carrier density to be tuned. Such an approach revealed the sensitivity of the normal and superconducting states to the carrier density. In particular, the electric field allows the tuning of the critical temperature between 200 mK and 0 K and thus the on-off switching of superconductivity. The system phase diagram reveals a superconducting pocket with an underdoped and an overdoped regime [3]. A large, interfacially generated, tunable spin-orbit coupling and a remarkable correlation between the spin-orbit coupling strength and the system phase diagram are other hallmarks of this fascinating system [4]. Recent developments allow high mobility samples to be grown. In such samples, resistance measurements display Shubnikov de Haas oscillations pointing to 2D electronic states [5]. I will present the status of the field and the perspectives opened by this new research area sometimes called “oxide interface engineering”.

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## A proper nonlocal formulation of the Quantum Maximum Entropy Principle in statistical mechanics

M. Trovato<sup>†</sup>, L. Reggiani

<sup>†</sup>Dipartimento di Matematica e Informatica Università di Catania, Italy

By considering the Wigner formalism [1], the quantum maximum entropy principle (QMEP) is here asserted as the fundamental principle of quantum statistical mechanics when it becomes necessary to treat systems in partially specified quantum mechanical states. From one hand, the main difficulty in QMEP is to define an appropriate quantum entropy [2] that explicitly incorporate quantum statistics. On the other hand, the availability of rigorous quantum HD (QHD) models is a demanding issue for a variety of quantum systems like, interacting fermionic and bosonic gases, confined carrier transport in semiconductor heterostructures, etc. Recently, in a series of two papers [3,4] we have presented a set of results addressing this problem by emphasizing the role played by a proper definition of a quantum entropy principle to close quantum hydrodynamic models. Here, we present a rigorous nonlocal formulation of QMEP by defining a quantum entropy that includes indistinguishability for a system of identical particles. Accordingly, we provide a general framework for the development of closed quantum hydrodynamic models in the case of Fermi and Bose gases at different level of degeneracy. Relevant results of the present approach are: (i) The development of a generalized three dimensional Wigner equation. (ii) The construction of Extended Quantum Hydrodynamic models evaluated exactly to all orders of  $\hbar$ . (iii) The definition of a generalized quantum entropy as global functional of the reduced density matrix. (iv) The formulation of a quantum version of the maximum entropy principle obtained by determining an explicit functional form of the reduced density operator, which requires the consistent introduction of nonlocal quantum Lagrange multipliers within a Moyal expansion. (v) The development of a quantum-closure procedure that includes nonlocal statistical effects in the corresponding quantum hydrodynamic system. (vi) The development of a closure condition for a set of relevant quantum regimes of Fermi and Bose gases both in thermodynamic equilibrium and nonequilibrium conditions. Finally, we show that all the results available from literature in the framework of a quantum Boltzmann gas are deduced as particular cases [5] and that the present formulation of the QMEP recovers its classical form in the limit  $\hbar \rightarrow 0$ .

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## A new physics-based approach to model quality assessment of protein structures

A. Trovato<sup>†</sup>, P. Cossio, A. Laio, F. Seno

<sup>†</sup>Physics Department - University of Padova and CNISM - Padova Unit, Italy

Knowledge-based approaches employ empirical relations to determine effective statistical potentials for protein models directly from protein databank structures. These approaches had a considerable success in computational protein science, and their use is widespread in state-of-the-art methods [1,2] able to discriminate correctly the native structure among a set of competitive decoys. In the case of model quality assessment [3], one searches for a good scoring function whose score correlates with decoy proximity to the native structure. We propose a strategy where we combine three different statistical potentials based on physical properties. One has been already introduced in the context of protein aggregation [4], and is based on pairwise interactions between backbone-backbone hydrogen-bonded residues. One is based on general pairwise interactions between residue side chains, in the spirit of quasi-chemical approximation [5]. One is based on the solvation properties of single residues. The resulting scoring function is implemented for a full atomistic description of the model protein structure. We employ the top500H database, a non-redundant specially refined set of 500 high resolution X-ray crystallographic structures of globular proteins, where hydrogen atoms were also reconstructed. A key ingredient in our approach is the recognition that physics-based potentials cannot cope with situations where the sequence of decoys differs from the sequence of the part of the native protein that is found to be structured. This is very common in CASP targets, due to part of the proteins that are not seen by crystallographers, but were modeled by the groups participating in the CASP structure prediction competition. We thus selected for our analysis only CASP targets where all residues of the target sequence were crystallized in the native structure. Moreover, we discarded decoys (predictions from alignment-based methods) which also do not share that sequence. Our statistical potential outperforms state-of-the-art methods on the selected subset of CASP8 and CASP9 targets, and at the same time drastically reduce the number of employed parameters. We will discuss possible applications of our new knowledge-based potential in the field of protein structure prediction and protein design.

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## Effect of overnight gap on financial return and volatility in Taiwan stock market

K.-T. Tsai<sup>†</sup>, J.-S. Lih, J.-Y. Ko

<sup>†</sup>Department of Physics, National Kaohsiung Normal University, Taiwan (R.O.C.)

We studied the statistical regularities among three components of stocks or indices: the daytime (trading hour) return, the overnight (off-hour session) return, and the total (close-to-close or open-to-open) return. Recently many studies on daily returns and volatilities [1,2] or high-frequency quotes [3,4] presented the response from investors during trading hours. However, the new events and information which happened in off-hour session would be reflected in the overnight gaps. Fengzhong Wang et al. studied the correlations among three components of the 2215 New York Stock Exchange (NYSE) stocks from 1988 to 2007. They found (1) the cross correlation between the daytime returns and the total returns was strong; (2) the daytime returns and overnight returns tended to be anti-correlated [5]. We assumed the length of off-hour session would affect the overnight gaps, so we chose a market with longer off-hour session to explore the correlation among the three components. Compared with the major exchanges in America and Europe, the Asian exchanges operate for longer off-hour session. Especially, the Taiwan Stock Exchange (TWSE) is one of the Asian major exchanges and regular operates from 9:00 a.m. to 1:30 p.m. The TWSE has the longest off-hour session among major world exchanges. Therefore, the TWSE is the most suitable market to study the effect of overnight gap on financial return. The analysis of Taiwan Stock Exchange Corporation (TSEC) weighted index showed that the cross correlation between daytime return and overnight return presented negative correlation. Additionally, the cross correlation between volatility of daytime return and the volatility of overnight return presented positive correlation. In order to realize the correlation between volatility of daytime return and the volatility of overnight return, we introduced the volatility rank which was to sort volatilities in order of values for characterizing the relative position of distribution of volatilities. We found that a small volatility rank of a certain component often followed a small volatility rank of the other components. Likewise, a large volatility rank of a certain component often came from a large volatility rank of the other components. Moreover, we analyzed the probability of the same direction of every component and its following components. Compared with other indices, the TSEC weighted index presented the most possibility of an opposite direction of overnight return to its following daytime return.

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## Enhanced stochastic resonance of individuals in complex networks

K.-T. Tsai<sup>†</sup>, J.-S. Lih, J.-Y. Ko

<sup>†</sup>Department of Physics, National Kaohsiung Normal University, Taiwan (R.O.C.)

We studied the enhancement of stochastic resonance (SR) of individual oscillator in different topologies of complex network with the degree inhomogeneity. SR is a counterintuitive phenomenon and interests researchers in many fields, such as physical [1] and biological sciences [2]. A traditional description of SR defines that a weak signal could be amplified by tuning the intensity of external noise. In order to optimize the enhancement of SR, many models of SR were investigated in the last several decades. A typical model of SR is a weak periodic driving bistable system. When a moderate noise intensity adds to the system, the output signal has an optimal signal-to-noise ratio (SNR). In other words, a SR system shows a peak on a plot of SNR versus noise intensity. Lindner et al. found the peak could be enhanced by coupling identical bistable systems into a chain [3]. Because the topology of networks might influence the cooperative behavior of the systems, there were many investigations on SR in different coupling structures [4]. In order to show the global response of SR, the SNR (or spectral power amplification, SPA) was calculated and averaged over different elements of the networks. However, little attention has been paid to enhanced SR in each oscillator [5]. The global mechanics of networks is formed by their interactions. Through the couplings an oscillator could spread its information to connected neighbors. When the number of connected neighbors increases, the effect of the oscillator on the network would also be greater. Understanding the relationship of local activity interacting with local couplings is helpful to realize the behavior of networks. We study how enhanced SR depends on the degree of each oscillator in complex networks with the degree inhomogeneity. Four different coupling topologies are considered: (1) the scale-free networks, (2) the deterministic scale-free networks, (3) the small-world networks, and (4) the random networks. We use the SNR peak, which is the maximum SNR at the optimal noise intensity, to characterize the response of SR of each oscillator. In the four types of networks, the relationship between the SNR peak and the degree of individual oscillator follows the same empirical fitting curve. The curve always arose and approached to an upper limit when the degree of individual oscillator increased. Additionally, the enhanced SR of individual oscillator in a network would be the same at extremely weak and extremely strong coupling strength.

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## Exploring the region where the hypothesis validating Boltzmann-Gibbs statistical mechanics are not satisfied

C. Tsallis<sup>†</sup>

<sup>†</sup>Centro Brasileiro de Pesquisas Fisicas - Rio de Janeiro and Santa Fe Institute - New Mexico

The Boltzmann-Gibbs entropy and its associated statistical mechanics are producing, along 140 years, uncountable contributions to our knowledge of systems whose collective dynamics satisfy simplifying hypothesis such as ergodicity, and independence or quasi-independence of relevant random variables. Nevertheless a fascinating world exists outside this (and related) hypothesis. It is the purpose of the nonadditive entropy  $S_q$  and its associated nonextensive statistical mechanics to theoretically approach this world. We shall illustrate some of its main grounding concepts, and exhibit recent realizations and verifications in natural, artificial and social systems. We shall present: (i) recent results for both pure and random magnetic systems in their fundamental states (i.e., at  $T=0$ ), illustrating the relevant difference between entropic additivity and entropic extensivity, thus reconciling anomalous quantum-entangled systems with classical thermodynamics; (ii) recent results concerning a  $q$ -generalized representation of Dirac delta, a new representation of the number  $\pi$ , and the inverse of the  $q$ -generalized Fourier transform (typical examples such as  $q$ -Gaussians and Hilhorst recent counterexamples will be explicitly discussed); (iii) Nonlinear generalizations of the Schroedinger, Klein-Gordon and Dirac equations for free particles, with their exact solutions in terms of  $q$ -generalized plane waves that are normalizable, in contrast with standard plane waves which are not. Finally we address industrial (ceramics like the Limoges ones, more precisely in experiments involving Cs atoms implanted in zirconia) and medical (mammograms, with special emphasis in microcalcifications; the true positives being improved from 80% to 97%, and the false positive being decreased from 8% to 0.4%) applications of the concepts involved in  $q$ -statistics, as well as verifications and predictions for transverse momenta distributions and rapidity in  $p$ - $p$  collisions as well as in heavy ion collisions, in the LHC (CMS Collaboration) and Brookhaven Laboratories (PHENIX Collaboration). The values of  $q$  in the transverse momenta distributions are close to 1.1-1.2, a value which coincides with that obtained a decade ago by Walton and Rafelski, but whose microscopic interpretation still remains as an open problem.

