Exact calculation of the probabilities of rare events in cluster-cluster aggregation

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Cluster-cluster aggregation (CCA) is a non-equilibrium, irreversible process, of importance in many physical systems, where a particle of mass i and a particle of mass j coalesce to form a larger particle of mass i+j, with a collision rate dependent on a collision kernel K(i,j). This problem has been studied avidly for more than a century, beginning with the Smoluchowski coagulation equation, postulated by Smoluchowski in 1917. This equation, which is a mean field, first order differential equation for the rate of change of number of particles of mass m, i.e., the typical mass distribution, with time, can only be solved exactly for a few kernels.

In our work, we study the probabilities of atypical or rare events in CCA analytically for arbitrary K(i, j). We develop a formalism which expresses the probability in terms of an action, which corresponds to a large deviation function for arbitrary kernels. From the large deviation function, we recover the result that the Smoluchowski equation describes typical trajectories. We have also explicitly calculated the action for the constant, sum and product kernels. Further, we have calculated the instanton trajectories for typical as well as atypical times. We find that the action and the instanton trajectories are in excellent agreement with numerical simulations, for the constant and sum kernels for all time. For the product kernel, the action matches with the numerical simulations when the fraction of particles remaining at the final time is greater than half, but deviates when the fraction of particles remaining at the final time is less than or equal to half.

References

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