A simple model of 1/f fluctuations from amplitude modulation and demodulation

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1/f fluctuations are ubiquitous. This fluctuation is characterized by the power-law behavior of index -1.5 to -0.5 in the very low-frequency region of the power spectrum density (PSD). Since its discovery in 1925 in the electric current flowing through a vacuum tube (Johnson), it has been widely observed in various fields: semiconductors, biological membranes, crystal oscillators, temperature changes over very long periods, etc. Various theories have been proposed for the origin of 1/f fluctuations, but a universal mechanism has not yet been discovered.

We propose a simple model of 1/f fluctuations (pink noise) based on the amplitude modulation of accumulated frequency waves (AMAF) and its demodulation (DM) (arXiv:2301.11176, 2104.08872). This mechanism is a simple beat of many waves with accumulated frequencies. The wave beats can yield an unlimited low-frequency signal. However, on top of this AMAF, the demodulation (DM) process is also needed for 1/f fluctuations to show up. The existence of the DM process characterizes our model and is essential for its verification.

Familiar examples of amplitude modulation are the electric musical instrument theremin and the AM radio receiver. Both use high-frequency waves in the form of electric oscillations or radio waves and extract audible signals by demodulation process. The frequency accumulation can spontaneously arise in the cases of a) synchronization, b) resonance, and c) infrared divergence. We explore each of them below. Some examples of a) synchronization are as follows. A typical frequency accumulation is exponential, and the PSD shows a perfect 1/f structure. Interestingly, the exponential yields power law. If the frequency accumulation is a power law, then the PSD shows a 1/f structure with slightly modified power. Dynamical examples are the Hamiltonian Mean Field (HMF) model and the coupled macroscopic spin model. In the case of b) resonance, earthquakes, solar flares, and variable stars are typical examples. For c) infrared divergence, electric currents, and the associated nerve system show 1/f noise.

Further, there are various demodulation methods from which the diversity of 1/f fluctuations arises. For music, HMF, and electric currents, the time series of the data show 1/f fluctuations only when the original signal is squared. For seismic waves, neurotransmission signals, and solar flares, the time series of the data show 1/f fluctuations only when demodulated in the form of thresholds. The threshold mechanisms are fault rupture, neuronal firing, and magnetic reconnection. If the 1/f fluctuations are interpreted as AMAF as we propose, then it would be natural that they often appear inconsistent with various basic principles of statistical mechanics. For example, the Wiener-Khinchin theorem linking PSD and time correlation function cannot conclude the existence of long-time memory. This is because the wave beat does not appear in the PSD unless the signal is squared. Furthermore, since the beat has nothing to do with dissipation, the fluctuation-dissipation theorem does not hold. Therefore, 1/f fluctuation can appear even in HMF, a fully conserved system. We can claim that ubiquitous 1/f fluctuations do not require any elaborate stage of statistical mechanics, but they are simple amplitude modulation.