# Structure and connectivity of solutions in non-convex continuous optimization problems

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The characterization of the structure of the manifold of low-energy lying states in neural networks is among the most fundamental theoretical questions in machine learning. In recent years, many empirical studies on the landscape of neural networks and constraint satisfaction problem (CSP) have shown that the low-lying configurations are often found in complex connected structures, where zero-energy paths between pairs of distant solutions can be constructed. In this talk, I will discuss the geometrical organization and the connectivity properties of solutions in the negative perceptron, a linear neural network model and a prototype of a continuous non-convex CSP. I will show that a wide flat minima arise as complex extensive structures from the coalescence of minima around “high-margin” (i.e. locally robust) configurations [1]. Moreover, I will introduce a novel analytical method for characterizing the typical energy barriers between groups of configurations sampled from the zero-temperature measure of the problem [2]. We find that, despite the overall non-convexity of the space of solutions, below a critical fraction of constraints the geodesic path between any solution and the robust solutions of the problem, located in the interior of the solution space, remains strictly zero-energy. The value of $\alpha$ where this simple connectivity property breaks down is compatible with the point at which the dense core of solutions fragments in multiple smaller pieces [3].

References

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