Nonlinear constitutive relations by using some deformed functions

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Constitutive relations are fundamental and essential to characterize physical systems. They are combined with the other equations of the physical laws to solve physical problems. Some well-known examples of linear constitutive relations are as follows: Hooke's law ($F = k\, x$) for the tensile or compressive force F of a spring with a spring constant k against the change in its length x; Ohm's law ($V = R\, I$) for the voltage V of an electrical conductor with resistance R under an electric current I; and the relation $p = mv$ in analytical mechanics between the momentum p and the velocity v of a classical particle. However, as a real spring deviates from Hooke's law and as a real electrical resistance deviates from Ohm's law, we know that any linear constitutive relation describes an idealized situation, and it is merely a linearized- and/or approximated-relation to describe some real physical properties. Hence, in general, a nonlinear constitutive relation plays a crucial role to describe more realistically physical systems and complex systems.

It is known that some deformed functions are useful to describe a physical phenomenon which obeys an asymptotic power-law distribution, which maximizes an appropriate deformed entropy, e.g., Tsallis- or Kaniadakis-entropy, under the relevant constraints. In our previous work, we studied a thermal particle under a velocity-dependent potential which can be regarded as a deformation of Rayleigh's dissipation function, from which we can obtain the nonlinear constitutive relation between the dissipative force and the velocity of the thermal particle. We showed that the probability distribution function (pdf) for the stationary-state of this thermal particle is a κ-deformed Gaussian pdf.

In this contribution we introduce the nonlinear constitutive relation which is deformed from a linear constitutive relation by using a deformed exponential or logarithmic function. For example, the linear constitutive relation of Ohm's law can be deformed as $V = R\, ln\, [exp_{\kappa}\, (I)]$ by using the κ-exponential function $exp_{\kappa}(x)$. For $\kappa \neq 0$, this constitutive relation is nonlinear and reduced the original $V = R\, I$ in the limit of $\kappa \to 0$. In addition we show some basics of Rayleigh's dissipative functions and the related variational principle based on the so-called Rayleighians. We show some examples from the different fields, e.g, the velocity-dependent dry friction coefficients and Toda's nonlinear oscillators.

References

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