Combining topological data analysis with equation-free methods to analyse macroscopic dynamics of a complex network neuronal model

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We combine topological data analysis with equation-free methods to identify and study the macroscopic dynamics of a complex network neuronal model. Initially, we project the network dynamics of activated neurons on a circle S^1. Then, the filtration process of witness simplicial complexes [1] is applied to reduce the dimensionality of the system dramatically and to compute the minimum filtration radius where the Betti one number appears [1-3]. The minimal filtration radius is related to the density of activated nodes in the network. Using simulating annealing [4] as a minimising procedure, a method to express the state of the network as a function of minimal filtration value is defined. Furthermore, using the equation-free framework [5-7], we identify the macroscopic network dynamics as a function of the minimal filtration radius of the underlying persistence topology. Additionally, we perform a numerical bifurcation and stability analysis of the macroscopic network dynamics. To our knowledge, this is the first time where such a type of analysis has been made, identifying the network behaviour in terms of topological properties of dimensionally reduced data that the network model produces.

References

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