Spatial and temporal cluster tomography

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As we address systems of increasing complexity, it becomes ever more challenging to identify the relevant order parameters to characterize different phases and the corresponding transitions. At the same time, pattern formation in complex systems often leads to distinct clusters or regions. The emerging cluster structures span from magnetic domains in classical and quantum systems through flocks and brain regions to motility-induced phase separation in active matter. Here, we discuss the concept of cluster tomography, a simple and efficient geometric approach to detect phase transitions and characterize the universality class in any classical or quantum system with a relevant cluster structure, both in- and out-of-equilibrium.

Such measurements have been inspired by studying entanglement properties at quantum phase transitions, like the von Neumann entropy. On many occasions, the critical point can be located even without access to an order parameter, solely based on quantum information patterns. Subsequently, using tools from conformal field theory, we have considered the classical 'cluster' counterparts of such quantum methods, revealing new universal insights in variations of the Potts model in both two and three dimensions.

In the simplest application of spatial cluster tomography, we consider the N(L) number of clusters intersected by a line of length L. As expected, to leading order, N(L)=aL, where 'a' depends on the microscopic details. In this talk, we will show that in a broad range of classical and quantum systems, critical points are indicated by an additional nonlinear 'corner term' in the form of b*ln(L), where 'b' is universal. We also introduce the analogous concept of temporal cluster tomography, motivated by the concept of burstiness in the area of complex network dynamics. These methods are just two aspects of a unified cluster tomography framework, characterizing the geometric complexity of a system via the statistics of low-dimensional cross-sections, akin to a geometric notion of susceptibility.

In spatial cluster tomography, the corner term is an integrated statistics over the so-called gap-size statistics, n(s). Indeed, each cross-section in each cluster can be viewed as a one-dimensional ON-OFF process, where ON events correspond to sites of a specific cluster and OFF periods to the lack of them. n(s) is then the inter-event 'time' statistics between the ON events along the cross-section. Analogously, in temporal cluster tomography, we study the inter-event time distribution between ON events, where ON events indicate that a pair of particles belong to the same cluster. By quantifying the relative width of this distribution over all particle pairs, we can detect different phases with or without dynamical complexity, quantified by the burstiness parameter $B=(\sigma-\tau)/(\sigma+\tau)$. Here τ and σ stand for the mean and standard deviation, and B quantifies the distance from a random, Poisson distributed time series (B=0) with no dynamical complexity, i.e., no memory. B>0 is known as bursty dynamics, e.g., with cascades and avalanches, leading to broad inter-event time distributions. B<0 indicates regular dynamics, with a characteristic inter-event time scale. Various phases and the corresponding transitions are expected to be characterized by different levels of burstiness.