## The predictable chaos of rare events in complex systems

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Many natural systems show emergent phenomena at different scales, leading to scaling regimes with signatures of chaos at large scales and an apparently random behavior at small scales. These features are usually investigated quantitatively by studying the properties of the underlying attractor. This multi-scale nature of natural systems makes it practically impossible to get a clear picture of the attracting set as it spans over a wide range of spatial scales and may even change in time due to non-stationary forcing.

Here we present a review of some recent advancements in characterizing the number of degrees of freedom and the predictability horizon of complex systems showing non-hyperbolic chaos, randomness, statedependent persistence and predictability. We compare classical approaches, based on Lyapunov exponents and correlation dimension, with novel approaches based on combining adaptive decomposition methods with concepts from extreme value theory. We demonstrate that the properties of the invariant set depend on the scale we are focusing on and that the proposed formalism can be generally helpful to investigate the role of multi-scale fluctuations within complex systems, allowing us to deal with the problem of characterizing the role of stochastic fluctuations across a wide range of physical systems as well as the role of different dynamical components in determining the predictability of rare events in complex systems.