Spin based Quantum Otto engines and majorization

Ramandeep Johal, Sachin Sonkar
Indian Institute of Science Education and Research Mohali, S.A.S. Nagar, India

Quantum thermal machines exploit new thermodynamic resources such as quantum entanglement, coherence, quantum interactions, and quantum statistics. A quantum Otto engine (QOE) has been widely studied for its possible quantum advantages—both in its quasistatic formulations as well as based on time-dependent constraints. It offers conceptual simplicity by virtue of a clear separation of heat and work steps in its heat cycle. The quantum working medium used in these models may be taken in the form of spins, quantum harmonic oscillators, interacting systems, and so on. One of the prominent analytical tools in this context is the notion of majorization, which was developed to quantify the notion of disorder, in a relative sense, when comparing probability distributions.

In the present work, we characterize the operation of a QOE through the notion of majorization. We focus only on quasistatic heat cycles in which the quantum adiabatic theorem is assumed to hold as well as a complete thermalization with heat reservoirs is involved. We show that a spin-based quantum working substance provides a natural platform by which the majorization conditions provide sufficient criteria for the operation of a spin-based Otto engine. In particular, for a working substance as a single spin of arbitrary magnitude, majorization yields necessary and sufficient conditions for the Otto engine, provided the canonical distribution of the working medium at the hot reservoir is majorized by its canonical distribution at the cold reservoir. It can be shown on general grounds that a greater Shannon entropy of the equilibrium distribution due to the hot temperature as compared to the cold temperature is a necessary, but not a sufficient condition for positive work extraction. Further, the total entropy generated in a quantum Otto cycle can be given as the sum of the two relative entropies. In this regard, a single spin has a special spectrum for which all the energy gaps change by the same ratio during a quantum adiabatic process. We consider a model three-level system in which the various ratios of energy gaps are not the same and have shown the usefulness of the majorization criterion to infer the regime of positive work.

We extend our analysis to a bipartite system consisting of a spin 1/2 interacting with an arbitrary spin via an isotropic Heisenberg exchange interaction. We find that while majorization condition implies positive work extraction, it only yields sufficient conditions for the engine operation. Finally, we study the local thermodynamics of spins in the case of the bipartite system and infer an upper bound on the quantum Otto efficiency using the majorization relation. This presentation thus illustrates the usefulness of the majorization heuristic to infer the performance of quantum Otto engines.

References:

[1] S. Sonkar, R. S. Johal, Spin-based quantum Otto engines and majorization, Phys. Rev. A 107, 032220 (2023).