Numerical investigation of spatiotemporal chaos in nonlinear lattice models

Haris Skokos
University Of Cape Town, Cape Town, South Africa

We discuss various numerical approaches for studying the chaotic dynamics of multidimensional Hamiltonian systems, focusing our analysis on the chaotic evolution of initially localized energy excitations in the disordered Klein-Gordon (DKG) oscillator chain in one spatial dimension. The system's linear modes are exponentially localized by disorder and consequently Anderson localization [1] is observed in the absence of nonlinearity. On the other hand, nonlinear interactions result to the destruction of the initial energy localization, leading to the eventual subdiffusive spreading of wave packets in two different dynamical regimes (the so-called 'weak' and 'strong chaos' spreading regimes), which are characterized by particular power law increases of the wave packet's second moment and participation number [2-6].

Quantifying the strength of chaos through the computation of the maximum Lyapunov exponent (MLE, see for example [7] and references therein), we observe that the index exhibits power law decays, with different exponents for the weak and strong chaos regimes, whose values are distinct from -1 seen in the case of regular motion [8-10]. The spatiotemporal evolution of the coordinates' distribution of the deviation vector used to compute the MLE (the so-called deviation vector distribution – DVD) reveals that chaos is spreading through the random oscillation of localized chaotic hot spots in the excited part of the wave packet [8-10]. Furthermore, the implementation of the SALI/GALI2 chaos indicator [11-13] permits the efficient discrimination between localized and spreading chaos, with the former dominating the dynamics for lower energy values, for which the system is approaching its linear limit [14]. In addition, by computing the time variation of the fundamental frequencies of the motion of each oscillator in the lattice, i.e. the so-called frequency map analysis (FMA) technique [15-17], we reveal several characteristics of the dynamics for both the weak and strong chaos regimes [18], related to the location of highly chaotic oscillators and the propagation of chaos.

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