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## Discrete nonlinear Schrödinger equation dynamics in complex networks

G. Tsironis<sup>†</sup>

<sup>†</sup>University of Crete and FORTH

The discrete nonlinear Schrödinger equation (DNLS) is a ubiquitous equation that appears in many areas of physics including condensed matter physics, optics, BEC, etc [1]. Its salient properties include a competition between propagation induced by the tunneling to usually nearest neighbor sites and localization induced by the nonlinear term. The latter introduces a form of self-focusing or selftrapping in the lattice. Most work in DNLS in the past has focused in regular lattices; we present here work in lattices that form small world networks [2]. These lattices incorporate two features, viz. increased transfer rates due to the presence of multiple links emanating from a given site and also some form of disorder due to the randomness of the links. We study the role of the competition of these two tendencies of the lattice in the presence of the cubic nonlinearity of DNLS. We start from a finite periodic chain with nearest neighbor interactions and insert randomly links connecting distant pairs of sites across the lattice. Using localized initial conditions we focus on the time averaged probability of occupation of the initial site as a function of the degree of complexity of the lattice and nonlinearity. We observe that selftrapping occurs at increasingly larger values of the nonlinearity parameter as the lattice connectivity increases, while close to the fully coupled network limit, localization becomes more preferred. For nonlinearity values above a certain threshold we find a reentrant localization transition, viz. localization when the number of long distant bonds is small followed by delocalization and enhanced transport at intermediate bond numbers while close to the fully connected limit localization reappears. Thus, for nonlinearity values above a certain threshold, the dynamics in the network is dominated by localization in the two extreme limits of nearest neighbor and fully connected coupling respectively and delocalization in the intermediate bond number regime. In this intermediate regime the increase of the number of bonds destroys nonlinear localization. We further analyze the dynamics in the fully connected limit of the network with arbitrary finite number of sites [3]. Using a localized initial condition we explore different behaviors that stem from the exact solution of the equation that is written in terms of the Weierstrass elliptic function. In this limit, we find that two dynamical transitions occur as a function of the nonlinearity parameter, a hyperbolic and a trigonometric one. In the latter the network behaves exactly as the corresponding linear one but with a renormalized frequency.

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## A description of the evolution of quantum states by means of the kinetic equation

Z. Tsvir<sup>†</sup>, V.I. Gerasimenko

<sup>†</sup>Taras Shevchenko National University of Kyiv

Experimental advances in the Bose condensation of dilute atomic gases and in the strong correlated Fermi systems have stimulated interesting problems on the quantum theory of many-body systems. Among them it is a description of collective behavior in such systems by quantum kinetic equations, i.e. by the evolution equations for a one-particle marginal density operator. Nowadays the considerable progress in the rigorous derivation of quantum kinetic equations in suitable scaling limits is observed. We develop a new formalism for the description of the quantum kinetic evolution based on usage of a non-perturbative solution of the quantum BBGKY hierarchy. We establish that in case of initial data, which is completely determined by a one-particle density operator, all possible states of infinitely many bosons or fermions at arbitrary moment of time can be described without any approximations by means of a one-particle density operator together with explicitly defined marginal functionals of this one-particle density operator. For this purpose we develop the method of the kinetic cluster expansions of cumulants of scattering operators, which define the evolution operators of every term of expansions of the marginal functionals of the state over the products of a one-particle density operator, and derive the generalized quantum kinetic equation. Thus, for initial states of many-particle systems obeying Fermi-Dirac or Bose-Einstein statistics, which are given in terms of a one-particle marginal density trace-class operator, the equivalence of the Cauchy problem of the quantum BBGKY hierarchy and the Cauchy problem of the generalized quantum kinetic equation is proved. The existence of a strong and a weak solution of the Cauchy problem of stated quantum kinetic equation is proved in the corresponding spaces of trace-class operators. The links between the specific quantum kinetic equations with the generalized quantum kinetic equation are discussed. One of the advantages of such approach is the possibility to construct the kinetic equations in scaling limits in the presence of correlations of particle states at initial time, for instance, correlations characterizing the condensate states. The specific quantum kinetic equations such as the Boltzmann equation and other ones can be derived from constructed generalized quantum kinetic equation in the appropriate scaling limits or as a result of certain approximations. For example, in the mean-field scaling limit we derive the quantum Vlasov kinetic equation for bosons or fermions. In particular case of pure states it reduces to the nonlinear Schrödinger equation.

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## Non-local models of swarming

M.S. Turner<sup>†</sup>, D.J.G.Pearce, G. Rowlands

<sup>†</sup>University of Warwick

Swarming is a non-equilibrium phenomenon observed throughout the animal kingdom; flocking in birds, swarming in insects [1], shoaling in fish and herding in mammals. Most previous attempts at understanding this phenomenon postulate that members of a swarm align their velocity with those of their immediate neighbours' [2]. Such local models seem plausible, mainly on the grounds that it is unfeasible for an individual to follow the positions and velocities of all  $N$  members of an arbitrarily large swarm. These models typically contain an alignment term and a noise term. This leads to phenomena reminiscent of phase transitions in statistical physics. However, both metric-based and metric-free versions of these models have undesirable features, e.g. the density  $\rho$  of the swarm decreases in time in the absence of an *ad hoc* long range attraction or a finite spatial system size. We analyse for the first time the possible role of a biologically plausible global measurement of the swarm in which each individual responds to a projection of the swarm. A simple class of candidate models arises naturally in which there is an additional parameter controlling the strength of response to the projection. We analyse this model in 2D and show that it leads to swarms that remain localized. By requiring the strength of the noise, alignment and projection terms to sum to unity we can examine the behaviour of the system through a two dimensional "phase diagram". We find that the order parameter  $\alpha = \sum_{i=1}^N |\mathbf{v}_i|$  has a maximum for some characteristic combination of alignment and projection (and hence, noise) strengths. Our model suggests a mechanism for swarms to self-select a particular density at which the swarm is marginally opaque. We show that the opacity reaches a plateau, with a characteristic value that is typically order unity, as the size of the swarm is increased. This leads to a scaling relationship between the number of individuals in a swarm and its density; in 2D this is  $\rho \sim N^{-1}$ , with the corresponding result in 3D being  $\rho \sim N^{-1/2}$ . We argue that some evidence for the latter scaling result already exists [3]. Our model therefore makes several experimentally testable predictions. It would also appear to provide emergent biological function given that marginal opacity provides for rapid, long range information transfer.

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## Digital ecologies for a green world

M. Ulieru<sup>†</sup>

<sup>†</sup>IMPACT Institute for the Digital Economy

Pervasive computing and communication environments link systems and people in unprecedented ways into a global socio-ICT "ecology" in which we and our 'things' co-exist in an atmosphere of information as a new kind of 'techno-social' system having the physical and Cyber - dimensions interwoven and juxtaposed with a social fabric (the users and their ability to form dynamic coalitions mediated via the Cyber dimension). By its hybrid techno-social nature a digital ecology constitutes a unique blend of socio-economic modeling and innovative ICT-enabled smart infrastructure design which considers user motivation in all its social relevance having people, in their many roles and personas, central in this respect, not only as "consumers" of applications, but also as producers, "players" and "inputs" that seamlessly drive the behavior of complex interdependent global-scale systems such as economies of scale and climate. We aim at designing digital ecologies as a framework for steering large scale techno-social systems by catalyzing and reinforcing beneficial collective user behavior. Our main concern is how to trigger the self-organization of users to achieve a particular goal (e.g. 'greening' the economy) using the digital ecology as a socially smart Cyber - controller. The digital ecology will be persuasive in that it will stimulate collaboration and provide incentives to facilitate socially flavored interactions with positive effects that support the objectives of the particular application. To this extent a digital ecology mirrors the 'invisible hand' of the market by acting like a 'social-network operating system' which leverages on the community via an incentive-driven mechanism. To improve the predictability and management of their consumption digital ecologies foster self-organization of groups around a particular trade-off of the incentive mix. To achieve such goals, it is essential for the digital ecology to obtain perceivable predictors of behavioral profiles of prosumers (aka the producers and consumers of services for a particular application) over different time horizons. For this we focus on designing market models and a distributed agent system that can incite people to engage in the particular (e.g. 'green') activity. Large empirical data sets collected in correspondence with various aspects of the particular management challenge will guide and support the modelling endeavor. Applications of digital ecologies to promote energy efficiency and reduce carbon and greenhouse gas emissions ranging from the deployment of smart power grids with renewable energy components to carbon-free network-enabled transportation and performance management in organizations will be revealed.

