About a curious equivalence: Boltzmann Entropy as measure of information production - its functional representations and ensuing process classes

Rudolf Hanel
Section For Science Of Complex Systems, Medical University Of Vienna, Vienna, Austria

In recent years so called Generalized Entropy Functionals were attracting considerable attention, with pertinent questions concerning the microscopic foundations of such functionals, their interrelations, and their correspondence to what one might identify as "thermodynamic entropy" being under debate. We will briefly discuss challenges statistical theory faces, when considering complex generative processes, [1,2]. One in particular.

Different entropy concepts, including e.g. axiomatic schemes, or Boltzmann's prescription, log number of states, may differentiate with distinct functional forms and operational meaning, as one applies them outside the realm of iid and equilibrium processes [3]. However, we can show very generally that Boltzmann entropy and the information theoretic concept of information production can be unified conceptually once one shifts attention from single states a process accesses towards suitable sets of path fragments a process takes. By doing so one can in principle reversibly translate any reasonably well behaved process into an adjoined iid process [4]. For iid processes we know the triplet Shannon entropy, cross-entropy, and Kullback-Leibler information divergence completely determines the MaxEnt principle (MEP) which can be pull back entirely from the adjoined process space to the marginal distribution space of the original process class. Generalized MEP entropies turn out to be the natural functional representations of Boltzmann entropy. They context sensitively depend on the considered process class and measure of information production.

Despite theoretical and practical issues that limit our capability of inferring optimal adjoined process representations from statistical data alone the existence of such not necessarily unique representations has stunning consequences. One being, that each suitable process is self-generalizing. It implicitly carries all the information about the particular process class it belongs to. Another, that we are being provided with theoretical tools to study how "thermodynamic relations" deform as we drive processes away from equilibrium conditions [5].

References

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