Lyapunov vectors and the energy levels of the directed polymer in random media

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The evolution of infinitesimal perturbations in spatially extended chaotic systems has been shown [1-6] to be generically described by the heat equation with multiplicative noise. After a logarithmic (Hopf-Cole) transformation, the statistical description of the dynamics of perturbations is captured by the prototypical stochastic surface growth equation of Kardar-Parisi-Zhang [7]. In the surface picture, erratic fluctuations, due to the chaotic nature of the trajectory, are treated as an effective noise. The surface picture has been shown to work also for covariant/generic Lyapunov vectors (CLVs) corresponding to sub-leading Lyapunov exponents (Oseledec splitting theorem). In Ref. [3] it was shown that surfaces associated with CLVs (other than the first one) exhibit scaling with the wavenumber k as $< |h(k)|^2 > \sim 1/k^{\delta}$ with exponent $\delta \simeq 1.20$ at long wavelengths ($k \to 0$). The crossover from KPZ scaling, $\delta = 2$, to the new universality with $\delta \simeq 1.20$ takes place at shorter length scales as one looks at higher order CLVs[3]. This new scaling exponent has been shown to be crucial to explain the universal scaling of Lyapunov-exponent fluctuations in space-time chaos [6]. While the correspondence between the main LV and KPZ universality can be understood through the multiplicative heat equation ansatz, the origin of the asymptotic $\delta \simeq 1.20$ scaling for sub-leading LVs has remained an open question for the last teen years.

Here we study the problem of the directed polymer in random media (DPRM) at zero temperature [8]. The ground state of the DPRM is known to be in the same universality class as KPZ, after a suitable correspondence between free energy of the minimal path and KPZ surface height [8]. We study, by means of numerical simulations, the excited states of the DPRM at $T = 0$, i.e. those paths with energies larger than the optimal path (ground state). We show that the DPRM energy profile E(x), which includes the energies of all paths (i.e. including excited states) starting at (0,0) and ending at (x,t), exhibits fluctuations that scale as $< |E(k)|^2 > \sim 1/k^{\delta}$, where the exponent crosses over from $\delta = 2$ for large k to $\delta = 1.2$ at long wavelengths $k \to 0$. Our results strongly support a link between the covariant LVs in space-time chaos and the excited states of the DPRM problem. We conjecture that free energies of the DPRM excited states map into surface heights of the CLVs.

References

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