Coordinate-dependent diffusion in a heat bath: Ito-process and its consequences

Arijit Bhattacharyay
Indian Institute of Science Education and Research, Pune, India, Pune, India

In this talk, I would present coordinate-dependent diffusion of a Brownian particle in equilibrium with a heat bath at constant temperature as a stochastic process following Ito-convention. A Brownian particle near a wall or interface, in general, gets its diffusivity re-normalized by the hydrodynamic effects arising due to the proximity to the wall. This makes the diffusivity of the particle coordinate dependent due to the fact that the diffusion is related to dynamic viscosity in equilibrium. This coordinate dependence (or state dependence) of diffusion makes the stochastic problem involve multiplicative noise in the dynamics.

The method of solving such stochastic differential equations with multiplicative noise, considering the process as a Markov process with uncorrelated noise, was developed by Kiyoshi Ito around the middle of last century. However, in the standard physics literature, in this area, this Ito-process is hardly considered as the process for thermal equilibrium of a Brownian particle under confinement. The reason being that, the distribution that results for an Ito-process does not apparently look like a Boltzmann distribution. Most likely, for this reason, to consider equilibrium of such a Brownian particle, one forces some form of a Boltzmann distribution by analyzing the stochastic problem using Stratonovich or Stratonovich-like conventions.

In this talk, I would make the case for the fact that Ito-distribution for equilibrium of a Brownian particle undergoing coordinate dependent diffusion is consistent with Gibbs/Boltzmann distribution. This fact could be identified by properly identifying the Hamiltonian of such a system which must be a function of coordinate dependent diffusivity of the particle because, otherwise, the Hamiltonian of the system will not be taking into account all the sources of in-homogeneity of space.

Following this, I would like to demonstrate a few consequences of the Ito-distribution for equilibrium of such systems. I would show that such a process can make structural broken symmetry of interacting Brownian particles result in motion of center of mass of the system through the uniform heat bath where all currents in the configuration space of the structured object, where inter-particle forces work, vanish. Such an average directed motion of center of mass of a structured object by filtering Brownian motion will violate no principle of physics because the heat bath is on average isotropic and homogeneous. There can exist far reaching consequences of such motion of structured objects appearing spontaneously in a heat bath for biological and other systems, and I will discuss some of those.