Measuring the importance of individual units to the structure integrity of a complex network

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A network may fail to function due to the depression or deterioration of certain individual nodes (e.g., the 2003 great blackout that darkened much of North America). How to identify the deteriorated/depressed unit(s) is thence of great importance. This study provides an easy and faithful way to fulfill the task, by generalizing information flow, a real physical notion which has just been rigorously formulated with causality naturally embedded (see Liang, 2016, and references therein), to cumulative cases. For a network, given the time series for the units, it is shown that a natural measure is the information flowing/transferring from the unit of concern to the rest units. This flow or transfer can be rigorously derived in the setting of a continuous-time dynamical system, either deterministic or stochastic. Under a linear assumption, a maximum likelihood estimator can be obtained, allowing for an estimation of it in an easy way. As expected, this "cumulative information flow" does not equal to the sum of the information flows to other individual units, reflecting the collective phenomenon that a group is not the addition of the individual members. For the purpose of demonstration and validation, we have examined a network made of Stuart-Landau oscillators. Depending on the topology, the computed information flow may differ. In some situations, the most crucial nodes for the network are not the hubs (i.e., those with highest degrees), in contrast to the traditional point of view; they may have low degrees, and, if depressed or attacked, will cause the failure of the entire network. This study provides an easy yet effective approach to measuring the importance of individual units in producing the collective behavior of a complex network. It can allow us to understand the potential damage to the structure integrity due to the failure of local nodes, and hence help diagnose neural network problems, control epidemic diseases, trace city traffic bottlenecks, identify the potential cause of power grid failure, build robust computer networks, and so forth.

References

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