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## Matching of RNA-type sequences and statistical analysis of random RNA

O. Valba, S.K. Nechaev, M.V. Tamm

†LPTMS, Paris Sud University

We have developed and implemented a new statistical algorithm for quantitative determination of binding free energy of two heteropolymer sequences under the supposition that each sequence can form a hierarchical cactus-like secondary structure, typical for RNA molecules. We have offered a constructive way to build a "cost function" characterizing the matching of two RNAs with arbitrary primary sequences. The substantial difference of this procedure with the conventional sequence comparison of linear chains is that in RNA case we should align not only the sequences of nucleotides which constitute pairs between two RNAs, but also take into account the secondary structure of the parts of RNA between the aligned nucleotides. The proposed algorithm is based on two facts: i) the standard alignment problem can be reformulated as a zero-temperature limit of more general statistical problem of binding of two associating heteropolymer chains; ii) the last problem can be generalized onto the sequences with hierarchical cactus-like structures (i.e. of RNA-type). Taking zero-temperature limit at the very end we arrive at the desired ground state free energy with account for entropy of side cactus-like loops. Using this algorithm we have studied various statistical properties of random RNA sequences such as average free energy, fluctuation and loop distribution for both linear and cactus-like structures formed by two interacting heteropolymers. We have found that without the limitation on the minimal loop length the cactus-like structures are characterized by very high binding energy per monomer, about 0.92. To explain this effect a new hierarchical model for determination average free energy of two interacting random RNAs have been proposed. This hierarchical model shows a good agreement with numerical experiments for cactus-like structures and allows us to suppose a critical behavior of binding energy per monomer. We have found that if the number of different nucleotide types is larger than 4, the average normalized binding energy of the infinite chain is always less than 1, while for number of nucleotides less than 4 the normalized binding energy is always 1 in the chains of finite length. Fluctuation for both linear and cactus-like structures belongs to the KPZ (Kardar-Parisi-Zhang) universality class. This allows us to detect phase transition for random RNA by temperature behavior of fluctuation.

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## Monte Carlo study of external noise influence on polymer translocation

D. Valenti†, G. Denaro, D. Persano Adorno, N. Pizzolato, S. Zammito and B. Spagnolo

†Dipartimento di Fisica, Group of Interdisciplinary Physics, Università di Palermo and CNISM

The translocation of biopolymers into or across membranes is one of the most important biological process in nature, and is central in processes such as: (i) protein translocation from the cytosol into and across the endoplasmatic reticulum membrane [1]; (ii) DNA and RNA translocations across nuclear pores [2]; (iii) virus (bacteriophages) injection through the pores of a membrane to fight bacteria resistant to antibiotic therapies [3]; (iv) gene therapy and controlled drug delivery [3]. Previous studies demonstrated that the translocation time can be influenced by several factors, such as different geometrical and physical characteristics of the pore-channel structure, thermal fluctuations, always present in a biological system because of its continuous interaction with environment, and presence of any other external forces which can perturb the dynamics of the polymeric chain. In this work a Monte Carlo algorithm is employed to explore the effects of external noise on the dynamics of a flexible linear chain molecule crossing a nanochannel in the presence of a static or oscillating driving electric field acting on the monomers of the chain. The polymer chain is modeled using the two-dimensional "bond fluctuation model" [3,4]. In analogy with the stochastic behavior of a Brownian particle, the polymer dynamics is generated by random monomer jumps, which originate from the thermal fluctuations affecting the polymer due to its interaction with environmental variables such as temperature. In addition, we consider that the polymer dynamics is influenced not only by a white Gaussian noise whose origin is connected with the thermal fluctuations, but also by the presence of noise sources characterized by different statistical properties. Within this context we model the dynamics of the polymeric chain by considering that the polymer moves through the channel so that the monomers of the chain interact with the walls by a discrete quasi-Lennard-Jones potential. Moreover, the interactions between adjacent monomers is modeled by a spring potential, and between non-adjacent beads through a Lennard-Jones potential [5]. Finally, in view of providing a more realistic description of the polymer dynamics, the chain is provided with a suitable stiffness obtained by including a bending recoil torque in the model, with a rest angle equal to zero between two consecutive bonds. In our study we concentrate on the Mean First Translocation Time (MFTT) of the polymer centre of inertia through the channel and analyze how this time is influenced by the presence of an external noise source, for different values of the strength and frequency of the driving field, varying the length of the chain and the dimension of the channel.

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## Kinetic theory and relativistic thermodynamics

P. Ván†

†KFKI Research Institute for Particle and Nuclear Physics,  
Budapest, Hungary

Relativistic thermodynamics is the less established among relativistic theories. The famous "imbroglio" of the temperature of moving bodies - the different but reasonable Einstein-Planck, Blanus-Ott, Landsberg and Doppler transformation rules - indicate, that basic thermodynamic concepts may have contradictory explanation in relativity theory. Here we argue that thermodynamics and special relativity alone does not determine relativistic thermodynamics. Moreover, statistical mechanics does not clarify the situation, too, one can give statistical explanations to all of the different views providing different interpretations of the macroscopic quantities. The reason is that the problematic point is somehow outside the usual realm of equilibrium theories. The key aspect is the interaction of moving bodies, the thermodynamics of the motion. Therefore the problem is best investigated in the framework of classical field theories - like continuum (hydrodynamic) or kinetic theories - where this aspect is the part of the theory and equilibrium and thermodynamic equilibrium is motion related. In our previous research we investigated stability and causality in relativistic hydrodynamics. A detailed analysis of the Second Law revealed that assuming acceleration independent entropy production the local rest frame entropy density may depend on the absolute value of the energy momentum vector, time-spacelike part of the energy-momentum density tensor [2]. The resulted hydrodynamic theory was proved to generic stable without the restricting conditions and additional parameters of the Israel-Stewart theory . On the other hand, with the help of the generalized hydrodynamic background we have interpreted and derived thermodynamic concepts. It was shown, that our generalization unifies the above mentioned different transformation rules and determine the physical conditions of their difference. In this presentation we show, that the generalized Jüttner distribution suggested in [1] is compatible with kinetic theory. Moreover, this compatibility requires a further generalization of our previous suggestion and we should introduce a Gibbs relation where the velocity field is an independent thermodynamic variable. Some consequences in relativistic dissipative hydrodynamics with heat conduction are calculated.

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## Passive-scalar decay in chaotic flows

J. Vanneste†

†University of Edinburgh

We discuss the behaviour of passive scalars released in smooth, divergence-free chaotic flows modelled by random processes. In the long-time limit, the variance of the scalar concentration decays exponentially, at a rate which can be thought of as the Lyapunov exponent of the (random) advection-diffusion equation. This decay rate tends to a non-zero value as diffusivity tends to zero (i.e., in the large Peclet-number limit). In order to understand what controls its value, we consider simple Markovian flows, including white-in-time flows and the randomised alternating sine flow which has been employed extensively in studies of mixing in two dimensions (Pierrehumbert 2000). Two distinct regimes of scalar decay are identified: a locally controlled regime, which applies when the fluid domain has a size comparable to the characteristic lengthscale of the velocity field, and a globally controlled regime, which applies when the domain is larger. Mathematically, they are distinguished by the fact that the variance decay rate is determined by either the continuous part or the discrete part of the spectrum of the linear operator governing the covariance evolution (Haynes and Vanneste 2005). An asymptotic analysis in the limit of zero diffusivity provides predictions for the variance decay rate in both regimes. In the locally controlled regime, the prediction is related to the large-deviation statistics of the flow stretching (Balkosky and Fouxon 1999); in the globally controlled regime, the prediction reduces to the effective diffusivity of homogenisation theory in the limit of very large fluid domain. Corrections to the decay rate associated with finite diffusivity are estimated and shown to be very different in the two regimes. The theoretical predictions are tested against high-resolution simulations of simple two- and three-dimensional flows (Ngan and Vanneste 2011). In three dimensions, consideration of both the forward flow and its time reverse makes it possible to compare the scalar evolution in flows with one or two expanding directions and confirm the prediction that the decay rate of the scalar is the same in both flows, despite the very different scalar field structures. The evolution of higher-order statistical moments, and the form of the concentration probability density function are also discussed.

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## Gauge theory of glass transition

M. Vasin<sup>†</sup>

<sup>†</sup>Physical-Technical Institute, Ural Branch of Russian Academy of Sciences

In this work a new analytical approach for description of glass transition in frustrated system is suggested [1]. The theory is based on the non-equilibrium dynamics technique [2, 3], and takes into account the interaction of the local order field with the massive gauge field, which describes frustration-induced plastic deformation. Glass transition according to the presented theory can be regarded as a phase transition interrupted because of caused by the frustrations, premature critical slowing down one of the freedom degrees. We described the critical behavior of the frustrated 3D Heisenberg model in terms of the gauge theory and showed that this critical behavior was attributed to vanishing the effective mass of the gauge field, which happens at the temperature  $Tg > Tc$ . As a result, the system freezes in a state with a disordered structure. The gauge symmetry breakdown is not spontaneous in the considered model. It is induced by the random sources of the gauge field, which is connected with the geometrical frustration. The symmetry breakdown leads to the appearance of the gauge field mass. However, the gauge field mass becomes small when the temperature reaches  $Tg$  ( $Tg > Tc$ ). Close to  $Tg$  the fluctuations of the local magnetization field are small, but the gauge field fluctuations dramatically increase, which leads to the critical slowing-down of the gauge field dynamics, as well as the dynamics of the local magnetization field. This slowing-down freezes the disordered structure of the local magnetization field with the finite correlation length and susceptibility, and can be interpreted as a glass transition. Thus, the glass transition represents the critical slowing-down of fluctuations growth by disorder-induced frustrations, that complies with the “frustration-limited domain theory” [4] and correlates with other approaches. It is shown that the freezing of the system appears when the correlation length and relaxation time of the gauge field diverge. The Vogel-Fulcher-Tammann relation for the transition kinetics and critical exponent for non-linear susceptibility are derived in the framework of the suggested approach. Expression for the temperature dependence of the heat capacity near to glass transition is derived. This dependence qualitatively is in good agreement with experimental data. The presented theory reproduces the characteristic form of the correlation function dependence on time, and explains the boson peak appearance on this curve. Besides, the function of the glass transition temperature value on cooling rate is derived, this dependence fully conforms with known experimental data.

