Decomposition of cross-correlation networks by means of the concept of q-MST

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The dynamics of complex systems is commonly accessible through the multivariate time series. They are then used to determine the correlation matrices. By introducing an appropriate distance matrix, such matrices - for transparency - are usually converted into networks which are typically reduced to the minimum spanning tree (MST) representation. Of course, the traditional correlation coefficients, by construction, considerably compress and reduce the amount of information contained in the original series. The related compression may result, at the first place, from the fact that such coefficients involve averaging correlations over the whole span of fluctuations and thus do not filter out some possible variability of the intensity of correlations at different values of their amplitudes. Within such an approach, in the case of strong correlations, the resulting MST may give rise to some false signals, such as promoting a peripheral node to play the role of a central hub. Here the generalization of the concept of cross-correlation coefficient to the q-dependent detrended cross-correlation coefficient $\rho(q,s)$ [1] is presented such that when varying the q-parameter, it acts selectively to cross-correlations between different fluctuation amplitudes at different time scales s of multivariate data. Following such a generalization, the family of q-dependent minimum spanning trees (q-MSTs [2]) is introduced, which allows to disentangle the composition and organization of correlations graphically and thus to study their varying network characteristics.

The utility of such a procedure in addressing the above indicated issues is illustrated on a recently vital subject of the world cryptocurrency market [3] and of the underlying cross-correlations [4]. During the periods of the relatively stable increases, accompanied by rather moderate cross-correlations, the overall structure of q-MSTs does not change significantly with varying q-values. This signals that the cross-correlations are rather uniformly distributed over the range of fluctuations. Also, the anticipated central hub - the Bitcoin - remains such at different values of q. On the other hand, during the periods of violent decreases and strong cross-correlations, the structure of q-MSTs sizeably varies, and for instance, in May 2021, it is the DASH which at q=2 is seen to constitute a node of comparable multiplicity to the Bitcoin. Even more, at q=4, the latter is seen as a peripheral node, and an overall structure of the corresponding MST is much more dispersed. This indicates that here the strength of cross-correlations is more diversified at the large amplitude of fluctuations and that q-MST offers a promising tool for a systematic study of such effects in many different areas.

References

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[4] Work based on cooperation with J. Kwapień and M. Wątorek.