Statistical-mechanical consequences of the spectral properties of size-invariant shape transformations

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Changing the shape of a domain while keeping its sizes fixed under the Lebesgue measure has been realized via a geometric technique called size-invariant shape transformation, which leads to so-called quantum shape effects in the thermodynamic properties of confined particles [1, 2, 3]. In this theoretical work, we provide a detailed analysis of the spectrum under a size-invariant shape transformation in quantum thermal systems. We show that the geometric couplings between levels generated by the size-invariant shape transformations cause a nonuniform scaling in the spectra. We find that the nonuniform level scaling is characterized by two distinct spectral features: ground state reduction and modification of the spectral gaps (energy level splitting or degeneracy formation depending on the symmetries). We explain the ground state reduction by the increase in local breadth (i.e. parts of the domain becoming less confined) that is associated with the sphericity of these local portions of the domain. We accurately quantify the sphericity using two different measures: the radius of the inscribed n-sphere and the Hausdorff distance. Level splitting or degeneracy, depending on the symmetries of the initial configuration, becomes a direct consequence of size-invariance due to Weyl law. Moreover, we find that the ground state reduction causes an effect called quantum thermal avalanche causing an unusual swapping in the thermal occupation probabilities. Thermal occupation of the ground state abruptly increases while the that of excited states decrease. This is the underlying reason for the peculiar effect of spontaneous transitions to lower entropy states in systems exhibiting the quantum shape effect [4]. We also identify avoided crossing effects in the spectra which are important in determining the peculiar behaviors in the entropy of the system. From the application perspective, such unusual spectral characteristics of size-preserving transformations could be utilized to design classically inconceivable quantum thermal machines.

References

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