Large scale simulations of the Ising quantum spin glass transition

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Although an Ising Quantum Spin Glass (QSG) is the prototype of disordered systems whose dynamics is affected strongly by quantum effects, there is not yet a clear picture of its critical phase, upon varying the transverse magnetic field at zero temperature whenever the spatial dimension D is ≥ 2. Currently there are two competing theories and we aim at finding out which is the correct one by providing the answer to four important questions: i) What is the value of the quantum-dynamical z exponent?; ii)How should the Finite Size Scaling analysis [2] be carried out when exponent z is unknown? iii)Does exponent z depend on the considered symmetry sector? iv) What are the critical exponents for this universality class [1,2]? These questions are addressed through a combination of exact diagonalization of the Transfer Matrix [1] (for small system sizes, up to L=6) which helps to control the $L \rightarrow \infty$ limit) and Quantum Monte Carlo (which reaches larger values of L). We highlight that a better understanding of the scaling of the spectral gap with respect to the system size, is instrumental in the assessment of the quantum computational complexity of the adiabatic quantum algorithm proposed for some classical optimization problems [3,4].

Our numerical approach pushes algorithms for Ising quantum spin glasses beyond the present limits and led us to develop two novel, highly tuned, multi GPU codes. CQSG is a Monte Carlo code for Quantum Spin Glass that relies on three levels of parallelism: multi-spin coding, multi (CUDA) threads, and multi-GPU (running simultaneously different values of the transverse field to speed up the dynamics, as required by the Parallel Tempering technique [5]). Special attention has been devoted to the implementation of a reliable and efficient parallel random number generator. EDQSG finds the first four eigenvalues (and the eigenvectors corresponding to the first two eigenvalues) of the Transfer Matrix of a 6x6 Ising Quantum Spin Glass by using up to 1024 GPU.

Our data-analysis is largely inspired by the exact diagonalization of smaller systems. In order to avoid any controversial assumption about the quantum dynamical exponent, we effectively reach the limit of zero temperature in our simulations. We find that the spin-glass susceptibility is barely divergent at the critical point, which is recognized as the crucial difficulty hampering previous works. Inspired by [6], we work-out practical alternatives to study the phase transition. In particular, we performed and now present a careful study of the energy gap.

References

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