From fireflies to power grids: The physics of spontaneous synchronization

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Crickets singing in synchrony, fireflies blinking in unison, audience clapping in unison, power grids operating at a common frequency with generator and motor shafts rotating in phase, and last, but foremost, pacemaker cells syncing up to send out electrical signals and generating regular heart beats are all examples of the fascinating phenomenon of spontaneous synchronization. In this talk, I will summarize how a physicist studies such an emergent collective behavior using tools of nonlinear dynamics and statistical physics.

In particular, in the context of the Kuramoto model of coupled oscillators with distributed natural frequencies interacting through a time-delayed mean-field, we derive as a function of the delay exact results for the stability boundary between the incoherent and the synchronized state and the nature in which the latter bifurcates from the former at the critical point. Our results are based on an unstable manifold expansion in the vicinity of the bifurcation, which we apply to both the kinetic equation for the single-oscillator distribution function in the case of a generic frequency distribution and the corresponding Ott–Antonsen (OA)-reduced dynamics in the special case of a Lorentzian distribution. Besides elucidating the effects of delay on the nature of bifurcation, we show that the approach due to Ott and Antonsen, although an ansatz, gives an amplitude dynamics of the unstable modes close to the bifurcation that remarkably coincides with the one derived from the kinetic equation. Further more, quite interestingly and remarkably, we show that close to the bifurcation, the unstable manifold derived from the kinetic equation has the same form as the OA manifold, implying thereby that the OA-ansatz form follows also as a result of the unstable manifold expansion. This may have important bearings on their inter-relationship to be unravelled in future. As an explicit physical effect of the presence of delay, we demonstrate with our exact results that for a sum of two Lorentzians as a representative example of a bimodal frequency distribution, while absence of delay leads to a bifurcation of the synchronized from the incoherent state that is subcritical, even a small amount of delay changes completely the nature of the bifurcation and makes it supercritical.