A generalized definition of cumulants, including operators, to obtain statistical information of a broad class of stochastic processes of interest

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The possibility of obtaining a simple Master Equation (ME), such as for example a Fokker Planck Equation (FPE), for the PDF of a stochastic process of interest, is linked to the applicability of the central limit theorem (CLT), albeit under hypotheses less stringent than those of the original formulation. Ultimately, this is possible only in the case of strong separation of the time scale between the dynamics of interest and the "environmental" fluctuations (noise), i.e. in the case of effective Markovianity of the process. Alternatively, the FPE also holds for fully linear models, where the noise is Gaussian. These classical approaches to ME can all be traced back to the exploitation of the characteristics of classical cumulants, introduced in statistical mechanics starting from the first half of the 19th century. Unfortunately, Markovianity, linearity and Gaussianity are practically exceptional, rather than recurrent, conditions for real processes, particularly in fields such as climatology or the dynamics of oceanic-atmospheric systems. The extension of the concept/definition of cumulants, introduced in a recent work, which draws inspiration and formalizes an old idea of Kubo's, allows to open new paths, which lead to simple MEs, even starting from nonlinear/non-Gaussian and/or non- Markovians models.

In this work, in addition to a didactic introduction to the definition and use of these new cumulants, we will present two significant applications. The first exploits the consequent natural extension of the CLT, already presented elsewhere, to obtain a FPE with memory kernel, valid for a very general class of diffusion processes. The second example shows how to obtain a general ME, valid under the sole hypothesis that asymptotically the relative dependence between noise instances, separated by a large "tau" time interval, is a function only of tau, and not of the intermediate times.