A dichotomy for planar loop systems with implications for classical and quantum spin models

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Random loop ensembles show up as a common mathematical scaffolding in a number of quantum and classical stat-mech models. In some cases a common random loop system is even found to simultaneously project onto seemingly different classical and quantum spin models.

For two-dimensional loop-soup configurations of finite local density, and distribution which is ergodic under shifts, there is a natural dichotomy: the number of loops encircling any specified site is either almost surely finite or almost surely infinite. The transition between the two is found to play a role for a number of different physics phenomena. These include:

i) The discontinuity in the phase transition in planar Q state Potts models at $Q > 4$
[explained in this manner by G. Ray and Y. Spinka (2020).]

ii) A pair of different manifestations of symmetry breaking in the ground states of two different extension of Heisenberg's quantum anti-ferromagnetic spin chains:
* dimerization in an antiferromagnetic model of spins $S > 1/2$ with Hamiltonian favoring the the singlet state for each pair of nearest-neighbors', and
* Néel long range order under an asymmetric HXXZ interaction with $\Delta > 1$.

[With (i) and (ii) linked and explained in this manner in a joint work with H. Duminil-Copin and S. Warzel (2020).]

iii) The Berezinskii-Kosterlitz-Thouless phase of slow decay of correlations in O(2) symmetric spin models.

iv) Pinning versus delocalization in a class of height functions formulated over R2.
[with (iii) and (iv) linked, and explained in this manner, in a joint work with M. Harel, R. Peled and J. Shapiro (2021).]

In the talk we shall recall a known representation of quantum Gibbs states in a manner reminiscent of (d+1) dimensional classical stat mech system, and discuss specific examples from the above list.