Innovation diffusion and Bass model on complex networks

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A milestone in the study of diffusion of innovation, the Bass model has played, since its first appearance in the Sixties, a constantly present role in marketing. It has been extensively applied for the prediction of diffusion patterns and it has also been variously generalized. The original version is expressed by a single ordinary differential equation, a Riccati equation which is, albeit non-linear, analytically solvable. Its solution describes the evolution in time of the number of adopters of a new product within a population, under the assumption that there are two types of potential adopters, innovators and imitators. The perspective is an aggregate one since, for a fixed product, the two parameters appearing in the equation, the "innovation" and the "imitation" coefficient, are the same for the whole population. A thorough investigation however (as can be tackled with today's computational tools) should take heterogeneity of individuals into account. One way to do that is to consider the network of interpersonal connections. Especially in relation to the imitative aspect of the process, it can make a big difference whether individuals who have already adopted have few or many contacts.

We here discuss how a network structure can be introduced into the model. We consider networks whose nodes have at most a number n of links. In particular, we consider scale-free networks with degree distribution of the form $P(k) = ck^{(-\gamma)}$ where $2<\gamma<3$, because it is into this category and with power law exponent into this range that many real-world networks fall. Of further fundamental importance in relation to network topology are the degree correlations $P(h|k)$, with $P(h|k)$ expressing the conditional probability that an individual with k links is connected to one with h links. In this connection, we devised an algorithm to build correlation matrices and get assortative or disassortative networks. Following a heterogeneous mean-field approach, we reformulate the Bass model as a system of n ordinary differential equations with each of the n equations governing the evolution in time of the number of adopters among those who have j connections (with $1\leq j \leq n$). We explore various aspects of the dynamics related to different classes of links for both correlated and uncorrelated networks. And we focus on the identification of two specific times, the takeoff time and the peak time, which play a significant role in the life cycle of an innovation/product.

References

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