Dynamical chaos in the integrable Toda chain induced by time discretization

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Integrability is one of the fundamental concept of Hamiltonian mechanics. Although rare within the space of Hamiltonian systems, integrable models play a highly relevant role in physics, as several experimentally achieved systems posses integrable limits – i.e. specific parameter regimes where an Hamiltonian turns integrable – and around such regimes novel physical phenomena have often been found. However, compute the long time propagation of integrable systems is less than trivial, as we show that time-discretization lifts integrability and induces dynamical chaos. We use the integrable Toda chain to show that even one of the most used and developed numerical method in the community – the symplectic integration scheme – induce a finite Lyapunov time TL (inverse largest Lyapunov exponent) and eventually a breakdown time TB ≫ TL where the simulations fail due to divergently large fluctuations. We discuss the dependence of these timescales on the chosen time step as well as on the chosen symplectic scheme.