Energy localisation and dynamics of a mean-field model with non-linear dispersion

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Ergodicity breaking and long term stability are not rare phenomena in many-body Hamiltonian systems. There are various forms of non-equilibrium states that appear in these types of systems, like the formation of metastable states, energy localization, synchronisation, and more, which are often accompanied by a weaker form of chaos indicated by Lyapunov exponents' numerical values. In particular, the Hamiltonian systems that involve long range forces have an even more perplexed and enigmatic behaviour: they exhibit out-of-equilibrium states that persist for longer integration times, when the size of the system increases. Such well-known systems are the Mean - Field Hamiltonian (HMF) [1,2] and the Fermi – Pasta – Ulam – Tsingou model with long-range interactions (FPUT-LRI) [3]. In both systems, numerical experiments show that chaoticity wanes towards the thermodynamic limit (namely as the number of particles N increases, while the specific energy or energy per particle $\varepsilon=E/N$ is kept constant). From Statistical Mechanics point of view, various studies show that the statistical behaviour of many-body systems Hamiltonian systems with LRI also deviates from the classical Boltzmann-Gibbs (BG) thermostatistical description. A further recent example is the ionic-crystal model which has been called "the modern form of the FPU problem" [4].

In this talk, we focus on energy localisation and non-equilibrium phenomena in a mean-field model with non-linear dispersion. In the absence of linear dispersion, the model exhibits very strong localisation phenomena, namely the initial excitation of a wave-packet will be almost preserved at all times. For generic initial conditions, we find that the maximum Lyapunov exponent decays as a power – law in terms of the system size, indicating an integrable-like behaviour in the thermodynamic limit. For a fixed N size and varying the energy, we derive an analytic estimate on the Lyapunov exponent's upper-bound $\lambda(\varepsilon)$, which approximates the energy dependence in the strong nonlinearity regime. This dynamical behaviour is also reflected in the statistical behaviour of the model: we evaluate and study the evolution of the momenta distributions (which initially form a uniform distribution) as well as their distance from a Gaussian distribution, while N increases. These numerics and their analysis quantify the model's detachment from ergodicity towards the thermodynamic limit.

References

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