Marginal stability in the spherical spin-glass: on the competition between disorder and (ordered) non-linearity.

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Since the 1970's spin glasses have been a rich source of techniques and ideas that provided a theoretical foundation and universal paradigm for the emergence of ergodicity broken phases at low temperature in many-body systems with frustration. What was evident from the beginning is that the behaviour of disordered models in statistical mechanics depends on the nature of the variables, whether they are continuous or discrete. Let us consider for instance the celebrated Sherrington-Kirkpatrick model, which is a sort of mean-field Ising model with random Gaussian couplings. What is remarkable is the difference between the low temperature equilibrium phase of this model, characterized by a fractal free-energy landscape and by the so-called full replica-symmetry breaking scenario [1], and the behaviour of a model with the same Hamiltonian, but where "spins" are locally unbounded continuous variables, known as spherical spin glass [2], where a transition to a low temperature non-ergodic phase occurs at the same critical temperature of Sherrington-Kirkpatrick, but the nature of this low-temperature phase is completely different. It is a trivial spin-glass phase with only one big connected component of the phase space, which is not broken down into an infinite hierarchy of sub-clusters as in Sherrington-Kirkpatrick. The stability analysis of this "trivial spin-glass" phase, which is "marginally stable", suggests that it might be driven to a different phase by arbitrarily small perturbations. On the top of a historical briefing on the differences between disordered models with discrete or continuous variables, the goal of the present communication will be the description of how the addition of non-linear terms to the spherical spin-glass solved by Kosterlitz et al. in 1976 modifies the nature of the low-temperature phase. Motivated by the idea of investigating the competition between disorder and non-linearity, we have analytically studied the effects of different kinds of non-linear perturbations, i.e. those corresponding to the following kind of distributions for the random coefficients of the non-linear couplings: first, purely ordered coefficients; second, purely disordered coefficients; and third, a competition between ordered and disordered interactions. The main outcome of our investigation, detailed by the presentation of complete phase diagrams, is that the marginally stable trivial spin-glass phase of a model with two-body interactions (soft spins) cannot be destabilized by any perturbation: randomness looks like a necessary ingredient. In particular, consistently with previous works [3], we find that the spherical spin glass with 2+4 body disordered interactions, is characterized by full replica symmetry breaking in its low temperature phase when the non-linearity is not too strong. We emphasize how and why the transition to this phase with an infinite hierarchy of nested disjoint ergodic components is different from the case of Sherrington-Kirkpatrick model.

References

[1] G. Parisi, Infinite number of order parameters for spin-glasses, Phys. Rev. Lett. 43, 1754 (1979).
[2] J. M. Kosterlitz, D. J. Thouless, R. C. Jones, Spherical Model of a Spin-Glass, Phys. Rev. Lett. 36, 1217 (1976).
[3] A. Crisanti, L. Leuzzi, Spherical 2+ p spin-glass model: An exactly solvable model for glass to spin-glass transition, Phys. Rev. Lett. 93, 217203 (2004).