Dynamic correlations in the conserved Manna sandpile

Anirban Mukherjee, Punyabrata Pradhan
Department of Physics of Complex Systems, S. N. Bose National Centre For Basic Sciences, Block-JD, Sector-III, Salt Lake, Kolkata, India

We study dynamic correlations for current and mass, as well as the associated power spectra, in the one-dimensional conserved Manna sandpile. We show that, in the thermodynamic limit, the variance of cumulative bond current up to time T grows subdiffusively as $T^{1/2-\mu}$ with the exponent $\mu \geq 0$ depending on the density regimes considered and, likewise, the power spectra of current and mass at low frequency f varies as $f^{1/2+\mu}$ and $f^{-3/2+\mu}$, respectively; our theory predicts that, far from criticality, $\mu = 0$ and, near criticality, $\mu = \frac{\beta+1}{2}\nu_\perp z > 0$ with β, $\nu_\perp$and z being the order-parameter, correlation-length and dynamic exponents, respectively. The anomalous suppression of fluctuations near criticality signifies a “dynamic hyperuniformity'', characterized by a set of fluctuation relations, in which current, mass and tagged-particle displacement fluctuations are shown to have a precise quantitative relationship with the density-dependent activity (or, its derivative). In particular, the relation, $D_{s(\bar{\rho})} = a(\bar{\rho}) / \bar{\rho}$, between the self-diffusion coefficient $D_{s(\bar{\rho})}$ activity $a(\bar{\rho})$ and density $\bar{\rho}$, explains a previous simulation observation [1] that, near criticality, the self-diffusion coefficient in the Manna sandpile has the same scaling behaviour as the activity. The main results of our work are summarized as following.

We show that, in the thermodynamic limit, $L \to \infty$ and density $\bar{\rho}$ fixed, the variance of the local (bond) current $Q(T)$ grows subdiffusively. Near criticality, the current fluctuation is further suppressed, $\langle Q^2(T)\rangle \sim T^{1/2-\mu}$. We find that the time-dependent (two-point) correlation function for the instantaneous current is long-ranged (power-law) and negative. The corresponding power spectrum $S_J(f) \sim f^{\psi_J}$, vanishes at low frequency, where $\psi_J = 1/2$ away from criticality (in the frequency regime $\frac{1}{L^2} \ll f \ll 1$ for finite L) and $\psi_J = 1/2 + \mu$ near criticality (in the frequency regime $1/L^z \ll f \ll 1$). On the other hand, the power spectrum $S_M(f)$ for subsystem-mass fluctuation, diverges $S_M(f) \sim f^{-\psi_M}$ at low frequency, where $\psi_M = 3/2$ away from criticality ($1/L^2 \ll f \ll 1$) and $\psi_M = 3/2 - \mu$ near criticality ($1/L^z \ll f \ll 1$). These two exponents are not independent, and they are connected by a scaling relation $\psi_M = 2 - \psi_J$.

The steady-state fluctuation of the subsystem current $\bar{Q}(I,T)$, i.e., the cumulative (summed over bonds) current in a subsystem of size I in the thermodynamic limit ($L \to \infty$) interestingly converges to twice the activity when the infinite-subsystem-size limit is taken first and then the infinite-time limit. By deriving a fluctuation relation, we express the scaled subsystem mass fluctuation as an exact ratio of current fluctuation to twice the bulk diffusivity.

Finally, we theoretically show that the self-diffusion coefficient $D_{s(\bar{\rho})}$ is identically equal to the ratio, $a(\bar{\rho})/\bar{\rho}$, of the activity to the global number density of the system, a fluctuation relation, which connects the (scaled) tagged-particle displacement fluctuation to the density-dependent activity.

References:

[1] S. D. da Cunha, R. R. Vidigal, L. R. da Silva, R. Dickman, Eur. Phys. J. B 72, 441 (2009).
[2] A. Mukherjee, P. Pradhan, Phys. Rev. E 107, 024109 (2023).