Bayesian expectation of the mean power of several Gaussian data

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The uniform distributions over the reals for the prior probability-density of the means of several normal data leads to an inconsistent inference of the data mean-power. For example, when estimating the power of a signal from samples affected by an additive white Gaussian noise. We reinvestigate the problem, note that the uniform prior delivers unrecognised information, and propose a solution looking at the problem in a novel way. We took the power limitedness into account by a sequence of priors converging to the uniform one, organised them into a hierarchical structure, and left the data to choose among them. We obtained an extended James-Stein estimator averaging out the hyper-parameters and avoiding empirical Bayes techniques.

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