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## Deformed statistics framework for deterministic annealing

R. Venkatesan<sup>†</sup>

<sup>†</sup>Systems Research Corporation

Deterministic annealing (DA) is a prominent tool in areas as diverse as machine learning and optimization [1, 2]. Given  $X$  as the source and  $\tilde{X}$  as the most optimal compressed representation, this work describes a DA model within the framework of a maximum Tsallis entropy model using normal average constraints, for data clustering. The measure of uncertainty is the joint Tsallis entropy subjected to the additive duality. The minimization of the generalized DA Helmholtz free energy is substituted with the maximizing of the generalized statistics DA Massieu potential. This results in a cost function of the form

$$\Phi = S_{q^*} (X, \tilde{X}) - \beta D,$$

where:  $S_{q^*} (X, \tilde{X})$  is the dual Tsallis joint entropy,  $D = \sum_x \sum_{\tilde{x}} p(x, \tilde{x}) d(x, \tilde{x})$  is the expected distortion with  $d(x, \tilde{x})$  being the Euclidean distance, and  $\beta$  is a Lagrange multiplier which tends to the inverse temperature as  $q^* \rightarrow 1$ . Here, (1) is subjected to the normalization constraint. Note that  $q^* = 2 - q$ . Decomposing the dual joint Tsallis entropy as:  $S_{q^*} (X, \tilde{X}) = S_{q^*} (X) + S_{q^*} (\tilde{X} | X)$  [3], and the first term depends only on the source vectors, it is discarded since clustering only requires the dependence of  $\tilde{X}$  on  $X$ . Retaining only the second term of the decomposed dual joint Tsallis entropy which is the dual Tsallis conditional entropy, yields a modified generalized Massieu potential for DA

$$\tilde{\Phi} = S_{q^*} (\tilde{X} | X) - \beta D.$$

A principled method to calculate the first and second order phase transition conditions to obtain the mean of the distributions and the criterion for splitting of the clusters is presented using  $q$ -deformed calculus [4]. The construction of a numerical scheme that solves the generalized statistics DA problem by extending the Blahut-Arimoto algorithm is presented [3,5]. Numerical examples for mixtures of highly correlated data distributions and power law distributions exemplify the efficacy of the generalized statistics model vis-à-vis equivalent models derived using Boltzmann-Gibbs-Shannon (B-G-S) statistics. This work concludes with a discussion of regimes involving mixtures of highly correlated data where the Tsallis theory provides better quantitative results for data clustering using DA vis-à-vis equivalent models within the B-G-S framework, and provides possible rationales where such behavior is exhibited.

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## Modeling urban housing market dynamics: can the socio-spatial

A. Vignes<sup>†</sup>, L. Gauvin, A. Vignes, J.-P. Nadal

<sup>†</sup>Université Paris 2, Pantheon-Assas

This paper is concerned with issues related to social segregation in urban environments. Going beyond the simplest models such as the one introduced by T. C. Schelling in the 70's, we introduce a spatial model of real estate transactions between agents that are heterogeneous in their income and thus in their willingness to pay. The housing market consists of assets differentiated by their location in the city and the agents choose according to these locations. A key feature of the model is the assumption that agents preferences for a location depend both on an intrinsic attractiveness of the location, and on the social characteristics of its neighborhood. An other important assumption is that people prefer to live with others who are at least as rich as they are. The demand for an asset thus depends on the local attractiveness and this is a crucial hypothesis of the model. Non-trivial buying/settling patterns emerge from the resulting dynamics. We first focus on the case of a monocentric city, i.e. with a highly attractive center. The agents can be buyers, sellers or housed. The stationary state of the market dynamics is analytically characterized and yields the distribution of income over space. We then show how these results extend to more complex non-monocentric cities. The analytical resolution of the model highlights the existence of a critical endogenous income threshold: agents with willingness to pay above this threshold can buy an asset wherever they demand. On the contrary, agents with a willingness to pay below the threshold can buy only in a restricted area. In a second part, the model is studied through numerical agent-based simulations. The joint analytical and numerical analysis reveals that, even if socio-spatial segregation does occur, some social diversity is preserved at most locations. A third part verifies the pertinence of these results through the empirical analysis of a database of real-estate transactions in Paris. Some general trends are fitted by the housing market model we have proposed: the distribution of agents by income inside the city is characterized by a dissimilarity index that shows variations in the space comparable to those observed through the arrondissements of Paris. We distinguish arrondissements with a low level of social mix, both with a high average price and a low average price and less segregated arrondissements.

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## Normal and anomalous transport in transcription kinetics

R. Voituriez<sup>†</sup>, O. Bénichou

<sup>†</sup>CNRS, France

How long does it take a "searcher" to reach a "target" for the first time? This first-passage time is a key quantity for evaluating the kinetics of various processes, and in particular chemical reactions involving "small" numbers of particles. A striking example is given by gene transcription, where specific proteins search for target sequences on DNA. The FPT distribution in realistic confined geometries has until now, however, not been obtained, and only results on the mean first passage time were available. First I will introduce a new method that enable the calculation of FPT distribution analytically and show that transport processes as varied as regular diffusion, anomalous diffusion, and diffusion in disordered media and fractals, fall into the same universality classes. Beyond the theoretical aspect, this result changes our views on standard reaction kinetics and we introduce the concept of 'geometry-controlled kinetics'. More precisely, we argue that geometry-and in particular the initial distance between reactants in 'compact' systems-can become a key parameter. These findings could help explain the crucial role that the spatial organization of genes has in transcription kinetics, and more generally the impact of geometry on diffusion-limited reactions. Next, I will discuss the case of proteins performing facilitated diffusion. Facilitated diffusion of DNA-binding proteins is known to speed up target site location by combining three dimensional excursions and linear diffusion along the DNA. I will explicitly calculate the distribution of the relocation lengths of such 3D excursions, and quantify the short-range correlated excursions, also called hops, and the long-range uncorrelated jumps. These results substantiate recent single- molecule experiments that reported sliding and 3D excursions of the restriction enzyme EcoRV on elongated DNA molecules. I will extend the analysis to the case of anomalous 3D diffusion, likely to occur in a crowded cellular medium. Last, in the context of transcription in eukaryots, I will present a theoretical model of facilitated diffusion of proteins in the cell nucleus. This model, which takes into account the successive binding and unbinding events of proteins to DNA, relies on a fractal description of the chromatin which has been recently evidenced experimentally. Facilitated diffusion is shown quantitatively to be favorable for a fast localization of a target locus by a transcription factor and even to enable the minimization of the search time by tuning the affinity of the transcription factor with DNA. This study shows the robustness of the facilitated diffusion mechanism, invoked so far only for linear conformations of DNA.

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## Dynamics and evolution of languages in the Indo-European and Austronesian language families

D. Volchenkov†

†CITEC, University of Bielefeld, Germany

Changes in languages go on constantly affecting words through various innovations and borrowings [1]. Although tree diagrams have become ubiquitous in representations of language taxonomies, they obviously fail to reveal full complexity of language affinity characterized by many phonetic, morphophonemic, lexical, and grammatical isoglosses; not least because of the fact that the simple relation of ancestry basic for a branching family tree structure cannot grasp complex social, cultural and political factors molding the extreme historical language contacts [2]. Virtually all authors using the phylogenetic analysis on language data agree upon that a network, or a web rather than trees can provide a more appropriate representation for an essentially multidimensional phylogenetic signal. Networks have already appeared in phylogenetic analysis [3-4] either as a number of additional edges in the usual phylogenetic trees representing contacts and combined interactions between the individual languages and language groups, or as the considerable reticulation in a central part of the tree-like graphs representing a conflict between the different splits that are produced in the data analysis. However, the more comprehensive the graphical model is, the less clear are its visual apprehension and interpretation [1]. We show how the relationships between different languages in the language family can be represented geometrically, in terms of distances and angles, as in Euclidean geometry of everyday intuition. Our method is fully automated and based on the statistical analysis of orthographic realizations of the meanings of Swadesh vocabulary containing 200 words essentially resistant to changes. First, we have tested our method for the Indo-European language family by construction of language taxonomy for the 50 major languages spoken in Europe, on the Iranian plateau, and on the Indian subcontinent selected among about 450 languages and dialects of the whole family. The method uncovers the origin, relationships, and migration chronology of Indo-European population. Second, we have investigated the Austronesian phylogeny considered again over 71 languages chosen among those 1,200 spoken by people in Indonesia, the Philippines, Madagascar, the central and southern Pacific island groups (except most of New Guinea), and parts of mainland Southeast Asia and the island of Taiwan. We thoroughly discuss the Anatolian and Kurgan hypotheses of the Indo-European origin, as well as the 'express train' model of the Polynesian origin.

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## Tumbling dice with classical music

D. Volchenkov†

†CITEC, University of Bielefeld, Germany

The Internet based economy calls for robust recommendation engines for appreciating and predicting the musical taste of customers, since any improvement in the accuracy of predictions might have an immense economic value. Studies of Markov chains aggregating pitches in musical pieces might provide a neat way to efficient algorithms for identifying musical features important for a listener. As with music, speech and written language also have the sets of rules (crucial for establishing efficient communication) that determine which particular combinations of sounds and letters may or may not be produced. While communications by the spoken and written forms of human languages have been paid much attention from the very onset of information theory [1,2], not very much is known about the relevant information aspects of music [3]. A Markov chain model allows to appraise tonal music as a generalized communication process, in which a composer sends a message transmitted by a performer to a listener. The applications of Markov chains in music have a long history dating back to 1757 when a system for using dice to compose music randomly known as the musical dice game (MDG) had proposed. The further developments of computer musical data formats also called for a formalization of musical events as the states of hierarchical Markov chains. Although much work using Markov chains for compositional purposes have been done in so far, including a real-time interactive control of the Markov chains, less researches have been focused on the detailed analysis of musical compositions by means of Markov chains. We report some results on the Markov chain analysis of MDG encoded by the transition matrices between pitches in the MIDI representations of the 804 musical compositions attributed to 29 composers. Contrary to human languages, entropy dominates over redundancy, in the musical dice games based on the compositions of classical music. The maximum complexity is achieved on the blocks consisting of just a few notes (8 notes, for the musical dice games generated over Bach's compositions). Statistics of complexity measured by the past-future mutual information suggests that pieces in classical music might contain a few melodic lines translated over the diapason of pitches by chromatic transposition. First passage times to notes can be used to resolve tonality and feature a composer. The names of composers that are contiguous in the tonality preferences are often found together in musical concerts and on records performed by commercial musicians.

