Tsallis distribution decorated with log-periodic oscillation

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The two parameter Tsallis distribution, \( f(X) = C \cdot \left[ 1 + \frac{X}{mT} \right]^{-m} \) with scale parameter \( T \) (identified in thermodynamical applications with the usual temperature) and with real power index \( m = 1/(q - 1) \) (\( q \) being known as the parameter of nonextensivity in statistical mechanical approaches) are nowadays very well known and applied in vast variety of situations. Actually, Tsallis distribution can be regarded as generalizations to real power \( m \) (or \( q \)) of such well known distributions as the Gosset-Student distribution (\( X = t^2, m = (\nu + 1)/2 \) with integer \( \nu \), which for \( \nu \to \infty \) becomes a Gaussian distribution and for \( \nu = 1 \) a Cauchy distribution). We discuss a Tsallis distribution with complex nonextensivity parameter \( q \). In this case usual distribution is decorated with a log-periodic oscillating factor. The presence of log-periodic features signals the existence of important physical structures hidden in the fully scale invariant description \cite{Sornette1998}. The system considered and/or the underlying physical mechanisms have characteristic scale invariance behavior, \( f[(1 + \alpha)X + \alpha nT] = (1 - \alpha n)f(X) \). This form follows quasi-power law (Tsallis distribution) with the complex power index \( m = [-\ln(1 - \alpha n) + 2\pi i]/\ln(1 + \alpha) \). It can also be shown that discrete scale invariance and its associated complex exponents can appear spontaneously, without a pre-existing hierarchical structure. We illustrate our point by example of transverse momentum distributions obtained for the highest presently available energy of 7 TeV. Albeit Tsallis distribution fits look pretty good (large \( p_T \) transverse momentum distribution exhibit apparently a power-law behavior), closer inspection shows clear visible log-periodic oscillations \cite{Wilk2014b,Wilk2014c}. Finally, complex \( q \) also means complex heat capacity, \( C = 1/(q - 1) \). In particular, it will be shown that results for heat capacity can be used to a new phenomenological interpretation of the complex \( q \) parameter. Namely, one can argue that \( q - 1 = \text{Var}(T)/\langle T \rangle^2 - iS(T)/\langle T \rangle^2 \) where \( S(T) = \int \omega e^{-i\omega t} \text{Cov}[T(0), T(t)] dt \) is the spectral density of temperature fluctuations (i.e., the Fourier transform of the covariance function averaging over the nonequilibrium density matrix). This can be regarded as a generalization of our old proposition for interpreting \( q \) as a measure of nonstatistical intrinsic fluctuations in the system (which corresponds to the real part of the above \( q \)) by adding the effect of spectral density of such fluctuations (via its imaginary part).