Higher-order interactions generate mixed order phase transition and Griffiths phases on heterogeneous complex networks

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In d > 2 dimensional, homogeneous threshold models, involving higher order interactions, discontinuous phase transition emerge, but the mean-field approximation provides 1/t power-law activity decay and other critical power laws, thus it is called mixed-order or hybrid type [1]. This is in contrast with simple two body interaction, spreading models, where continuous transition occur [2]. Furthermore, quasi-static network heterogeneity can cause dynamical critical behavior around the transition point if the graph dimension is d < 4 [3]. We derive the mean-field approximation analytically for the K=2 threshold model and show the occurrence of the prototype of higher order interaction models. We compare these results with extensive simulations by putting this model on hierarchical modular graphs similar to the rat brain connectome. We provide numerical evidence that even in case of the high graph dimensional hierarchical modular networks a Griffiths phase in the K=2 threshold model is present below the hybrid phase transition. That means we can observe control parameter dependent, non-universal power-laws in dynamical quantities, activity avalanche sizes and duration. This happens in an extended control parameter region, where the susceptibility also diverges in the whole Griffiths phase, thus sensitivity for external input is maximal, as required by optimal neural systems. This is due to the fragmentation of the activity propagation by the modules, which are connected via single links, while the interaction requires higher number (K>1) of active neighbors. This delivers a widespread mechanism in the case of the threshold type of heterogeneous systems, modeling the brain, socio or epidemic spreading for the occurrence of dynamical criticality in extended Griffiths phase parameter spaces [4] even in the presence of discontinuous phase transitions.

References

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