Double-replica theory for evolution of genotype-phenotype interrelationship

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The relationship between genotype and phenotype plays a crucial role in determining the function and robustness of biological systems. Here we study this relationship within the context of an evolution model, where phenotypes are given by spin configurations; genotypes are interaction matrix for spins to give the Hamiltonian, and the fitness depends only on the configuration of a subset of spins called target. We describe the interplay between the genetic variations and phenotypic variances by noise in this model by our new approach that extends the replica theory for spin-glasses to include spin-replica for phenotypes and coupling-replica for genotypes. We obtain a phase diagram of the evolved phenotypes against the noise and selection pressure, where each phase is distinguished by the fitness and overlaps for genotypes and phenotypes. Among the phases, robust fitted phase, relevant to biological evolution, is achieved under the intermediate level of noise (temperature), where robustness to noise and to genetic mutation are correlated, as a result of replica symmetry. We also find a trade-off between maintaining a high fitness level of phenotype and acquiring a robust pattern of genes as well as the dependence of this trade-off on the ratio between the size of the functional (target) part to that of the remaining non-functional (non-target) one.