The gradient-flow equations in information geometry and electric circuits.

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Information geometry (IG) is a useful method exploring the fields of information science by means of modern differential geometry. It has been applied to some different fields including statistical physics, statistics, information theory, dynamical systems and so on. IG is invented from the studies on invariant properties of a manifold of probability distributions. The dually-flat structures are important and a statistical manifold (M, g, ∇, ∇*) is characterized by a (psuedo-) Riemannian metric g, and torsion-less dual affine connections∇ and ∇*. For a given convex function $\psi(\theta)$ together with its dual convex function $\psi^*(\eta)$, one can construct a dually-flat structures as follows. The affine coordinates θ^i (η_i) are obtained as the derivative of $\psi(\theta)$ ($\psi^*(\eta)$). The convex functions are Legendre dual to each to other. The positive definite matrices $g_{ij}(\theta)$ and $g^{ij}(\eta)$ are obtained from the Hessian matrices of the convex function $\psi(\theta)$ and $\psi^*(\eta)$. The product of these matrices is Kronecker's delta.The θ- and η-coordinate systems are dual affine coordinates. Since connections are not tensors, there exists a coordinate system in which the connection becomes zero and such a coordinate system is called affine coordinate.

The gradient-flow equations with respect to the η-potential function are given by $d\eta_i/dt = - g_{ij}(\eta)(\partial\psi^*(\eta)/\partial\eta_j)$, in the η-coordinate system. They are equivalent to the linear differential equations $d\theta^i / dt = - \theta^i$, in the θ-coordinate system. It is worth emphasizing that the two sets of the differential equations describe different processes in general. The gradient-flow equations are useful for some optimization problems. The gradient flows on a Riemann manifold follow the direction of gradient descent (or ascent) in the landscape of a potential functional, with respect to the curved structure of the underlying metric space. In this contribution, we consider the gradient-flow equations in an electrical circuit based on the correspondence between the thermodynamic processes in equilibrium thermodynamics and the gradient-flows in IG. Specifically we have constructed the gradient-flow equations in a simple electrical resistor-capacitor (RC) circuit based on the first law of thermodynamics, which is a conservation law of energy in thermodynamical systems, and Telegen's theorem, which is a conservation law of electric powers in electric circuits. The corresponding η-potential function (negentropy) is constructed from the entropy production rate in the electrical resistor (R) by using Joule's law. Remarkably it is shown that the associated gradient-flow equation is consistent with the conventional transient analysis of the RC circuit when the dynamical evolution parameter is regarded as the time parameter.

References

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