# Preserving Bifurcations through Moment Closures

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Moment systems arise in a wide range of contexts and applications as high- or even infinite-dimensional systems of coupled equations. Hence, an indispensable step to obtain a low-dimensional representation that is amenable to further analysis is in many cases to apply a moment closure. The latter is a set of approximations that express certain higher-order moments in terms of lower-order ones, so that applying those breaks the hierarchical structure of the equations and leads to a closed system of equations for only the lower-order moments. Closures are frequently found drawing on intuition and heuristics in trying to come up with quantitatively good approximations. Apart from that, from a dynamical systems' point of view, a key consideration when deriving closures has to be whether dynamical features such as bifurcations are preserved. Hence, we propose a change of perspective where we focus on closures giving rise to certain qualitative features such as bifurcations. Importantly, this provides us with the possibility to classify moment closures rigorously and makes the design and selection of the same more algorithmic, precise, and reliable. In this talk, we will revisit two paradigmatic network dynamical systems, the SIS epidemic and the adaptive voter model, and derive conditions that a moment closure has to satisfy so that the corresponding closed systems exhibit the transcritical bifurcation that one expects in these systems coming from the stochastic particle model. Finally, we examine existing moment closures for both systems in the light of these results and show that they indeed satisfy all the conditions.