Searching for universal formula for percolation threshold on two-dimensional lattices with complex neighbourhoods

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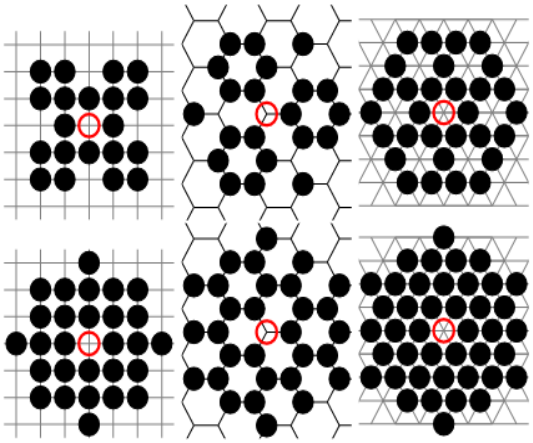
In this paper we present our recent attemps [1-6] to identify the universal formula allowing for predicting percolation thresholds on regular two-dimensional lattices. We consider complex neighbourhoods as presented in figure below. Our results [1-3] base on the scaling hypothesis regarding the probability of belonging to the largest cluster vs the occupation probability p obtained by Newman--Ziff algorithm [7]. For complex but compact (also called extended) neighbourhoods, the p_c is proportional to $1/z$, where z is the total number of sites in the neighbourhood [8]. For complex (not-compact) neighbourhoods and triangular lattice p_c follows the power-law with exponent close to 0.71 when ploted vs. $\xi = \sum_i z_i \times r_i^2/i$, where z_i and r_i are the number of sites and radius of the i-th coordination zone, respectively [3]. On the other hand, similar dependence is reocvered for honeycomb lattice but in dependence on $\zeta = \sum_i z_i \times r_i$ (with exponent close to 0.5) [2]. Similarly to the honeycomb lattice, the percolation thresholds for complex neighborhoods on a square lattice follow the power law $p_c(\zeta) \propto pow(\zeta, -\gamma)$ with $\gamma = 0.5454(60)$ [1]. The shape of universal formula for all these above mentioned complex neighbourhoods on regular two-dimensional lattices remains an open question. In this paper we present recent (more or less successful) attempts to identify such formula.



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