Global in time existence theorem for the full revised Enskog equation

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I prove global in time existence of solutions to the full revised Enskog equation. This equation generalizes the Boltzmann theory to dense gases in two ways:

1. by taking into account the fact that the centers of two colliding spheres are at a distance a, equal to the diameter of hard spheres.

2. by increasing the collision frequency by a factor Y_0 which nowadays is identified with the radial pair correlation function g_2 for the system of hard spheres at a uniform equilibrium.

In contrast to the dilute gas mode described by the Boltzmann equation, the Enskog equation includes spatial pair correlation function for hard-spheres potential and depends in a highly nonlinear way on the local density of dense gas. The full revised Enskog Equation refers to the case where g_2, the pair the correlation function (for non-uniform equilibrium of hard-spheres) is in general form. In terms of the virial expansion (in local density n, spatially dependent) at contact value, g_2 reads:

$$g_2\,(n) = 1 + V_1(n) + V_2(n) + \ldots. + V_N(n) + \ldots,$$

where the term $V_1(n)$ depends on n linearly, $V_2(n)$ depends on n quadratically, $V_N(n)$ depends on n as n^N, ad so on.

Circa 30 years ago Arkeryd-Cercignani proved the result for the truncated i.e., $g_2 = 1$ (no density dependence). The case with $g_2 = 1$ refers to the so called Boltzmann-Enskog equation. It differs from the Boltzmann equation only by existence of the shifts in the spatial variable in the collisional integral. Since then, many researchers tried/wanted to prove the result for general form of g_2. Dependence of g_2 on n requires a different approach and new tools as compared to Arkeryd-Cercignani proof ([1]). Additionally, this result finally completes and fulfills the existence result for the revised Enskog Equation.

The proof of existence of solutions to the revised Enskog equation is based on two constructions:

1. Construction of an H-functional (see [2]), where the full expansion of g_2 is used, but convergence of the series was not addressed.

2. Construction of a special sequence of stochastic kinetic equations (studied in [3]) and the proof that their solutions converge to weak solutions of the revised Enskog equation.

References

[1] L. Arkeryd, C. Cercignani, Global existence in L1 for the Enskog equation and convergence of solutions to solutions of the Boltzmann equation, J. Stat. Phys. 59, 845–867 (1990).
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