On ensemble dependence of fluctuation-induced forces: Exact results for Casimir and Helmholtz forces

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Fluctuations are ubiquitous: they unavoidably appear in any matter either due to its quantum nature or due to the nonzero temperature of the material bodies and of the confined medium. Immersing bodies of given shapes and materials into a fluid changes its fluctuation spectrum, which has to be in accord with the geometry, the relative positions and orientations, and the material properties of the bodies. If these fluctuations are correlated in space, the dependence of their spectrum on the relative positions and orientations of the bodies generates an effective force and torque acting between them. If the excitations of the fluctuations lack an energy gap, as it is the case, e.g., for photons, Goldstone bosons, and the fluctuations of an order parameter at criticality, the fluctuation-induced force acquires an algebraic decay and, thus, becomes long-ranged.

When the degrees of freedom can enter and leave the region between the interacting objects one speaks about Casimir force. In the case of the electromagnetic Casimir force the medium is the vacuum, and the underlying mechanism is the set of quantum zero point or temperature fluctuations of the electromagnetic field. The critical Casimir force (CCF) results from the fluctuations of an order parameter. Recently, a review on the exact results available for the CCF has been published in Ref. [1].

In a recent Letter [2] we introduced the term Helmholtz fluctuation-induced force. It is a force in an ensemble in which the order parameter value is fixed. We stress that in customarily considered applications of the equilibrium Ising model to binary alloys or binary liquids the case with order parameter fixed must be addressed. In Ref. [2] via deriving there exact results on the example of Ising chain with fixed magnetization and under periodic boundary conditions, we have shown that the Helmholtz force has a behaviour very different from that of the Casimir force. It is interesting to note that the studied Helmholtz force has a behaviour similar to the one appearing in some versions of the big bang theory --- strong repulsion at high temperatures, transitioning to a moderate attraction for intermediate values of the temperature, and then back to repulsion, albeit much weaker than during the initial period of the highest temperature.

We stress that the definition and existence of Helmholtz force are by no means limited to the Ising chain and can be addressed, in principle, in any model of interest. We note that the issue of the ensemble's dependence of fluctuation-induced forces pertinent to the ensemble has yet to be studied. In the envisaged talk, we review some recent and present some new both exact and numerical results for the behaviour of the Casimir and Helmholtz force. We find that all significant results are consistent with the expectations of finite-size scaling theory.

Acknowledgments

The financial support via Grant No BG05M2OP001-1.002-0011-C02 financed by OP SESG 2014-2020 is gratefully acknowledged.

References

[1] D. Dantchev and S. Dietrich, Physics Reports 1005, 1 (2023)
[2] D. Dantchev and J. Rudnick, Phys. Rev. E 106, L042103 (2022)