A key observable that guarantees linear thermalization of all macroscopic observables

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It has been attracting much attention to determine whether a given quantum many-body system thermalizes or not. However, such studies have usually examined thermalization of only a small number of observables, and it was unclear whether other observables thermalize. In this talk, we investigate 'linear thermalization,' which is many-body thermalization against a small change of a physical parameter of the system, and show that there exists a key observable whose linear thermalization guarantees linear thermalization of all macroscopic observables. Suppose that an isolated quantum many-body system is prepared in an equilibrium state, and then a parameter f of the Hamiltonian is changed by a small amount Δf, which induces the unitary time evolution. We say linear thermalization occurs for an additive (macroscopic) observable when its expectation value and fluctuation after a long time are consistent with thermodynamics up to the linear order in Δf.

We find that the additive observable B that is conjugate to f is the key observable for linear thermalization: If the long time average of the expectation value of B coincides with its equilibrium value predicted by thermodynamics up to $O(\Delta f)$, so do all additive observables. In addition, we find that, under a reasonable condition on the energy eigenvalues, the time fluctuations of the expectation values of all additive observables are macroscopically negligible up to $O(\Delta f)$. We also find that, under a reasonable condition that the fluctuations of additive observables in the initial state are sufficiently small, they remain so up to $O(\Delta f)$, for all additive observables. Thus linear thermalization of B guarantees linear thermalization of all additive observables.

Furthermore we consider the dynamics induced by small changes of other parameters, which are not conjugate to B. We find that linear thermalization of B occurs against the change of any other parameter if it occurs against the change of the conjugate parameter f. Moreover, we investigate the generalized susceptibilities for cross responses, and their consistency between quantum mechanics and thermodynamics. We demonstrate the main results numerically in nonintegrable and integrable spin models.

Our results will dramatically reduce the costs of experiments and theoretical calculations of linear thermalization and cross responses because testing them for a single key observable against the change of its conjugate parameter gives much information about those for all additive observables and about those against the changes of any other parameters.

The time scale of linear thermalization will be discussed in Chiba's presentation of this conference.

References

[1] Y. Chiba, K. Asano, A. Shimizu, Phys. Rev. Lett. 124, 110609 (2020).
[2] Y. Chiba, A. Shimizu, arXiv:2303.03868 (2023).