# The transition to synchronization of networked systems

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Synchronization of networked units is the collective behavior characterizing the normal functioning of most natural and man made systems. As an order parameter (typically the coupling strength in each link of the network) increases, a transition occurs between a fully disordered and gaseous-like phase (where the units evolve in a totally incoherent manner) to an ordered or solid-like phase (in which, instead, all units follow the same trajectory in time).

The transition between such two phases can be discontinuous and irreversible, or smooth and reversible. The first case is known as Explosive Synchronization, which refers to an abrupt onset of synchronization following an infinitesimally small change in the order parameter, as in a thermodynamic first-order phase transition. The second case is the most commonly observed one, and corresponds instead to a second-order phase transition, resulting in intermediate states emerging in between the two phases. Namely, the path to synchrony is here characterized by a sequence of events where different functional modules (or clusters) emerge, each one evolving in unison. The structural properties of the graph are responsible for the way nodes cluterize. In particular, it was argued that the clusters formed during the transition are to be connected to the symmetry orbits and/or to the equitable partitions of the graph.

In my talk, I will provide a full elucidation of the transition to synchronization in a network of identical systems. Namely, I will introduce a (simple, effective, and limited in computational demand) method which, with the only help of the eigenvalues and eigenvectors of the Laplacian matrix of a network, is able to:
i) predict the entire sequence of events that are taking place during the transition;
ii) identify exactly which graph's node is belonging to each of the emergent clusters, and
iii) provide a rigorous calculation of the critical coupling strength value at which each of such clusters is observed to synchronize.

I will also demonstrate that such a sequence is in fact universal, in that it is independent of the specific dynamical system operating in each network's node and depends, instead, only on the graph's structure. Moreover, I will clarify that the emerging clusters are those groups of nodes which are indistinguishable at the eyes of any other network's vertex. This means that all nodes in a cluster have the same connections (and the same weights) with nodes not belonging to the cluster, and therefore they receive the same dynamical input from the rest of the network.

As such, synchronizable clusters in a network are subsets more general than those defined by the graph's symmetry orbits, and at the same time more specific than those described by equitable partitions. Finally, I will show extensive numerical simulations with both synthetic and real-world networks, which demonstrate how high is the accuracy of the predictions, and also report on synchronization features in heterogeneous networks showing that the predicted cluster sequence is maintained even for networks made of slightly non identical dynamical units.