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## Relationships between the Legendre structure in the $S_{2-q}$ formalism and the dually flat structure in the space of escort distributions

T. Wada, A. Ohara, A.M. Scarfone

†Department of Electrical and Electronic Engineering, Ibaraki University, Hitachi, Ibaraki, Japan

Information geometry [1] is a powerful framework for studying the families of probability distributions or statistical models by applying differential geometry. Two kinds of information functionals play fundamental roles in this framework: the Kullback-Leibler information, or relative entropy, that is a measure of distance, or divergence, among two distributions; and the Fisher information, which takes the role of curvature in the distribution space. Information geometry provides us a useful tool for treating many important structures in probability theory like structures in differential geometry by regarding a space of probability distributions as a differentiable manifold endowed with a Riemannian metric.

Recently, it has been shown [2] that a dually flat structure on the space of the escort probabilities can be obtained by applying  $\pm 1$ -conformal transformation to the  $\alpha$ -geometry, which is an information geometry structure with constant curvature, related with Tsallis relative entropy [2].

On the other hand, as shown in [3], the  $(2-q)$ -formalism is a natural framework in the  $q$ -generalized thermostatistics based on Tsallis entropy  $S_q$  because the associated Legendre structures are derived in a similar way as in the standard thermostatistics theory based on the Boltzmann-Gibbs-Shannon entropy. Moreover, the so-called escort probability distribution function naturally appears in the  $(2-q)$ -formalism from a  $q$ -exponential probability distribution which maximizes  $S_{2-q}$  entropy under the constraint of linear average energy  $U$ . The generalized Massieu potential  $\Phi_{2-q}$  associated with  $S_{2-q}$  and  $U$  is related to the one  $\Phi_q^N$  associated with the normalized Tsallis entropy  $S_q^N$  and the normalized  $q$ -average energy  $U_q$ , which is the average of the energy with respect to the escort probability. Furthermore, the  $(2q)$ -formalism has also provided the connections among some different versions of Tsallis nonextensive thermostatistics, a non self-referential expression of Tsallis distribution and the relation between the Boltzmann temperature and the Lagrange multiplier in nonextensive thermostatistics [4].

In this work we explore some relations between the information geometrical structures associated with this dually flat structure and the Legendre structures in the  $(2-q)$ -formalism. We show that the Legendre dual potential functions in the information geometry with this dually flat structure are the generalized Massieu potential  $\Phi_q^N$  and  $S_q^N$ . Several relationship among the potential functions, the dual affine coordinates and the relevant divergence functions between the information geometry of the dually flat structure and  $S_{2-q}$ -formalism are also examined.

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## Graviational interaction among nodes of complex networks - The case of Japanese firms

H. Watanabe†, H. Takayasu, M. Takayasu

†Department of Computational Intelligence and Systems Science, Interdisciplinary Graduate School of Science and Engineering, Tokyo Institute of Technology

We consider transport phenomena on complex networks using the trading network data of Japanese firms. The date is provided by Research Institute of Economy, Trade and Industry, a Japanese governmental research institute containing about 870 thousand firms covering almost all active firms in Japan, from very small firms of size a few employees to international firms. The trading network is a directed network, in which nodes are firms and links represent flows of money. In our network data we have information about the existence of money flows between pairs of business firms without their quantity. We have sales data which are expected to correspond approximately to the sum of inflows of money. We already analyzed the data and confirmed the following empirical statistical properties. (1)The cumulative distribution function (in short CDF) of Sales obeys a power law distribution with exponent about -1, which belongs to the Zipf's law. (2)Both CDFs of in-degrees and out-degrees of links obey power law distributions with exponent about -1.3. (3) The conditional mean of sales for given in-degrees follows a scaling law, that is, the mean sale is proportional to the power of in-degrees with the exponent about 1.3. In order to clarify the relation among sales and money flows we introduce two transport models called Model-A and Model-B. Model-A is a linear model in which money flow is given by superposition of money flows proportional to the in-degrees for each node. Model-B is a nonlinear model in which the amount of money flux between connected pair of nodes is assumed to be proportional to the product of sales of the pairs analogous to gravitational interaction. For given network structure we simulate the growth of sales of the nodes starting with the simple initial condition that all firms have the same size. We restrict our attention to a strongly connected network and we assume both injection and dissipation to realize a statistically steady state. The injection is uniform for all nodes at each time step, and dissipation is applied to all nodes proportional to the sales with a constant rate. According to our numerical simulation both Model-A and Model-B satisfy the above basic properties, (1)-(3), in the steady state. We compare the steady state values of sales with the actual values of sales. It is found that both models' results are roughly proportional to the real sales, especially the nonlinear model B gives better fit with the real data for wider range of sales. From these simulation results we find that the value of sales are strongly correlated.

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## Volatility clustering, 'mild' multifractals and the Kesten process

N. Watkins<sup>†</sup>, S.C. Chapman, G. Rowlands and C. Franzke

<sup>†</sup>British Antarctic Survey

The Kesten multiplicative process is a very simple linear dynamical stochastic system. It modifies the form of the familiar AR(1) stochastic process, so rather than being a constant the correlation time becomes a random variable. Perhaps surprisingly, it nonetheless has a rich set of nonlinear fractal properties. Sornette [1] has given an extensive account of these, including its intermittency, and the simple method it offers for generating power law distributions. Intriguingly, it mimics some of the properties of self-organised critical systems while displaying others that are sometimes taken to be hallmarks of turbulence. Its "SOC-like" aspects include the power law pdfs, and perhaps the interplay of the additive and multiplicative terms in controlling activity, while the resemblance to turbulence includes intermittency and apparent multifractality. To what extent SOC and turbulence are fundamentally different has been a perennial topic of study, particularly in solar physics (see [2,3,4] and the contributed presentation at Sigma Phi by Chapman et al "Order and control parameters in driven, dissipating systems- parallels from avalanching systems, fluid turbulence and ecology"), and interest continues in models which might give SOC and turbulence as "by-products". My talk will address the question of the extent to which the Kesten process can be said to be truly multifractal. I will present some numerical investigations of structure functions and approximate scaling collapse both for the Kesten process and for random walks based on it. These investigations posit the question of whether full-blown multifractality is in general due not only to departures from power law tails in amplitude pdfs (a kind of nonlinear modification to the monoscaling seen in Mandelbrot's "Noah effect") but also to nonlinear transformations in temporal scaling (effectively thus a nonlinear "Joseph effect")? I also compare the autocorrelation behaviour of the Kesten process with that of its square. The latter shows some volatility clustering [c.f. Fig 4 (right panel) of [5]], a familiar property of many financial series. Again, this is one of the key diagnostics in ongoing debates about the relative merits of self-similar and multifractal models in finance.

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## Steady-state topologies of adaptive networks

S. Wieland<sup>†</sup>, A. Nunes, T. Aquino, A. Parisi

<sup>†</sup>University of Lisbon

Since its introduction in 2006, the adaptive SIS model of Gross et al. ([1]) has triggered further research into the interplay of dynamics on and of complex networks. It incorporates disease awareness into conventional SIS dynamics by additionally allowing population's susceptibles to evade infection through retracting links from infected neighbours and rewiring them to randomly selected susceptibles. Using the moment closure from [2] to obtain a pair-approximation model of the process, the time evolution of the fraction of infected is coupled to that of the three different link densities (of links connecting either types of nodes), with overall mean degree being conserved. Subsequent works either applied the original pairwise model to slightly altered scenarios ([3]) or extended it to higher moments in order to numerically obtain observed topological steady-states ([4]). In the latter case, steady-state degree distributions from individual-based simulations were reproduced by numerically integrating a set of degree-class equations beyond what was deemed to be the system's transient. The novelty of that particular ansatz was to model the time evolution of star motifs and compute en route a vital mean field from them. This infection mean field, coupling the immediate neighbourhood of a node to the rest of the network, will reappear in our framework. We describe the topological equilibria observed in [1] and [4] by analytically calculating the steady-state degree distribution in the original model's dominating phase. Considering dynamics on adaptive networks as random walks of a typical node in each of its state, we use generating functions to obtain a new class of degree distributions characterising the system's steady state. We show that this steady state is unique and stable, and proceed by deriving the classic steady-state degree distributions of both node subensembles (as defined by the node state), also inferring existence and uniqueness for them. Moreover, lifetime distributions of nodes in each state are computed, as are steady-state densities of any kind of links, link triples and higher-order star motifs. With three parameters under three constraints tying our framework to the actual process, we use least squares to establish correspondence. Since our approach does not depend on the specific dynamics of the process, it is easily generalisable to other adaptive dynamics featuring more node states or a variable number of links.

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### Experimental investigations of power laws and self organized criticality in granular piles, including a Casimir-like effect.

R.J. Wijngaarden<sup>†</sup>, K.A. Lorincz, Y.Y. Villanueva, S. May and D. Denisov

<sup>†</sup>VU University Amsterdam

Power laws are ubiquitous in a number of natural phenomena, ranging from earthquakes and rain fall patterns to stock market fluctuations. A rather detailed framework for establishing the detailed consequences of spatio-temporal fractal behavior has been formulated by P. Bak and co-workers, and by others and is designated "Self Organized Criticality" (SOC). However, an experimental investigation into the causes and origins of the required spatio-temporal fractal behavior has been largely lacking up to now. Here we discuss experiments (1) that investigate the origin of the spatio-temporal fractal behavior in granular piles and (2) that investigate the consequences of such behavior. We performed experiments on rice (oblong grains), quinoa (close to spherical), mung beans (*Vigna radiata*, close to spherical) and lentils (oblate). Quinoa showed a very clear deviation from power law behavior, while the other grains have an avalanche size distribution that is close to a power law, although with different exponents. Rice is power law over more than 3 orders of magnitude [1]. We present these experimental distributions and compare them with various distribution functions, to ascertain to what extent one can be certain of real power law behavior. In the SOC framework a number of exponents are defined (roughness exponent, avalanche size distribution exponent etc.). Also a number of exponent relations have been derived. We present the experimental determination of these exponents and verify experimentally their scaling relations [2]. We discovered that the avalanche behavior is qualitatively different if the foot of the pile is on a ledge or if it rests on a flat surface [3]. For experiments on rice and with the foot of the pile at a ledge, we observe a power law avalanche size distribution, but with an extra peak due a predominance of system wide avalanches. Clearly this has consequences for real life situations. Taking all our experimental observations together, we discuss the possible origins for self organized criticality. The avalanches on our granular piles are fluctuations and it is clear for some time that fluctuations can cause Casimir-like effects. The Casimir force is caused by electromagnetic fluctuations of the vacuum and drives two parallel conducting plates in a vacuum together. Similar forces have been observed due to the fluctuations close to a continuous phase transition. We study the force between two parallel plates embedded in a granular pile with avalanches and do find a force that increases sharply when the plates are put closer together [4].

