High-dimensional central limit theorem by Stein's method

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High-dimensional central limit theorem has attracted much attention both in theory and applications since the work of Chernozhukov et al. (2013, 2017). They proved that, under mild regularity assumptions, Gaussian approximation in multivariate Kolmogorov distance is valid even when the dimension d is much larger than the sample size n. Such results are crucial in high-dimensional statistical inference (Belloni et al. (2018)).

The currently best known convergence rate is $n^{-1/4}$ subject to logarithmic terms on dimension d. It has been an open problem whether this rate can be improved. In this talk, we will introduce recent progress (Fang and Koike (2021, 2022)) in improving the convergence rate to the optimal one, namely, $n^{-1/2}$. In particular, for sums of independent and identically distributed (i.i.d.) random vectors, we obtain the optimal convergence rate assuming the correlation matrix has off-diagonal entries bounded away from 1. For sums of i.i.d. log-concave random vectors, we obtain the optimal convergence rate without any assumption on the covariance matrix.

To prove our main results, we use the approach of Götze (1991) in Stein's method, together with modifications of an estimate of Anderson, Hall and Titterington (1998) and a smoothing inequality of Bhattacharya and Rao (1976).

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