## Rényi entropy of quantum anharmonic chain at non-zero temperature

## Miha Srdinsek<sup>1,2,3</sup>, Michele Casula<sup>2</sup>, Rodolphe Vuilleumier<sup>3</sup>

<sup>1</sup>ISCD, Sorbonne Université, Paris, France, <sup>2</sup>IMPMC, Sorbonne Université, Paris, France, <sup>3</sup>PASTEUR, ENS, Paris, France

The interplay of quantum and classical fluctuations in the vicinity of a quantum critical point (QCP) gives rise to various regimes or phases with distinct quantum character. In this work, we show that the Rényi entropy is a precious tool to characterize the phase diagram of critical systems not only around the QCP but also away from it, thanks to its capability to detect the emergence of local order at finite temperature. For an efficient evaluation of the Rényi entropy, we introduce a new algorithm based on a path integral Langevin dynamics combined with a previously proposed thermodynamic integration method built on regularized paths. We apply this framework to study the critical behavior of a linear chain of anharmonic oscillators, a particular realization of the  $\phi^4$  model. We fully resolved its phase diagram, as a function of both temperature and interaction strength. At finite temperature, we find a sequence of three regimes - para, disordered and quasi long-range ordered -, met as the interaction is increased. The Rényi entropy divergence coincides with the crossover between the para and disordered regime, which shows no temperature dependence. The occurrence of quasi long-range order, on the other hand, is temperature dependent. The two crossover lines merge in proximity of the QCP, at zero temperature, where the Rényi entropy is sharply peaked. Via its subsystem-size scaling, we confirm that the transition belongs to the two-dimensional Ising universality class. This phenomenology is expected to happen in all  $\phi^4$ -like systems, as well as in the elusive water ice transition across phases VII, VIII and X.