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### Recent developments on the KPZ equation

H.S. Wio<sup>†</sup>, J.A. Revelli, C. Escudero, R.R. Deza

<sup>†</sup>Universidad de Cantabria

It is well known that in the literature associated to growth processes there is a reiterated claim indicating that "the KPZ equation is in fact a genuine kinetic equation describing a nonequilibrium process that cannot be derived from an effective free energy;..". In opposition to such a statement, here we present a variational formulation for the Kardar-Parisi-Zhang (KPZ) equation leading to a nonequilibrium potential (a kind of thermodynamic-like potential in a far from equilibrium situation) for the KPZ as well as a general form for such thermodynamic-like potential of generalized KPZ and other related kinetic equations [1]. Its knowledge allows us to prove some global shift invariance properties previously conjectured by other authors, and also discuss a few results about the form of the stationary probability distribution function for arbitrary dimensions. In addition, we can extract some strong constraints for the choice of real-space discrete representation schemes, by means of the exploitation of the known fact that the KPZ equation results from a diffusion equation (with multiplicative noise) through a Hopf-Cole transformation. It implies a tight relation between the discrete forms for the diffusive and the nonlinear terms, i.e. they cannot be independent. Whereas the nearest-neighbor discrete representation passes the resulting consistency tests, several known examples in the literature do not. We propose a consistent and highly accurate scheme, and emphasize the importance of the Lyapunov functional as a natural starting point for a real-space discrete representation. As an extremely relevant byproduct, in the light of these findings, the mainstream opinion on the relevance of Galilean invariance in determining the KPZ scaling properties, as well as the fluctuation-dissipation theorem (peculiar of 1D) is challenged [2,3,4]. The results of thorough numerical analysis strongly indicates, in good agreement with some recent theoretical arguments, that the Galilean invariance does not seem to play the relevant role usually assumed in the literature for determining the KPZ universality class. Finally, we discuss a new perspective from a path integral point of view that can shed some light over the KPZ dynamics, and the existence of a critical dimension.

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## The imprints of superstatistics in multiparticle production processes

Z. Włodarczyk<sup>†</sup>, G. Wilk

<sup>†</sup>Institute of Physics, Jan Kochanowski University, Kielce, Poland

We provide an overview of imprints of Tsallis statistics (as a special case of superstatistics which is a superposition of two different statistics relevant to driven systems with a stationary state and intensive parameter fluctuations) seen in multiparticle production processes. In particular, we review, from the point of view of the nonextensive statistics, the ubiquitous presence in the hadronic and nuclear collisions power law distributions [1]. The power-like behavior of different variables, is in this case attributed to intrinsic fluctuations of temperature in the hadronizing system and is accounted for by the nonextensivity parameter  $q$ . However, it was shown recently that similar power-law spectra can be also obtained by introducing some specific volume fluctuations. We demonstrate that, when total energy is kept constant, these volume fluctuations are equivalent to the temperature fluctuations and can be derived from them [2]. We also investigate the multiplicity fluctuations observed in nuclear collisions and evaluate the corresponding nonextensivity parameter  $q$  and its dependence on the size of hadronizing system deduced from the experimentally observed collision centrality dependence of the mean multiplicity and its variance  $Var(N)$ . We attribute the observed system size dependence of  $q$  to the finiteness of the hadronizing source, with  $q = 1$  corresponding to an infinite, thermalized source with fixed temperature, and (actually observed)  $q > 1$  corresponding to a finite source in which both the temperature and energy fluctuate [3]. We have demonstrated that fluctuations of temperature, together with fluctuations of other thermodynamic variables, result in a specific sum rule, connecting different  $q$ 's obtained from analyses of different distributions [4]. This allows us to understand why parameters  $q$  evaluated from distributions of different observables are different. We analyze next [5] the ensemble in which the energy ( $E$ ), temperature ( $T$ ) and multiplicity ( $N$ ) can all fluctuate and propose a relation connecting these fluctuating variables. It generalizes the Lindhard's thermodynamic uncertainty relations known in the literature. As an example, we compare fluctuations extracted from distributions of different observables and indicate a possibility of connecting fluctuations observed in different parts of phase space [5]. Finally, we advocate that for the standard Tsallis entropy with degree of nonextensivity  $q < 1$ , the corresponding standard Tsallis distribution is described by  $q' = 2 - q > 1$ . We show that experimental data from nucleus-nucleus collisions (treated as quasi-superposition of nucleon-nucleon collisions) confirm such conclusion.

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## Linear response of SRB measures to time-dependent perturbations

J. Wouters<sup>†</sup>, V. Lucarini, T. Kuna, D. Faranda

<sup>†</sup>University of Reading

In the current work, we study numerical and theoretical aspects of the linear response of steady states of non-equilibrium systems to weak time-dependent perturbations. We follow the lines of the response theory formulated by D. Ruelle [1], which describes the impact of small perturbations of the dynamics on the statistical properties of a measurable observable of a dynamical system, whose attractor defines an SRB measure. In recent times, interest in such dynamical systems has been increasing steadily, motivated from the point of view of application to e.g. climate systems [2], among others. The addition of a time-dependent perturbation results in a non-autonomous dynamical system with a time-dependent SRB measure. Our approach to this problem is a practical one, as can be of interest to researchers looking to apply these results in physical systems far from equilibrium. First of all, we give a derivation of the formula described by Ruelle on time-dependent linear response. We present this material in a manner that is accessible to readers who are less familiar to the technical literature on dynamical systems. We furthermore explore the problems involved in numerically calculating the linear response of a time-dependent SRB measure. We show that a numerically interesting method of averaging time-dependent responses can lead to incorrect results. The subtlety lies in the definition of the time-dependent SRB-measure. The measure defined by Ruelle involves ergodic averaging over ensembles that have evolved over a certain time  $T$  in the past, letting  $T$  go to infinity. Numerically, it would be advantageous to instead evolve the ensemble of initial conditions to a certain point in time  $T$  in the future and take the average, as this requires a less intensive integration. However, as we demonstrate, such averaging can lead to a number of problems, such as non-convergence or convergence to incorrect averages. Finally, we show how a method that has been used to efficiently calculate the linear response of dynamical systems to time-periodic perturbations [3] can be extended to this setting of more general perturbations. This method involves integrating a single trajectory in time and directly calculating from this the Fourier transform of the response. Hence this method is numerically attractive and we show that it is mathematically justified.

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## Collective human behavior in the cyber space communication

K. Yamada<sup>†</sup>, Y. Sano, H. Takayasu, M. Takayasu

<sup>†</sup>Waseda University

Collective human behavior becomes a major target of the twenty-one century science, thanks to development of computer technology [1][2]. Especially the huge data of articles in the cyber space written by human such as blogs and twitters is attracting attention because the data directly reflects trends and topics in human society. For example, the number of blog entries including "earthquake" have a clear peak when a large earthquake occurs. By using huge Japanese blog data base with author's ID, we can observe not only the number of entries per day for any words, but also personal dynamics about blog entry. In this presentation, we report statistical properties and modeling for three major categories of words. The first category is "ordinary words" which are used daily in our life, for example "soon". The number of entries of this category shows a steady fluctuation with a scaling relation found by Sano, Kaski and Takayasu [3]. The second category is "news words", for example "Michael Jackson". We can observe a clear jump and a power law decaying in the number of entries containing "Michael Jackson" after the news of his sudden death. It is remarkable that the function of decay follows a power law. We assume that this decay reflects quantitatively defocusing interest for Michael Jackson. Similar power laws of decay can be observed for many other words after big news. The third category is "trendy words", for example, "twitter". The number of entries of "twitter" increased exponentially from Oct. 2008 to Jun. 2010. Before 2008, most people did not know this new word, however, now twitter becomes very famous social network tool. In order to clarify the relationship between microscopic bloggers' dynamics and the macroscopic statistical properties of word appearance, we introduce a simple agent based model. We find that there are two crucial effects to reproduce statistical properties of real data by this simple agent based model. One is time dependence of the probability including the specific word in his/her blog and the other is repeating tendency which means that once a blogger uses a new word then he/she tends to use the same word again. The model we introduce in this paper includes these two effects and it can reproduce all major statistical properties of the words.

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## Ising model on clustered complex networks: belief propagation algorithm

S. Yoon<sup>†</sup>, A.V. Goltsev, S.N. Dorogovtsev, J.F.F. Mendes

<sup>†</sup>Department of Physics, I3N, University of Aveiro, Portugal

Real-life networks are clustered. There are numerous triangles and loops of different sizes. However, the role of loops in cooperative phenomena in complex networks is still poorly understood. In order to answer this question, we study the Ising model on clustered uncorrelated random networks recently proposed by Newman [1]. These networks can be described as supergraphs with a tree-like structure. We generalize the belief-propagation algorithm [2,3,4] to the case of clustered networks and find the exact solution of the ferromagnetic Ising model. It was shown that the belief-propagation algorithm is an effective numerical method for solving inference problems on sparse graphs. It was originally proposed by Pearl [2] for tree-like graphs. Among its numerous applications are computer vision problems, decoding of high performance turbo codes, and many others. Empirically it was found that it works surprisingly good even for graphs with loops. Yedidia et. al. [3] discovered that the belief-propagation algorithm actually coincides with the minimization of the Bethe free energy. This discovery renews interest in the Bethe-Peierls approximation and related methods. The recent progress in the survey propagation algorithm, which was proposed to solve some difficult combinatorial optimization problems, is a good example of interference between computer science and statistical physics [5]. The belief-propagation algorithm enables us to find a general solution of physical models with discrete or continuous variables on a complex network. We are considering the configuration model of clustered networks correspond to a class of sparse uncorrelated random graphs. A network under consideration consists of uncorrelated motifs (loops) which are superedges of a supergraph. One can prove that the probability that different motifs have a common edge tends to zero in the infinite size limit  $N \rightarrow \infty$ . It means that in the thermodynamic limit this supergraph has a tree-like structure. We analyze critical properties of the Ising model on a supergraph and show that clustering increases the critical temperature of ferromagnetic phase transition in comparison with non-clustered networks. We demonstrate that the critical temperature of ferromagnetic phase transition strongly depends on the size of loops. We also show that the Ising model on considered clustered networks has critical properties similar to ones found for non-clustered networks. We show that at zero temperature the solution of the Ising model permits us to recover results of the percolation threshold obtained recently for this kind of clustered networks.

