Consistent thermodynamic framework for a nonextensive system and the concept of effective temperature

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Recent works have associated systems of particles, characterized by short-range repulsive interactions and evolving under overdamped motion, to a nonlinear Fokker-Planck equation within the class of nonextensive statistical mechanics, with a nonlinear diffusion contribution whose exponent is given by $\nu = 2 - q$. These identifications were reached through both a coarse-graining approach and molecular-dynamics simulations of the equations of motion and were achieved for $\nu \geq 2$. The particular case $\nu = 2$ applies to interacting vortices in type-II superconductors, whereas $\nu > 2$ covers systems of particles characterized by short-range power-like interactions, where correlations among particles are taken into account. In the former, several studies presented a consistent thermodynamic framework based on the definition of an effective temperature θ, conjugated to a generalized entropy S_ν (with $\nu = 2$), typical of nonextensive statistical mechanics. In these type-II superconductor analyses, the variable θ presents values much higher than those of typical room temperatures T, so that the thermal noise was neglected ($T/\theta \approx 0$). Herein, the whole scheme is extended to systems of particles interacting repulsively, through short-ranged potentials, described by an entropy S_ν, with $\nu > 1$, covering the $\nu = 2$ (vortices in type-II superconductors) and $\nu > 2$ (short-range power-like interactions) physical examples. The thermodynamic framework follows from the equilibrium state of the Fokker-Planck equation, obtained after a sufficiently long-time evolution, and approached due to a confining external harmonic potential, $\phi(x) = \alpha x^2/2$ ($\alpha > 0$). The main results achieved are: (a) The definition of an effective temperature θ conjugated to the entropy S_ν; (b) The construction of a Carnot cycle, whose efficiency is shown to be $\eta = 1 - (\theta_2/\theta_1)$, where θ_1 and θ_2 are the effective temperatures associated with two isothermal transformations, with $\theta_1 > \theta_2$; (c) Applying Legendre transformations for distinct pair of variables, different thermodynamic potentials are obtained, and furthermore, Maxwell relations and response functions are derived.

[l] Nobre et al, Phys. Rev. E **91**, 022135 (2015).

[2] Curado et al, Phys. Rev. E **89**, 022117 (2014).

[3] Nobre et al, Phys. Rev. E **86**, 061113 (2012).