On a multiscale mean-field spin glass

Emanuele Mingione, Pierluigi Contucci
Università Di Bologna, Department Of Mathematics, Bologna, Italy

We will consider a mean-field disordered system with Sherrington-Kirkpatrick model Hamiltonian study its thermodynamic properties in presence of multiple thermal equilibria, namely assuming that the random coupling can be divided into a finite number of families having their own effective equilibrium temperature.
The generating functional (thermodynamic pressure) of the model is constructed trough a hierarchical sequence of annealed averages, reminiscent of the Replica Symmetry Breaking interpolation [1]. The above construction can be also seen as a multiscale decomposition of the Hamiltonian viewed as a gaussian process. The measure induced by the above pressure is not Gibbsian and can be used to describes quasi equilibrium properties of systems in the small entropy production regime. Moreover it has a dynamical interpretation given in terms of Langevin dynamics where different degrees of freedom are subject to different thermal baths and have widely different timescales [2].

We will discuss the recent result obtained in [3]: the thermodynamic limit of the pressure per particle can be represented as a solution of an infinite dimensional variational principle of the Parisi type.
In particular we will show that the multiscale structure acts as constraint in the space of functional order parameter. We will briefly discuss the two main ingredients of the proof: the replica symmetry breaking interpolation and synchronization mechanism. We will show that one can add a suitable perturbation to the model in order to force the overlap to be synchronized with the different thermal baths acting on the system. Finally we will briefly discuss some dynamical aspect of the model.

References

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[3] P. Contucci, E. Mingione, A multi-scale spin-glass mean-field model. Comm. Math. Phys. 268, no. 3, 1323–1344 (2018).