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## Observation of market potentials by using a particle filter method

Y. Tokar†, H. Takayasu, M. Takayasu

†Tokyo Institute of Technology

The PUCK (potentials of unbalanced complex kinetics) model is introduced to the analysis of financial time series [1]. This model supposes that the market price is described by a random walker in a temporally deforming potential force whose center is given by the trace of the walker. Moreover, the continuum limit of PUCK model is shown to be equivalent to Langevin equation with time-dependent coefficients [2]. It is important to estimate the functional form of potential forces with high accuracy as quick as possible in order to characterize the real time statistical properties of the market. The idea of potential force is derived from the following statistical property of the market. In financial technology the square of the market price fluctuations in a unit time called the volatility is the key concept for estimation of the financial risk. However, it is shown that there are cases that two time series with almost the same volatility behave very differently in a large time scale, implying that volatility is not sufficient for describing large scale price motions[3]. The concept of potential forces in PUCK model is suitable for quantitative description of financial risks caused by large price changes. In this paper we adopt Particle Filter Methods[4] also known as Sequential Monte Carlo Methods for estimation of the the functional form of potential forces of foreign exchange markets. In the Particle Filters method we prepare thousands of parameter sets in parallel and replace parameters according to their fitness with the observed time series of the exchange rates. Here, we assume a quadratic function for the potential force term and we search the best algorithm for quick and accurate estimation of the quadratic coefficient. By applying this Particle Filter Methods for PUCK model it becomes possible to calculate the probability density of estimated values of quadratic coefficients. The calculation of probability density of the quadratic coefficient was difficult with conventional methods, and it became possible to measure the estimation error. We show results of analysis for the day when the Bank of Japan intervened on USD-JPY market in the Autumn of 2010 to compare particle filter methods with conventional methods as the market properties should have changed suddenly by the intervention on this day.

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## Finite difference methods with variable timesteps for fractional diffusion problems

S.B. Yuste†, J. Quintana

†Universidad Extremadura

Anomalous diffusion processes are ubiquitous in Nature. This is particularly true in biological environments where the presence of traps and obstacles often leads to mean square displacements that grow sublinearly with time. A well-known model for describing this kind of subdiffusion processes is the so-called Continuous Time Random Walk model in which waiting time distributions between successive steps have a power-law tail. A convenient property of this mesoscopic model is that leads to macroscopic fractional subdiffusion equations that describe how the concentration of walkers evolves in space and time. Some analytical methods of solution of these equations are known (method of images, separation of variables, Laplace transform methods). However, as for the normal diffusion case, the availability of numerical methods is most desirable, especially when no analytical solution is available. Finite difference methods are particularly convenient for solving fractional diffusion equation and, consequently, they have been extensively studied. In almost all the cases considered the timesteps are constant. Certainly, this way these methods are simpler. On the other hand, methods with variable timesteps have the great advantage that the size of the timesteps can be tailored so that numerical solutions are accurate in any circumstance. Besides being more reliable, these methods are usually faster because they can use large timesteps wherever the solution changes smoothly. Moreover, one can dynamically adjust the size of the timestep so that the error is smaller than a prefixed value (adaptive method). Here we investigate how to extend this kind of methods to deal with fractional differential equations. This goal is especially relevant because, for fractional equations, the number of operations required to get a solution after  $N$  timesteps scales as the square of  $N$  (for normal equations this number simply scales as  $N$ ). This fact makes these methods slow and quite memory demanding when  $N$  is large. Therefore, the reduction of the number of timesteps, but not at the expense of losing reliability and accuracy, is of paramount importance. For the sake of being specific, we will consider the fractional diffusion equation in the Caputo form although our procedure can be readily extended to other equations with other terms (fractional Fokker-Planck equations, fractional diffusion-wave equations) or even with other fractional operators such as the Riemman-Liouville derivative. To test the method, we solve the problem of a flux of subdiffusive particles arising from a point source, a problem that is related to the diffusion of morphogens in subdiffusive media.

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## Universal relation between skewness and kurtosis in complex dynamics

A. Zaccaria<sup>†</sup>, L. Pietronero, M. Cristelli

<sup>†</sup>"Sapienza" University of Rome and ISC-CNR

Complex time series are characterized by a probability density function which shows large deviations from a gaussian behavior [1]. In order to quantify the asymmetry and the weight of the tails one usually introduces the normalized third and fourth moments, called respectively skewness and kurtosis. The parabolic relation between the two is a known property of experimental time series coming from different fields of physics and its origin is debated (see [2] and references therein). As far as we know, this issue has never been investigated in finance, despite its relevance for risk management. We analyze the correlation between skewness and kurtosis in financial time series and, in particular, price variations. We find two different regimes of non gaussianity. The first one is common in Physics and shows a parabolic behavior and small deviations from gaussianity. The second one is, as far as we know, present only in financial data and exhibits a power law scaling with an experimentally robust  $4/3$  exponent. Moreover, this exponent is the same for all the data considered (eight stocks from NYSE and the S&P 500 index). This is a rather surprising result, as financial time series are usually characterized by a quite clear, albeit discussed, absence of universality in the empirical exponents [1]. This evidence calls for a common mechanism for all stocks. Regarding the first regime, we critically analyze the argument given by Sattin et al. [2] and we propose a further explanation based on the known presence of parabolic lower bounds in the skewness-kurtosis plane. Moreover, we introduce a simple model, based only on the fact that an extreme event dominates the statistics, which is able to quantitatively reproduce the second regime. In fact, in this framework the value of the moments are approximatively given by the suitable power of the largest element and we are able to derive a functional relation between skewness and kurtosis which matches very well with the empirical data. We study the range of validity of our approximation and in particular its dependence on the length of the series. We show that it is possible to derive other scaling relations for moments of higher order. We argue that the presence of this second regime in finance and its absence in physics is due to the predominance of the agents' psychological components when a large fluctuation occurs. Finally, we believe that this new empirical evidence represents a new stylized fact of financial markets and that it will be useful to discriminate among the various agent based models which have been proposed to reproduce the experimental data. Our work will be presented in [3].

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## Self-organized criticality in a model of trade: aggregate fluctuations from independent sectoral shocks investigated on a mixture of randomly distributed and scale free topologies

N. Zachariou<sup>†</sup>, P. Expert, K. Christensen

<sup>†</sup>Imperial College London

A long standing problem in macro-economy is explaining the mechanisms leading to the large fluctuations in aggregate production across different sectoral activities. This question can be rephrased to "How is it that many different industries decide to produce more synchronously or why they do the sectors go down at the same time and the economy as a whole reaches a crisis?" Even though this phenomenon is observed, and is crucial in the creation of business cycles, it is rather counter-intuitive, as independent sectoral excitation would tend to average out. Bak et al. [1] proposed a simple directed lattice model exhibiting self-organised criticality to explain these high fluctuations. The key ingredient is that industrial sectors are not independent, but correlated via indirect interaction coming from the structure of the consumer-producer network, hence potentially leading to large fluctuations. The production cycles are initiated by stochastic demands from end consumers, at the top layer of the lattice. As pointed out in Carvalho [2] the layered and directed lattice topology is highly unrealistic. In this study, we generalise the model and systematically investigate the effect of topology on the behaviour of aggregate production. More specifically we try to establish the conditions under which self-organised criticality is realised in these models [3]. The system consists of a large number of production units each, where the dynamics evolve from simple trade rules. Each production unit buys goods from its supplier neighbouring units (k-out degree). It then produces goods proportionally to the number of units to which it sells goods (k-in degree). In Bak's original model,  $k\text{-in}=k\text{-out}=2$ . As we change the topology of the interaction network, we adapt the rules of the model, such that demand and production are conserved. We start with a higher coordination number on directed lattices and then generate networks with Erdos-Renyi randomly distributed in and out-degrees, whilst keeping a layered organisation. We then generate networks with scale-free distributed in and out-degrees in an overall layered organisation. Finally, we marry the two topologies to produce a more realistic economy consisting of networks with Erdos-Renyi in-degree and scale-free out-degree distributions, as is observed in real systems.

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## Model gene regulatory networks under mutation-selection balance

M. Zagórski†

†Jagiellonian University, Institute of Physics

Modern DNA sequencing methods help to reveal genomes of animals, plants and microbes. What strikes from these studies is that complex organisms do not have many more genes than simple ones. For instance, human genome consists of about 22000 protein-coding genes, fruit fly has approximately 14000 and baker's yeast has around 6000. Hence, human beings do not even have an order of magnitude more genes than simple eukaryotes. Still we are far more complex than them. The reason behind this mystery might be the way genes work together instead of their number. In particular, there are strong indications that in eukaryotes and prokaryotes with increasing genome size the number of regulatory genes grows faster than linearly in the total number of genes [1]. Thus we consider biological complexity in the framework of interaction networks. We propose a gene regulatory network (GRN) model [2] which incorporates the microscopic interactions between genes and transcription factors [3]. In particular the gene's expression level is determined by deterministic synchronous dynamics with contribution from both excitatory and inhibitory interactions. We study the structure of networks that have a particular "function" and are subject to the natural selection pressure. The question of network robustness against point mutations is addressed, and we conclude that only a small part of connections defined as "essential" for cell's existence is fragile. Additionally, the obtained networks are sparse with narrow in-degree and broad out-degree, properties well known from experimental study of biological regulatory networks. Furthermore, during sampling procedure we observe that significantly different genotypes can emerge under mutation-selection balance. All the preceding features hold for the model parameters which lay in the experimentally relevant range. Finally, using formulated framework we analyse certain subgraphs of interactions or "motifs" [4] that appear at anomalously high frequencies in GRNs having a prescribed function. We ask whether this phenomenon may emerge because of the functions carried out by these networks. In the case where the regulatory networks are constrained to exhibit multi-stability, we find a high frequency of gene pairs that are mutually inhibitory and self-activating. In contrast, networks constrained to have periodic gene expression patterns (mimicking for instance the cell cycle) have a high frequency of bifan-like motifs involving four genes with at least one activating and one inhibitory interaction.

