Dimensional measures of generalized entropy for statistical physics

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Entropy is useful in statistical physics as a measure of irreversibility, randomness, mixing, dispersion, and number of microstates. However, there remains ambiguity over the generalization of entropy beyond the additive definition pioneered by Boltzmann, Gibbs, and Shannon. For generalized entropies to be applied to their full potential in nonequilibrium statistical mechanics, there is a need for a physically interpretable (dimensional) framework that can be connected to dynamical processes operating in phase space. To this end, I will present dimensional measures of entropy that admit arbitrary invertible weight functions (subject to curvature and convergence requirements). These ``dimensional entropies'' represent the phase-space volume occupied by level sets of the distribution function. For sufficiently structured distributions, they are sensitive to perturbations at a similar phase-space scale. Dimensional entropies may be useful as a diagnostic (for irreversibility) and for theoretical modeling (if the underlying irreversible processes in phase space are understood) in chaotic and complex systems.