## Approximate integral of motion for macroscopic lattice systems

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It is a well-established fact, and relevant for the workshop itself, that classical results of Hamiltonian perturbation theory, like KAM and Nekhoroshev theorems, become ineffective as the number of degrees of freedom, denoted by $N$, increases. Indeed, for example, the threshold in the perturbative parameter, below which the results are valid, vanishes very rapidly with $N$; but more generically, several of the constants which are present in the perturbative estimates exhibit unfavorable behavior with respect to the dimension of the system. Therefore, within the context of the workshop, it would be interesting to present the ideas of an adapted version of Hamiltonian perturbation theory that is capable of dealing with macroscopic systems and produce results applicable in the thermodynamic limit. The existing findings of this approach pertain to the existence of approximate conserved quantities in nonlinear lattices that are finite but have arbitrarily large dimensions; these results can be proven constructively and with estimates that are valid in a weak (in measure) sense. In particular, for a Klein Gordon lattice with periodic boundary conditions with $N$ particles, denoted by $a$ the nearest neighbour coupling constant, by $\epsilon$ the specific energy $E/N$ and by $\beta$ the inverse temperature of the Gibbs measure, for any $N$ and for a fixed and sufficiently small value of the coupling constant $a$ there exists an approximate constant of motion whose evolution is controlled up to a time scale of $\beta^{1/a}$ for any sufficiently large values of $\beta$. If the coupling constant is allowed to vanish jointly with the specific energy, the time scale becomes a stretched exponential. We demonstrate the adiabatic invariance by showing that the variance along the dynamics, as calculated using time averages, is significantly smaller than the corresponding variance over the entire phase space, as calculated using the Gibbs measure, for a set of initial data with a large Gibbs measure. A key point in the construction is the possiblity to exploit, within the perturbation construction and the measure estimates, the extensive nature shared by the Hamiltonian of the system as well as by the adiabatic invariant itself. If time permits, we will also discuss the relationship between the above mentioned result and the other directions being explored in the workshop.