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## Quantum engineering: From artificial atoms to quantum metamaterials

A. Zagoskin†

†Loughborough University

Some of the very significant progress in our understanding of quantum mechanics over the last 20 years is due to the experimental and theoretical investigation of mesoscopic structures, with an especially strong boost provided by all the activity in quantum computing. Among various solid-state based qubit prototypes, the superconducting devices currently take the special place, since they have low decoherence rates, good control, coupling and readout methods, well developed fabrication techniques, flexibility of design, and scalability, and their theory is well developed. Thus, they constitute novel and very interesting building blocks for designing meso- and hopefully macroscopic, fully quantum coherent and controlled devices. The usefulness of such devices goes beyond their possible applications to quantum computing (e.g., as quantum analog solvers [1]). A metamaterial built of such elements, which can be maintained in a superposition of quantum states - a quantum metamaterial - is expected to have some pretty unusual properties [2], e.g., a superposition of two different refractive indices. Research in this direction can bring us tantalizingly close to the quantum-classical boundary, and the quantum measurement problem. It could let us see - almost literally - how a Schrodinger cat looks like. The first steps towards the creation of quantum metamaterials lies in checking the optical properties of artificial atoms, i.e., constituent qubits. Recent experimental results [3,4] show that a superconducting flux qubit placed in a 1D transmission line behaves (in the microwave range) as a quantum point-like scatterer should, with an excellent agreement between the experiments and theory. Combining such "atoms" in 1D and 2D structures would be a straightforward (at least, to a theorist) way to creating true quantum metamaterials. Here we are moving towards the metamaterials community, which in its turn also sees the development of quantum metamaterials as an attractive goal [5]. In my talk I will review the current status of this emerging field and indicate what seem to be the most promising research directions. I will also discuss some implications, which the successful creation of a true quantum metamaterial (if it happens to be possible) will have for the more fundamental questions of quantum-classical transition and our understanding of quantum reality.

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### Microscopic analysis of protein oxidative damage: effect of carbonylation on structure, dynamics and aggregability of villin headpiece

B. Zagrovic<sup>†</sup>, D. Petrov

<sup>†</sup>Max F. Perutz Laboratories, University of Vienna

One of the most important irreversible oxidative modifications of proteins is carbonylation, a process of introducing the carbonyl group in reaction with reactive oxygen species. Importantly, carbonylation increases with the age of cells and is associated with the formation of intracellular protein aggregates and the pathogenesis of age-related disorders such as neurodegenerative diseases and cancer. However, it is still largely unclear how carbonylation affects protein structure, dynamics and aggregability on the atomic level. Here, we use classical molecular dynamics simulations to study structure and dynamics of the carbonylated headpiece domain of villin, a key actin-organizing protein. In addition to it being one of the most widely studied single domain proteins, both theoretically and experimentally, our interest in villin headpiece is also stimulated by the fact that actin cytoskeleton is an important target of oxidative damage in cells. Using the GROMOS 45A3 force field, we perform an exhaustive set of molecular dynamics simulations of native villin headpiece together with every single combination containing carbonylated versions of its seven lysine, arginine and proline residues, the quantitatively most important carbonylatable amino acids, for a total of over 14 microseconds of simulated time. Surprisingly, our results suggest that high levels of carbonylation, far above those associated with cell death *in vivo*, may be required to destabilize and unfold protein structure through the disruption of specific stabilizing elements, such as salt bridges or proline kinks, or tampering with the hydrophobic effect. On the other hand, by using thermodynamic integration and molecular hydrophobicity potential approaches, we quantitatively show that carbonylation of hydrophilic lysine and arginine residues is equivalent to introducing hydrophobic, charge-neutral mutations in their place, and, by comparison with experimental results, demonstrate that this by itself significantly increases intrinsic aggregation propensity of both structured, native proteins and their unfolded states. Finally, our results provide a foundation for a novel experimental strategy to study the effects of carbonylation on protein structure, dynamics and aggregability using site-directed mutagenesis.

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### Localized states of electrons and holes in graphene Fabry-Perot interferometer in electrostatic and magnetic fields

V. Zalipaev

<sup>†</sup>Loughborough University, UK

Recent development of semiclassical analysis has been demonstrated in various fields of modern physics such as nano-structures, electronic transport in mesoscopic systems [1], quantum chaotic dynamics of acoustic, optical and electronic resonators [2]. One of the examples of application of semiclassical analysis are quantum electronic waveguides with resonators. Conductance oscillations of electronic transport of waveguide through resonator in ballistic regime is analysed by total transmission coefficient, Landauer formula for the zero temperature conductance of a structure (see [3,4,5], for example). It is effected dramatically by excitation of localised eigen-modes inside resonator. It is of particular interest to study in the semiclassical approximation the role of periodic orbits in electronic transport in waveguide-resonator system. In this talk a semiclassical analysis of eigen-states of an electron-hole motion inside a strip graphene resonator with zigzag boundary conditions is developed. Similar to [6], an asymptotic method of construction of the energy spectrum and eigenfunctions, localized in the small neighborhood of a periodic orbit. The isolated periodic orbit is confined between two interfaces of graphene strip. Such a system represents a quantum electronic resonator or Fabry-Perot interferometer, an analog of the well-known high-frequency optical or acoustic resonator with eigen-modes called "bouncing ball vibrations". The first step in the asymptotic analysis involves constructing a solitary localized asymptotic solution to the Dirac equations (electronic Gaussian beam - wavepackage). Then, the stability of a closed continuous family of periodic trajectories confined between two reflecting surfaces of the resonator boundary is studied. The asymptotics of the eigenfunctions were constructed as a superposition of two electronic Gaussian beams propagating in opposite directions between two reflecting points of the periodic orbits. The asymptotics of the energy spectrum are obtained by the generalized Bohr-Sommerfeld quantization condition derived as a requirement for the eigenfunction asymptotics to be periodic. For one class of periodic orbits, localised eigen-states were computed numerically by the finite element method using COMSOL, and proved to be in a very good agreement with the ones computed semiclassically. There is a hope that the current model may be used in a study of corresponding transport problems of 2D electronic gas in graphene system of waveguide-resonator.

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## The sarcomere as a structured ensemble of molecular motors

L.R. Zastrow<sup>†</sup>, A. DiCarlo

<sup>†</sup>LaMS, Università Roma Tre & CASPUR

Striated muscles are hierarchically structured, extremely sophisticated biochemical actuators. Each muscle fibre contains a bundle of myofibrils. Each myofibril is a long chain of sarcomeres, the basic functional units of striated muscle tissue. Each sarcomere, whose size is of the order of a few micrometres, is made up of two intercalated arrays of parallel filaments: thick filaments, composed of the protein myosin, and thin filaments, composed chiefly of the protein actin (along with two regulatory proteins, troponin and tropomyosin). At both ends of each sarcomere, thin filaments are held together by relatively stiff transverse structures (the so-called Z-disks). It is well established that muscle contraction is produced by the relative sliding between thick and thin filaments within each sarcomere. However, the underlying regulatory mechanisms are still poorly understood. In the last twenty years, experimental techniques have been developed to test biological tissues on very fine length scales, down to nanometres and beyond [1, 2]. Micromanipulation experiments are done with optical tweezers or microneedles on huge protein complexes, measuring nanometre displacements and piconewton forces. Theoretical models are required to help understanding the experimental data so obtained, and simplified models are especially well suited to this endeavour [3, 4]. Such models, purposefully discarding molecular details, may be characterized by a relatively small number of parameters, which can be calibrated by comparison with available experimental results. Moreover, the use of simplified models makes it possible to compute the evolution of large collective systems for physiologically significant time lapses. In our approach, the molecular motor myosin II, responsible for the muscular function at the nanoscale, is mimicked by a 1D two-state model. This Brownian ratchet model, inspired by that introduced by Juelicher et al [3] to describe the processive motors acting in the cytoskeleton, is extended to allow for partial activation levels and time-varying stimuli. The basic unit of muscle cells at the microscale, namely one half-sarcomere, is represented as a collective system of myosin motors arranged on a bundle of mock thick filaments, each of which interacts with a mock thin filament. All thin filaments are harmonically connected to the Z-disk, while the thick filaments are rigidly held together at the sarcomere centre (the so-called M-line). This model, properly implemented into a computer code, holds promise to produce a virtual lab where different assumptions on the activation-contraction coupling could be tested out.

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## Dynamics of episodic transient correlations in financial time series and their predictability

M. Žukovič<sup>†</sup>

<sup>†</sup>SORS Research a.s., Košice, Slovakia

Some recent empirical studies suggested that returns of financial time series depart from the random walk hypothesis by showing a certain degree of long-term or short-term dependent relationships, thus violating the weak-form efficient market hypothesis [1,2]. The efficiency/non-efficiency can, for example, be assessed by evaluation of the Hurst exponent or the approximate entropy, as measures of long-term memory and randomness in time series. However, the character of the dependencies is not known and they are of transient nature, showing up in random intervals just for a short time. This makes it difficult to exploit them for prediction purposes [3]. We study dynamics of the linear and non-linear serial dependencies in financial time series, using windowed portmanteau test procedure by Brooks and Hinich [4]. In particular, we focus on detection of episodes of statistically significant two- and three-point correlations in returns of various financial time series that could offer some potential for their predictability. In several previous studies the windowed-test procedure was done by splitting the data into a set of non-overlapping windows the length of which was set to some ad-hoc fixed value. However, considering the episodic and transient nature of the correlations, these may or may not be detected, depending on the window length used. Furthermore, the window length also influences the onset and offset of the significant correlations. Shorter lengths facilitate quicker response to changes in the correlation strength and can help pinpoint the arrival and disappearance of the transient dependences, but on the other hand, they may lack adequate statistical power. We employ an approach of moving window in order to capture the correlations' dynamics for different window lengths. Then we analyze distributions of the periods with statistically significant correlations and relate them with other measures of the predictability of the returns, such as the Hurst exponent and the approximate entropy. We also measure the predictability itself by a hit rate, i.e. the rate of consistency between the signs of the actual returns and their predictions, obtained from a simple correlation-based predictor. It is found that during these brief periods the returns are predictable to a certain degree and the predictability depends on the selection of the window length.

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